## 2-бөлім <br> Механика <br> Раздел 2 <br> Механика

## Section 2

Mechanics

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# About the separation of finely divided particles during centrifugation in liquid media 

Kydyrbekuly A.B., Al-Farabi Kazakh National University, Almaty, Kazakhstan, +77071143341 , E-mail: almatbek@list.ru, Rahmetolla A.Sh., Science Research Institute of Mathematics and Mechanics, Almaty, Kazakhstan, +77476193631 , E-mail: abzhan.rakhmatolla@mail.ru, Ibraev G.E., Al-Farabi Kazakh National University, Almaty, Kazakhstan, +77051111745, E-mail: ybraev.alysher@mail.ru, Rakhimzhanova A.Zh., Al-Farabi Kazakh National University, Almaty, Kazakhstan, +77078191246 , E-mail: anarar.88@mail.ru


#### Abstract

This work is devoted to the study of centrifugation processes, which are one of the most complex processes of technology. The use of the action of the centrifugal force field for the separation of heterogeneous liquid systems in a rotor system (centrifuge) is very effective. Separability of centrifuged materials, multiphase dispersion, the relationship between phases cause ambiguous processes of centrifugation, and therefore it is not always possible to accurately predict and evaluate the main characteristics of the separation of heterogeneous systems in the centrifugal force field. For a qualitative assessment of the main separation characteristics, such as the angular velocity of rotation, the dependence of the angles of inclination of the glasses on the angular velocity, the settling time of particles, etc., is created a mathematical model of the vertical rotor system. The rotor represents a round disk on which cups (vial) with a multiphase liquid are symmetrically suspended. A particular case of a fixed rotor is considered. Nonlinear differential equations of motion of the suspension particle, which have no exact solution, are obtained. The research is conducted by analytical and numerical methods. The dependence of the slope angles of the tubes from the angular rotational speed of the rotor, sedimentation curves are obtained, which allow estimating the time of particle deposition and the effect of the particle size distribution on the separation process. The results of this research work allow us to determine with sufficient accuracy all the necessary characteristics of the sedimentation working process, and also, in certain cases, it is possible to exclude the conduct of experimental work. The results of the work confirm the physical meaning of the process, which can serve as a justification for the use and introduction of this mathematical model in industrial production.


Key words: separation, settling, centrifuge, multiphase fluid, rotor system, demetallization of oil.

Сұйық ортадағы ұсақ бөлшектердің центрифугирлеу кезіндегі сепарациясы
Қыдырбекұлы А.Б., Әл-Фараби атындағы Қазақ ұлттық университеті, Алматы, Қазақстан, +77071143341 , E-mail: almatbek@list.ru

[^0]Берілген жұмыс технология процесстерінің ең күрделілердің бірі болып табылатын центрифугирлеу үдерісін зерттеуге арналған. Роторлық жүйеде (центрифугада) біртексіз сұйық жүйелерді бөлу үшін центрден тепкіш күштер өрісінің әсерін қолдану өте тиімді. Центрифугирленетін материалдардың бөлінгіштігі, дисперсті жүйелердің көпфазалығы, фазалардың өзара қатынасы центрифугирлеу үдерісінің бір мәнді емес болуын туғызады, сондықтанда центрден тепкіш күштер өрісіндегі гетерогенді жүйелердің негізгі бөліну сипаттамаларына әрқашан нақты болжамдар мен бағалау жасау мүмкіншілігі бола бермейді. Бөлшектердің тұну уақыты, сынауықтардың көлбеулік бұрышы мен бұрыштық жылдамдықтың арасындағы қатынас, айналудың бұрыштық жылдамдығы және т.c.c. сепарацияның негізгі сипаттамаларына сапалық баға беру үшін вертикаль роторлық жүйенің математикалық моделі құрылды. Ротор ретінде симметриялы түрде дөңгелек дискке ораналастырылған көпфазалы сұйықтықтығы бар сынауықтар алынған. Фиксацияланған ротордың дербес жағдайы қарастырылып отыр. Нақты шешімі жоқ суспензия бөлшектерінің сызықсыз дифференциал қозғалыс теңдеулері алынды. Зерттеу аналитикалық және сандық әдістер арқылы жүзеге асырылды. Бөлшектердің тұну уақытын бағалауға мүмкіндік беретін седиментация қисықтары, сынауықтардың көлбеулік бұрышы мен ротордың бұрыштық айналу жылдамдығының арасындағы, жалпы сепарация үдерісіне деген гранулометрлік құрамның әсерін ескеретін қатынастар алынды. Зерттеу нәтижелері седиментация жұмыс үдерісінің барлық қажетті сипаттамаларын қажетті дәлдікпен анықтауға мүмкіндік береді, және де кейбір жағдайларда тәжірибелік жұмыстарды жүргізу қажеттілігінен құтылуға мүмкіндік береді. Жұмыс нәтижелері есепте қарастырылып отырылған үдерістің физикалық мағынасын дәлелдейді, ал ол өз кезегінде берілген математикалық модельді өндірістік өнеркәсіпке енгізуге және қолдануға негіздеме бола алады.
Түйін сөздер: сепарация, тұну, центрифуга, көпфазалы сұйықтық, роторлық жүйе, мұнай деметаллизациясы.

## О сепарации тонкоизмельчённых частиц при центрифугировании в жидких средах

Кыдырбекулы А.Б., Казахский национальный университет имени аль-Фараби, г. Алматы, Республика Казахстан, +77071143341 , E-mail: almatbek@list.ru,
Рахметолла А.Ш., Научно-исследовательский институт математики и механики, Алматы, Республика Казахстан, +77476193631 , E-mail: abzhan.rakhmatolla@mail.ru,
Ибраев Г.Е., Казахский национальный университет имени аль-Фараби, г. Алматы, Республика Казахстан, +77051111745 , E-mail: ybraev.alysher@mail.ru,
Рахимжанова А.Ж., Казахский национальный университет имени аль-Фараби, г. Алматы, Республика Казахстан, +77078191246 , E-mail: anarar.88@mail.ru

Данная работа посвящена исследованию процессов центрифугирования, являющихся одними из наиболее сложных процессов технологии. Использование действия поля центробежных сил для разделения неоднородных жидких систем в роторной системе (центрифуге) весьма эффективно. Разделяемость центрифугируемых материалов, многофазность дисперсной системы, соотношение между фазами обуславливают неоднозначность процессов центрифугирования, и поэтому не всегда существует возможность точного прогноза и оценки основных характеристик разделения гетерогенных систем в центробежном поле сил. Для качественной оценки основных характеристик сепарации, такие как угловая скорость вращения, зависимость углов наклона стаканов от угловой скорости, время оседания частиц и т.д., построена математическая модель вертикальной роторной системы. Ротор представляет собой круглый диск, на котором симметрично подвешены стаканчики (пробирки) с многофазной жидкостью. Рассматривается частный случай фиксированного ротора. Получены нелинейные дифференциальные уравнения движения частицы суспензии, которые точного решения не имеют. Исследование проводится аналитическими и численными методами.


#### Abstract

Получены зависимости углов наклона пробирок от угловой скорости вращения ротора, кривые седиментации, позволяющие оценить время осаждения частиц, а также влияние гранулометрического состава на процесс сепарации в целом. Результаты исследования данной работы позволяют с достаточной точностью определить все необходимые характеристики рабочего процесса седиментации, а также позволяют в определенных случаях исключить проведение экспериментальных работ. Результаты работы подтверждают физический смысл процесса, рассматриваемого в задаче, что может служит обоснованием использования и внедрения данной математической модели в промышленном производстве. Ключевые слова: сепарация, оседание, центрифуга, многофазная жидкость, роторная система, деметаллизация нефти.


## 1 Introduction

Today the purification of oil and the extraction of foreign bodies from it have several methods. These methods are conventionally divided into chemical, physical and physico-chemical. Such extraction methods as chemical impact, the impact of various fields and radiations, treatment with sorbents can well fit into the technological process of oil preparation for transportation and processing. However, currently used methods of demetallization of oil from foreign impurities with chemical and thermally treated are ineffective, they have low productivity and imperfect technology (the degree of purification is rather low, high cost, etc.) and quite harmful to the environment. One of the solutions to these problems is the using of special centrifuges for the separation of different densities' substances, in particular, for the separation of a mixture of solid particles of substances and liquids.

## 2 Literature Review

Centrifugation processes is the most complex processes of technology, and centrifuges are the most complex technological mechanism. Not all the questions of the theory of the centrifuges operation have been sufficiently developed. It is not always possible to accurately predict the separation of heterogeneous systems in centrifuges [1-3]. Fundamentals of theoretical and experimental research of these machines are laid in the works of G.I.Bremer, V.I. Sokolov, P.G. Romankova, N.N. Lipatova, E.M. Goldina. Research of U.N. Bochkova, S.A. Plyushkina, E.V. Semenova, and V.Shlau significantly developed the theory of centrifugation processes and contributed to the creation of new effective centrifuges [4]. Despite the complexity and ambiguity of the centrifugation processes, they still obey the corresponding mathematical dependencies. The flow of fluid inside the rotors of centrifuges can not be considered with its flow in the field of gravity. Coriolis forces play an important role in this case. Stability of flows in the field of centrifugal forces is not characterized by the measures which used to estimate the stability of flows in the field of gravity [3,5-6]. Modern trends in centrifugal technology have caused the appearance of multiple designs of centrifuges, some of which are difficult to estimate by calculation [7-8]. Numerous scientific works are devoted to this topic, patents and author's certificates have been obtained, methods of mechanically separating solid particles from liquids by means of centrifugation and devices for their implementation have been proposed [9-15]. In [16,17], where the particle motion accounted only for the tube axis, the results were insufficient to completely analyze the sedimentation process of the suspension. So, the spatial motion of a particle is studied in this work, where it is also necessary to take into account that the values of the angular rotational speed of the rotor, the tubes and the
angle of their inclination are variable depending on the time for a complete assessment of the separation of solid particles from the liquid, which also complicates the search general solution of the differential equations of motion of a particle and the mechanical system as a whole[18]. We propose a solution to this problem under certain assumptions in this work. A special case, which takes place in industrial production, has been given and analyzed. Analytical research methodology, which used for technical rotary installation used for slurry treatment in the industry was developed and presented in [19].

## 3 Material and methods

Consider a vertical rotor mounted on a flexible shaft symmetrically relative to the supports, which rotates with an angular velocity $\omega$. The rotor represents the circular disk on which tubes with a suspension are symmetrically suspended. Tubes can rotate around their own horizontal axis (Figure 1).


Figure 1 - Disc with test tubes
The rotation angle of the tubes from the vertical determines by the angle $\alpha ; \Omega=\dot{\alpha}$ - the angular speed of tubes rotation, $L_{s}$ - the distance from the axis of rotor rotation to the axis of tubes rotation, $L_{M}$ - the distance to the particle $M$ of the suspension with mass $\Delta m, r$ the radius of the cups, $g$ - the acceleration of gravity, $L$ - the length of the tubes. The position of the rotor defines in the fixed coordinate system $O_{1} x_{1} y_{1} z_{1}$ (Figure 2). The position of the investigated particle $M$ is given in the moving coordinate system $O x y z, M(x, y, z)$. The axis $O y$ is directed along the axis of symmetry of the tube downward. The axis $O z$ is directed along the axis of rotation of the tube.


Figure 2 - Scheme of the rotor system
When compiling the equations of motion, the following assumptions were used:

1. the angular velocity of the rotor is sufficiently large, so the gravity can be neglected $\left(g \ll \omega^{2} r\right)$;
2. the particles have a spherical shape;
3. the length of a free particle is much larger than its size;
4. there is no turbulence in the suspension;
5. forces of interaction of particles between themselves and the walls of the glass (tubes) are absent;
6. the tube is narrow enough what allow take a diameter much smaller than the length;
7. after deposition on the tube wall, the particles stop their movement;
8. the frictional force between the tubes and their rotation axes, as well as the change in the severity of the tubes with suspensions and the frictional force of the particle with the tube wall is neglected.

To compose the equation of the particle motion, it is necessary to determine the transport $\overline{F_{e}}$ and Coriolis forces $\overline{F_{c}}$ of inertia of the suspension particle. The transport acceleration of a particle $M$ determines by the formula (Figure 2)

$$
\begin{equation*}
\overline{a_{e}}=\overline{\Omega_{0}}\left(\overline{\Omega_{0}} \cdot \bar{r}\right)-\bar{r} \Omega_{0}^{2}+\bar{\varepsilon} \times \bar{r}, \tag{1}
\end{equation*}
$$

where

$$
\begin{equation*}
\overline{\Omega_{0}}=\bar{\omega}+\bar{\Omega} \tag{2}
\end{equation*}
$$

an instantaneous angular velocity of the rotor system.
Projecting the vector of the transport acceleration, which defined by the formula (1), on the moving coordinate axes $O x, O y, O z$, we obtain:

$$
\left\{\begin{array}{l}
a_{e x}=\omega \sin \alpha \cdot[\omega(x \sin \alpha-y \cos \alpha)-\Omega z]-\left(L_{s} \cos \alpha+x\right)\left(\omega^{2}+\Omega^{2}\right)+  \tag{3}\\
+\dot{\Omega}\left(L_{s} \cos \alpha+x\right)-z \dot{\omega} \sin \alpha, \\
a_{e y}=-\omega \cos \alpha \cdot[\omega(x \sin \alpha-y \cos \alpha)-\Omega z]-\left(L_{s} \sin \alpha+y\right)\left(\omega^{2}+\Omega^{2}\right)+ \\
+\dot{\Omega}\left(L_{s} \cos \alpha+x\right)-z \dot{\omega} \sin \alpha, \\
a_{e z}=-\Omega \cdot[\omega(x \sin \alpha-y \cos \alpha)-\Omega z]-z\left(\omega^{2}+\Omega^{2}\right)+\dot{\omega} \cdot\left(L_{s}+y \sin \alpha+x \cos \alpha\right)
\end{array}\right.
$$

The transport force of inertia is equal $\overline{F_{e}}=-\Delta m \overline{a_{e}}$, the Coriolis force of inertia is $\overline{F_{c}}=$ $-\Delta m \overline{a_{c}}$. Coriolis acceleration of particle $M$ of the suspension, taking into account the fact that $\Omega_{O x}=\omega \sin \alpha, \Omega_{O y}=-\omega \cos \alpha, \Omega_{O z}=\Omega$, has the form

$$
\left\{\begin{array}{l}
a_{c x}=-2(\omega \cdot \dot{z} \cos \alpha-\dot{y} \Omega),  \tag{4}\\
a_{c y}=-2(\omega \cdot \dot{z} \sin \alpha+\dot{x} \Omega), \\
a_{c z}=-2 \omega(\dot{y} \sin \alpha+\dot{x} \cos \alpha),
\end{array}\right.
$$

After using formulas (3) and (4), taking into account the assumed assumptions, the motion equations of suspension particle $M$ have the form:

$$
\left\{\begin{array}{l}
\ddot{x}=-g \sin \alpha-6 \pi \eta_{1} r_{0} \dot{x}+\left(L_{s} \cos \alpha+x\right)\left(\omega^{2}+\Omega^{2}\right)-\omega \sin \alpha[\omega(x \sin \alpha-  \tag{5}\\
-y \cos \alpha)-\Omega z]-\dot{\Omega}\left(L_{s} \sin \alpha+y\right)+2(\omega \cdot \dot{z} \cos \alpha-\dot{y} \Omega), \\
\ddot{y}=g \sin \alpha-6 \pi \eta_{1} r_{0} \dot{y}+\left(L_{s} \sin \alpha+y\right)\left(\omega^{2}+\Omega^{2}\right)+\omega \cos \alpha[\omega(x \sin \alpha- \\
-y \cos \alpha)-\Omega z]-\dot{\Omega}\left(L_{s} \cos \alpha+x\right)+2(\omega \cdot \dot{z} \sin \alpha-\dot{x} \Omega), \\
\ddot{z}=-6 \pi \eta_{1} r_{0} \dot{z}+z\left(\omega^{2}+\Omega^{2}\right)+\Omega[\omega(x \sin \alpha-y \cos \alpha)-\Omega z]-\dot{\omega}\left(L_{s}+y \sin \alpha+\right. \\
+x \cos \alpha)-2 \omega(\dot{y} \sin \alpha+\dot{x} \cos \alpha), \\
\eta_{1}=\frac{\eta}{\Delta m}=\frac{\eta}{\frac{3}{4} \pi r_{0}^{3} \rho_{r}}=\frac{3 \eta}{4 \pi r_{0}^{3} \rho_{r}} .
\end{array}\right.
$$

Here $r_{0}$ is the particle radius of the suspension $M$ by mass $\Delta m ; \eta$ - dynamic viscosity of the liquid (suspension); $\rho_{r}$ - particle density; $6 \pi \eta r_{0}$ - the friction coefficient of the particle $M$ (the coefficient of resistance of the liquid of the medium) when it moves.

The system of equations (5) is nonlinear, because of $\omega=\omega(t), \alpha=\alpha(t), \Omega=\Omega(t)$, in this case, the rotation angle $\alpha$ is an argument of trigonometric functions. In addition, the coefficients of $x, y, z, \dot{x}, \dot{y}, \dot{z}$ are variables. In this connection the system (5) does not have an exact solution.

Let us consider a special case when the rotation of the rotor occurs with a constant angular velocity ( $\omega=$ const), and $\alpha \neq 0=$ const, i.e. $\dot{\alpha}=\Omega=0$, which corresponds to the case of a fixed rotor. Also it is necessary to determine the dependence of the angle $\alpha$ from the angular velocity $\omega$ of the rotor. From the balance equation of forces acting on a tube

$$
\begin{equation*}
g \sin \alpha=\omega^{2}\left(L_{s}+l \sin \alpha\right) \cos \alpha . \tag{6}
\end{equation*}
$$

we find

$$
\begin{equation*}
\omega=\sqrt{\frac{g \cdot \operatorname{tg} \alpha}{L_{s}+l \sin \alpha}}, \tag{7}
\end{equation*}
$$

where $l$ - distance from the axis of rotation to the center of tube gravity with suspension. When $\omega \rightarrow \infty, \alpha \rightarrow \frac{\pi}{2}$ or when $\alpha \rightarrow \frac{\pi}{2}, \omega \rightarrow \infty$ which follows from the physical meaning of the problem.

From the relation (7) it is obvious that if the angular velocity $\omega$ of the rotor is constant, the angle of rotation $\alpha$ of the cups is also constant. Each value $\omega$ has a corresponding specific value $\alpha$ (Figure 3). For example, for the slope angle $\alpha_{1}=15^{\circ}$ corresponds the angular velocity $\omega_{1}=30,5 \mathrm{rad} / \mathrm{s}$ or 292 rpm . For the angle $\alpha_{2}=30^{\circ}-382 \mathrm{rpm}$, for $\alpha_{3}=45^{\circ}-463 \mathrm{rpm}$. For other angles of slope $\alpha_{4}=60^{\circ}, \alpha_{5}=70^{\circ}, \alpha_{6}=80^{\circ}, \alpha_{7}=85^{\circ}, \alpha_{8}=88^{\circ}$ correspond to angular velocities of $577 \mathrm{rpm}, 820 \mathrm{rpm}, 1005 \mathrm{rpm}, 1422 \mathrm{rpm}, 1640 \mathrm{rpm}$.


Figure 3 - Dependence between the angular velocity and the angle of inclination
Accordingly, the faster the angular velocity of rotation, the faster the process of particle sedimentation. But in general case it is better to avoid the acceleration of the rotor system to huge angular velocities, it is necessary to determine the optimum slope of the tubes and the corresponding rotational speeds that would satisfy the conditions of the problem of sedimentation of solid emulsion particles.
Now, when it is established that each value corresponds to a specific value, it is possible to obtain from the system (5) with $\omega=$ const, $\alpha=$ const, $\Omega=0, \varepsilon=0$

$$
\left\{\begin{array}{l}
\ddot{x}+2 n \dot{x}-\omega^{2} \cos ^{2} \alpha \cdot x=-g \sin \alpha+\left(L_{s}+y \sin \alpha\right) \omega^{2} \cos \alpha+2 \omega \cos \alpha \dot{z}  \tag{8}\\
\ddot{y}+2 n \dot{y}-\omega^{2} \sin ^{2} \alpha \cdot y=g \cos \alpha+\left(L_{s}+x \cos \alpha\right) \omega^{2} \sin \alpha+2 \omega \sin \alpha \dot{z} \\
\ddot{z}+2 n \dot{z}-\omega^{2} z=-2 \omega(\dot{y} \sin \alpha+\dot{x} \cos \alpha)
\end{array}\right.
$$

Where $2 n=6 \pi \eta_{1} r_{0}, n=3 \pi \eta_{1} r_{0}=\frac{9 \eta}{4 r_{0}^{2} \rho_{r}}=\frac{9 \nu}{4 r_{0}^{2} \rho_{r}}$ - coefficient of friction, $\rho_{r}$ - fluid density. The equations system (8) can be solved analytically, assuming certain assumptions that were made in [19]. In this case, taking into account the spatial motion of the deposited substance in the cavity, it is possible to calculate the trajectory of the particle by a numerical method, to estimate the time and the trajectory of settling the particles in the suspension for different values of the slope of the tube, the particle size and the coefficient of resistance.

## 4 Results and discussion

The calculations were performed for a centrifuge rotating at a speed of 300 to 1700 rpm . During the calculation varies three basic parameters, the radius of the particles, the slope of the tubes and the coefficient of resistance of the environment, which are the most important and are fundamental in industrial production.

Figure 4 shows the results of calculations for different inclinations of glasses (tubes). As was shown above, when the angular velocity of the rotor tends to an infinitely large value, the angle of inclination of the tubes tends to an absolutely horizontal position, i.e. to a value of $90^{\circ}$. In the production process, it is impossible to achieve an absolutely horizontal position of the glasses (test tubes), which is confirmed by the mathematical model.


Figure 4 - Movement of the particle at different values of the slope of the tube
Therefore, it is necessary to determine the angle of inclination, which would provide a sufficient degree of purification and at the same time would be optimal from the constructive point of view. For industrial centrifuges with operating speeds from corners of $80^{\circ}$ are suitable. For industrial centrifuges with operating speeds from 1500 rpm , the angles are from $80^{\circ}$. When tilted at $88^{\circ}$, the particles settle more quickly than at smaller tilt angles of the glass (test tube). Here cases with a larger slope are not considered in connection with design changes, which, given the parameters, can lead to a decrease in productivity, as follows from formula (7). Particle settling occurs already at the 9 th second of the operating mode of the fixed rotor (Figure 5). In cases where the inclination angles are less than $80^{\circ}$, the sedimentation process passes more slowly and subsidence occurs along the spatial curves with increasing amplitudes due to the increase in the centrifugal force (Fiures 4-5). The rotor needs rotate at a speed of 1640 rpm to achieve a tilt angle of $88^{\circ}$. Thus, if the working speed of the centrifuge is 3000 rpm or more, as in many modern centrifuges of this class, then the goal of separation is achieved at much lower speeds. This, obviously, will reduce certain economic costs for
centrifuges with a specified degree of purification. The rate of particle sedimentation also depends on the nature of the multiphase fluid, and, therefore, it is appropriate to consider the resistance of the medium as the next important parameter.


Figure 5 - Movement of a particle at different values of the inclination of the tube along the longitudinal axis as a function of time

For the calculation, the resistance coefficients for crude oil and some more viscous liquids were used. As expected, the resistance is higher, the longer the separation time, respectively, the lower the coefficient of resistance, the smaller the settling time of the particles (Figure 6), which confirms the correctness of the proposed mathematical model. The magnitude of the emerging resistance depends mainly on the driving mode and the shape of the streamlined body. The mode of precipitation of a solid phase particle can be taken as laminar as long as the condition $R e<1 \div 1,6$ is satisfied. In practical centrifugation cases, the Reynolds number is less than the transition value from laminar to turbulent.

In laminar motion, the body is surrounded by a boundary layer of liquid and flows smoothly. If a precipitating particle, having reached a certain distance from the axis of rotation, continued to experience the effect of a constant centrifugal inertia force with further precipitation, then the deposition rate of the particle would soon become constant. At the same time, the resistance of the liquid would increase to the value of the centrifugal force. The increase in resistance is due to the fact that the fluid molecules in the layer near the body are densified, and when the distance between them decrease, the energy of mutual repulsion increases. But in reality the centrifugal force of a particle inertia is always greater than the resistance force of a liquid medium because of its increase (force) as the particle moves away from the axis of rotation [20].

The last parameter is the granulometric (or dispersed) composition of the particles, which must be separated. This characteristic is of decisive importance when choosing a centrifuge. For a separator of this configuration, its required productivity is less than the finer particles of the solid phase in the processed suspension, which results in a higher separation factor, which formally represents the Froude number. Since it is initially assumed that the particles
have the shape of a sphere, their radius is accordingly changed to consider different particle sizes. It should be noted that in this model in one cycle of the system operation the particle sizes were assumed to be equal, so, in each case all solid fractions in the suspension have the same size.


Figure 6 - Motion of a particle at different values of the coefficient of resistance of the medium

Also, colloidal disperse systems are not considered, because of the particle size plays not the most important role in the separation process as in a certain range (Figure 7). For example, particles with a size $r_{0}=0.01-0.05$ settle along an almost identical curvilinear trajectory, although here the particle size increases 5 times (Figure 7). Initially, all particles up to the fourth second of the operating mode of the fixed rotor have practically very close trajectories of subsidence. Until the ninth second, particles of size $r_{0}=0.01-0.05$ move along very close curves. The particles of size $r_{0}=5 \cdot 10^{-3}-1 \cdot 10^{-6}$ until the ninth second settle on more curved trajectories, which further diverge even more. For these particles, these sedimentation curves are natural, because they are small in size, which means the need for higher operating speeds for micro- and nanoparticles. Since the separability of the suspension depends on the degree of dispersion, it becomes necessary to use the sedimentometric method to estimate the nature of the particle distribution. When applying this method, the duration of precipitation is plotted along the abscissa axis, and the ordinate is the total amount of solid phase deposited at the bottom of the centrifuge cup or passed through the reference level for a certain time. This amount is composed of the whole precipitated particles and still precipitated [20]. This method requires the presence of experimental data, which is not always possible. At a certain stage of the research, using the above sedimentation data for a qualitative assessment, it is possible to avoid the need for an experimental method.


$$
\begin{array}{ll}
\alpha=88^{0}, & \eta=1000 \\
\square & \mathrm{r}_{0}=10^{-6} \\
& \mathrm{r}_{0}=0.01 \\
\square & \mathrm{r}_{0}=0.05 \\
\mathrm{r}_{0}=0.005
\end{array}
$$

Figure 7 - Motion of a particle at different values of the particle radius

## 5 Conclusion

Important results were obtained in this work. A qualitative analysis of the results is carried out on the basis of classical numerical methods for solving differential equations. The time of particle deposition at different values of the tube inclination angle, the resistance coefficients and the size of these particles is taken into account when taking into account the spatial motion. Optimum parameters of operating modes for the a fixed rotor case, i.e.when each specific value of the inclination corresponds angle to a specific angular rotation velocity of the disk on the flexible shaft, are determined and proposed. The results of the performed work confirm the physical meaning of the problem, which can serve as a justification for the use and implementation of this mathematical model in industrial production. Obtained dependences with sufficient accuracy for engineering practice allow to determine such operational characteristics of the process as the time of sedimentation, the trajectory of precipitation and the dependence of the rotational speeds on the slope angles, and also to predict the efficiency of the centrifuge. In addition, these results allow to avoid the need of using the sedimentometric method and to exclude additional costly experimental work.

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[^0]:    Рахметолла А.Ш., Математика және механика ғылыми зерттеу институты, Алматы, Қазақстан Республикасы, +77476193631 , E-mail: abzhan.rakhmatolla@mail.ru,
    Ибраев Ғ.Е., Әл-Фараби атындағы Қазақ ұлттық университеті, Алматы, Қазақстан, +77051111745 , E-mail: ybraev.alysher@mail.ru
    Рахымжанова А.Ж., Әл-Фараби атындағы Қазақ ұлттық университеті, Алматы, Қазақстан, +77078191246 , E-mail: anarar.88@mail.ru

