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THE STATE OF THE PROBLEM OF THE JOINT MOVEMENT OF FLUID IN THE PORE SPACE

This article discusses the problems of studying the issue of joint motion of liquids in the porous space. The article provides the construction of a mathematical model of the theory of filtration, which describes phase transitions. The main difficulty in constructing this model is associated with the fact that free interphase boundaries create regions that change over time, and it is required to find the temperature or concentration fields of substances in them. In this case, the coordinates of the considered phase boundaries are not initially specified and must be calculated already in the process of solving. For this, a derivation of the averaged equation for the problem of finding the rupture surface during the movement of two incompressible viscous liquids in the pores of the soil skeleton was proposed. The article deals with the case when the skeleton is an absolutely rigid body. The rationale was given for the choice of an averaged filtration model instead of a microscopic one. The main research methods are classical methods of mathematical physics, functional analysis and computation methods of the theory of partial differential equations, as well as difference methods. The formulation of the problem is given, and the definition of a generalized solution for solving the problem is provided. Next, an averaged model is derived and the existence of at least one generalized solution to the problem is proved.

Key words: Stefan problem, difference scheme, numerical methods, phase boundary, sorption, adsorption, surfactant, relaxation time, averaged model, microscopic model, macroscopic model, joint motion of liquids.

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Кеуек кеңістігіндегі сұйықтықтардың бірлескен қозғалысы туралы

Мақалада сипатталған зерттеудің мақсаты кеуекті кеңістіктегі сұйықтықтардың бірлескен қозғалысы туралыі мәселені зерттеу болып табылады. Мақалада фазалық ауысуларды сипаттайтын сүзу теориясының математикалық моделінің құрылысы қарастырылады.
Бұл модельді құрудағы басты қиындық бос интерфазалық шекаралар уақыт өте келе өзгеретін айма қтарды құрайтындығына байланысты және олар температура өрістерін немесе заттардың концентрациясын табуды қажет етеді. Бұл жағдайда фазалық бөлімнің қарастырылған шекараларының координаттары бастапқыда орнатылмаған және шешім барысында есептелуі керек.
Ол үшін топырақ қаңқасының тесіктерінде екі сығылмайтын тұтқыр сұйықтықтың қозғалысы кезінде сыну бетін табу есебінің орташа теңдеуін алу ұсынылды. Бұл модельді құрудағы басты қиындық бос интерфазалық шекаралар уақыт өте келе өзгеретін аймақтарды құрайтындығына байланысты және олардағы заттардың температурасы мен концентрациясының өрістерін табу керек. Бұл жағдайда фазалардың қарастырылған шекараларының координаттары бастапқыда көрсетілмеген және оларды шешу барысында есептелуі керек. Ол үшін топырақ қаңқасының тесіктерінде екі сығылмайтын тұтқыр сұйықтықтың қозғалысы кезінде жыртылу бетін табу үшін орташа теңдеуді алу ұсынылды. Мақалада қаңқа мүлдем қатты болған жағдайы қарастырылған. Негізгі зерттеу әдістеріне математикалық физиканы классикалық әдістері, функционалдық талдау және дербес дифференциалдық теңдеулер теориясының есептеу әдістері, сонымен қатар айырымдық әдістері жатады.

Түйін сөздер: Стефан есебі, айырмашылық схемасы, сандық әдістер, фазалық шекара, сорбция, адсорбция, беттік-белсенді зат, релаксация уақыты, орташа модель, микроскопиялық модель, макроскопиялық модель.

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Состояние вопроса совместного движения жидкостей в поровом пространстве

Целью исследования, описанного в данной статье, является изучение вопроса о совместном движении жидкостей в пористом пространстве. В статье рассматривается построение математической модели теории фильтрации, описывающей фазовые переходы. Основная трудность при построении этой модели связана с тем, что свободные межфазные границы образуют области, изменяющиеся во времени, и они требуют нахождения полей температуры или концентрации веществ.
При этом координаты рассматриваемых границ раздела фаз изначально не заданы и должны быть рассчитаны в ходе решения. Для этого был предложен вывод усредненного уравнения задачи о нахождении поверхности разрыва при движении двух несжимаемых вязких жидкостей в порах скелета грунта. В данной статье рассматриваются вопросы изучения совместного движения жидкостей в пористом пространстве. В статье приведено построение математической модели теории фильтрации, описывающей фазовые переходы.
Основная трудность при построении данной модели связана с тем, что свободные межфазные границы образуют области, изменяющиеся во времени, и необходимо найти в них поля температуры и концентрации веществ. При этом координаты рассматриваемых границ фаз изначально не указаны и должны быть вычислены в процессе их решения. Для этого было предложено получить усредненное уравнение для задачи нахождения поверхности разрыва при движении двух несжимаемых вязких жидкостей в отверстиях почвенного скелета. В статье рассмотрен случай, когда скелет является абсолютно твердым телом. Основными методами исследования являются классические методы математической физики, функциональный анализ и вычислительные методы теории уравнений частных производных, а также разностные методы.

Ключевые слова: Задача Стефана, разностная схема, численные методы, граница раздела фаз, сорбция, адсорбция, поверхностно-активное вещество, время релаксации, усредненная модель, микроскопическая модель, макроскопическая модель.

1 Introduction

For a better and more complete understanding of the processes that occur during oil production, it is necessary to simulate liquid flow in porous media. Modeling is commonly used to develop optimal reservoir development methods, as well as to select suitable well locations, and of course to test various oil recovery technologies. Mathematical models of filtration are based on the laws of mechanics of multiphase media and contain systems of partial differential equations.
As a rule, the mathematical model is also supplemented with auxiliary equations depending on the properties of the porous medium.
A numerical study of liquid filtration has been carried out in many works. It can be pointed out that the main problems of such problems are associated, first, with the nonlinearity of the obtained systems of equations. If we turn to the definition, then the theory of poroelasticity studies the joint mechanism of fluid flow and the change in porous media. In this case, the

main mathematical models of the theory of filtration, as a rule, are supplemented by the Lame elasticity equation for the displacements of the medium. [1] Such mathematical models of poroelasticity contain systems of nonlinear, nonstationary systems of partial differential equations. For the approximate solution of boundary value problems, as a rule, numerical methods are used.
The equations of poroelasticity, which were obtained by M. Biot and C. von Terzaghi, for a certain time served as the basis for solving problems in the field of poroelasticity. These equations take into account the movement of not only the fluid in the pores, but also the solid skeleton. Later, some authors such as R. Burridge and J. Keller, E. Sanchez-Palencia and T. Levy, proposed the derivation of the poroelasticity equations, which are based on the laws of continuum mechanics and averaging methods. First, using the classical laws of continuum mechanics, the joint motion of the elastic skeleton and fluid in the pores is described at the microscopic level, and then approximating models are found using the averaging theory.

2 Materials and methods

The main methods of this research are the classical methods of mathematical physics, computational methods of the theory of partial differential equations, functional analysis, as well as difference methods. In practice, methods are also widely used that explicitly track the movement of interphase boundaries. All these methods are based on the use of the finite difference method, in this case, the calculations are carried out on uniform or non-uniform grids. [2] It is always determined between which nodes of the computational grid the moving border is at the moment, or through which node the border passes. The joint motion of elastic skeleton and fluid in pores in the area Ω is described by R. Burridge and J. Keller, T. Levy using the following mathematical model:

$$\frac{\partial}{\partial t}(\rho v) + \nabla(\rho v \oplus v - \chi P_f + (1-\chi)P_s) = \rho F, \tag{1}$$

$$\frac{\partial \rho}{\partial t} + \nabla(\rho v) = 0 \tag{2}$$

where $\nabla \cdot u$ is the divergence u, the matrix $a \oplus b$ is defined as $(a \oplus b)c = a(bc)$ for vectors a, b and c, χ is the characteristic function of the pore space, Ω, P_f, P_s is the stress tensors of the liquid and solid components, v is the velocity of the medium, ρ is the density of the medium and F is the given vector distributed mass forces. Equations (1) and (2) are understood as integral identities and contain dynamic equations for the liquid component:

$$\rho \frac{dv}{dt} = \nabla P_f + \rho F, \;\; \frac{d\rho}{dt} + \rho \nabla v = 0 \tag{3}$$

in Ω_f for $t > 0$, the dynamic equations for the solid component are given below:

$$\rho \frac{dv}{dt} = \nabla P_s + \rho F, \;\; \frac{d\rho}{dt} + \rho \nabla v = 0 \tag{4}$$

in Ω_s) for $t > 0$, and the condition for the continuity of normal stresses then looks like this:

$$(P_s - P_f) \cdot n \ = \ 0$$

on the common boundary "porous space rigid skeleton"$\Gamma(t)$, where $n$is the unit normal to $\Gamma(t)$. To describe the joint motion of two inhomogeneous fluids in an elastic skeleton, we will supplement our dynamic system with the transport equation for the density $\rho^{\epsilon}(x,t)$ of a mixture of liquid and solid components:

$$\frac{d\rho}{dt} = 0. \tag{5}$$

We also supplement this system with the initial condition:

$$\rho(x,0) = \rho_s, x \in \Omega_s, \quad \rho(x,0) = \rho_f^{\pm}, \quad x \in \Omega_{if}^{\pm}. \tag{6}$$

The resulting problem is highly nonlinear and contains an unknown quantity, that is, the interface between the pore space and the rigid skeleton. [3] In our case, the solid and liquid components do not mix. Therefore, the free boundary $\Gamma(t)$ is a contact discontinuity surface, and it can be determined from the Cauchy problem:

$$\frac{\partial \ \chi}{dt} \ \equiv \frac{\partial \chi}{\partial t} + \nabla \chi v \ \ = \ 0, \ \ \chi(x,0) \ = \ \chi_0(x) \tag{7}$$

is true for the characteristic function χ in the region Ω for $t > 0$.

Theorem Let $B_0, B.B_1$ be three Banach spaces, where

$$B_0 \subset B \subset B_1.$$

B_0, B_1 are reflective. Nesting $B_0 \subset B$is compact Then let

$$W = \left\{ v \middle| v \in L_{p_0}(0,T,B_0), \frac{\partial v}{\partial t} \in L_{p_1}(0,T,B_1) \right\}.$$

Proof. We use the norm of the space W

$$\|v\| \, L_{p_0}(0,T,B_0) + \left\| \frac{\partial v}{\partial t} \right\| L_{p_1}(0,T,B_1)$$

Then we get a Banach space. It's obvious that $W \subset L_{p_0}(0,T,B)$ Then the nesting $W \subset L_{p_0}(0,T,B)$ is compact.

3 Problem statement

If problem (1), (6), (7) can be solved, then such a given mathematical model will be useless for practical use, since the function χ changes its values from 0 to 1 on a scale of several microns. Although, the problem, in general, should be considered in an area of about several tens or hundreds of meters [4,5] In this case, you can consider and apply the averaging of this model. But then our problem (1), (2), (7) will become unsolvable. In this case, we propose to apply the linearization of the main dynamical system according to the scheme proposed by *R*. Burridge and E. Sanchez-Palencia, that is, we approximate the characteristic function χ of the liquid part Ω_f by its value at the initial moment of time, as well as the free boundary $\Gamma(t)$ by its initial position Γ_0. In what follows, we suppose that $v \sim \frac{\partial w}{\partial t}$, where w is the vector of displacement of the medium, we get:

$$\frac{\partial}{\partial t}(pv) \simeq \rho_f \chi_0 \ + \ \rho_s(1-\chi_0)\,\frac{\partial^2 w}{\partial t^2}$$

where ρ_f, ρ_s are the densities of the liquid in the pores and the solid skeleton:

$$P_f \ = \ 2\mu D(x,v) - \rho II.$$

$$P_s \ = \ 2\lambda D(x,w) - \rho II.$$

Here $D(x,v)$ is the symmetric part of ∇v, II is the unit tensor, w is the vector of displacement of the medium, as μ we denote the dynamic viscosity, through v we denote the bulk viscosity, and λ is the Lame elastic constant. [6] Let $\xi(x)$ be the characteristic function of the region Ω. Then the resulting $\chi^\epsilon(x) \ = \ \xi(x)\chi(\frac{x}{\epsilon})$ will be the characteristic function of the liquid region Ω_f^ϵ in dimensionless variables

$$x \to \frac{x}{L}, \quad w \to \frac{w}{L}, \quad t \to \frac{t}{\tau}, \quad F \to \frac{F}{g}$$

where L is the characteristic size of the physical area, τ is the characteristic time of the physical process, and g is the value of the acceleration of gravity. In this case, our dynamic system will take the following form:

$$a_\tau \varrho^\epsilon \frac{\partial^2 w}{dt^2} = \nabla P + \varrho^\epsilon F. \tag{8}$$

$$P \ = \ \chi^\epsilon a_\mu D(x, \frac{\partial w}{\partial t}) \ + \ (1-\chi^\epsilon) a_\lambda D(x,w) - \rho II. \tag{9}$$

$$\nabla w \ = \ 0. \tag{10}$$

Special cases of linearization of problem (1) (2), (7) have been studied by many scientists, such as, for example, Buckingham, Buchanan-Gilbert-Lin, Keller, Levy, Sanchez-Hubert, Sanchez-Palencia. The problem of averaging for compressible mean linearized systems was most fully investigated in the works of the scientist A.M. Meirmanov. [3, 7, 8] He proposed a classification based on the dependence on the values of dimensionless criteria, which are presented below:

$$\lim_{\epsilon \to 0} a_\tau(\epsilon) \ = \ \tau_0.$$

$$\lim_{\epsilon\to 0} a_\mu(\epsilon) \;=\; \mu_0.$$

$$\lim_{\epsilon\to 0} a_\lambda(\epsilon) \;=\; \lambda_0.$$

Filtration of a liquid is a very slow process, the medium speed is usually between 3 and 5 meters per year. Therefore, the process time is just very long and $a_\tau \sim 0$. And, for example, for fast processes such as water hammer, $a_\tau \sim 1,\ or\quad a_\tau \sim \infty$.
In this case, we can neglect the inertial terms in (9) and restrict ourselves to the following equation:

$$\nabla P \;+\; \varrho^\epsilon F \;=\; 0. \tag{11}$$

In order to describe the joint motion of two inhomogeneous fluids, we supplement the system of equations (9) - (11) with the following transport equation:

$$\frac{\partial \rho^\epsilon}{\partial t} \;+\; v\nabla\rho^\epsilon \;=\; 0, \quad v = \frac{\partial w}{\partial t}. \tag{12}$$

Supplement with the initial condition the equation for the density ρ^ϵ of a mixture of liquid and solid components:

$$\rho^\epsilon(x,0) \;=\; \rho_s, \quad x \in \Omega_s, \quad \rho^\epsilon(x,0) = \rho_f^\pm, \quad x \in \Omega_f^\pm \tag{13}$$

The simplest case of our system (9) - (11) will consider the case when a rigid skeleton is an absolutely rigid body. Then it is characterized by the following equality:

$$\lambda_0 \;=\; \infty.$$

Then the system of equations consists of the Stokes equations:

$$\nabla v \;=\; 0, \tag{14}$$

$$\nabla(a_\mu D(x,v) - \rho II) + \varrho_f F \;=\; 0 \tag{15}$$

for the pressure ρ and velocity v of the fluid in the region Ω_f at $t > 0$ and the equality

$$v \;=\; 0 \tag{16}$$

in a solid skeleton Ω_s.

4 Formulation of main result

Combining all the results, we formulate them together in the form of one theorem.

Theorem *Let the triple* $(\omega^\epsilon(x,t), \rho^\epsilon(x,t), \rho^\epsilon(x,t))$ *be a generalized solution of the MM model.*Then:

1) the sequences $\{\omega^\epsilon\}$, $\{\nabla\omega^\epsilon\}$, $\{v^\epsilon\}$, $\{\nabla v^\epsilon\}$, $\{\rho^\epsilon\}$, and $\{\nabla\rho^\epsilon\}$ converge weakly in $L_2(\Omega_T)$ to the functions ω,$\nabla\omega$, $v = E_{\Omega_f^\epsilon}(\partial\omega/\partial t)$, $\nabla v = \nabla(E_{\Omega_f^\epsilon}(\partial\omega/\partial t))$, ρ, p, $\nabla\rho$ respectively; 2) the limit functions are a solution of the averaged system of equations in the Ω_T region, consisting of the continuity equation

Let $\Omega \in R^2$ be a bounded region with boundary S, which was obtained by periodic repetition of the unit cell ϵY, where $\epsilon > 0$ is a small parameter,

$$Y = Y_f \cup Y_s \cup \gamma \cup \partial Y, \;\; Y = (0,1)\times(0,1), \;\; \epsilon Y = (0,\epsilon)\times(0,\epsilon)$$

where $\gamma = \partial Y_f \cup \partial Y_s$ is the Lipschitz boundary between two sets Y_f and Y_s. Let $\overline{\Omega}_f^\epsilon$ be the periodic repetition of unit cell $\epsilon\overline{Y_f}$, and $\overline{\Omega_s^\epsilon}$ is the periodic repetition of $\epsilon\overline{Y_s}$. Then

$$\Omega = \Omega_f^\epsilon \cup \Omega_s^\epsilon \cup \Gamma^\epsilon$$

where $\Gamma^\epsilon = \partial\, \Omega_f^\epsilon \cap \partial\, \Omega_s^\epsilon$ is a periodic repetition of the boundary $\epsilon\gamma$. Let the region Y_s be completely surrounded by the region Y_f, that is

$$Y_s \cap \partial\, Y = 0.$$

In the region Ω, the mathematical model of the joint motion of an incompressible fluid and an elastic incompressible skeleton at the microscopic level has the form

$$\nabla\cdot(\chi^\epsilon\mu_0 D(x, \frac{\partial\omega^\epsilon}{\partial t}) \; + \; (1-\chi^\epsilon)\lambda_0 D(x,\omega^\epsilon) - \qquad \rho^\epsilon I) + \rho^\epsilon F \; = \; 0. \tag{17}$$

$$\nabla\cdot\omega^\epsilon \; = \; 0, \;\; x\in\Omega, \;\; t>0. \tag{18}$$

$$x\in\Omega, \;\; t>0.$$

$$\frac{d\rho^\epsilon}{dt} \equiv \frac{\partial\rho^\epsilon}{\partial t} \; + \; \frac{\partial\omega^\epsilon}{\partial t}\cdot\nabla\rho^\epsilon \; = \; 0, \;\; x\in\Omega, \; t>0. \tag{19}$$

$$\chi^\epsilon\omega^\epsilon(x,0) \; = \; 0 \;\; at \;\; x\in\Omega. \tag{20}$$

$$\omega^\epsilon(x,t) \; = \; 0 \;\; at \;\; x\in S=\partial\Omega, \;\; t>0. \tag{21}$$

$$\chi^{\epsilon}\rho^{\epsilon}(x,0) = \rho_0(x), \quad x \in \Omega. \tag{22}$$

where $\omega^{\epsilon}(x,t) = (\omega_l^{\epsilon}(x,t), \omega_2^{\epsilon}(x,t))$ is the vector of displacement of the continuous medium, $\rho^{\epsilon}(x,t)$ is the pressure in the continuous medium, $D(x,\omega)$ is the symmetric part of the gradient of the vector ω (stress tensor), I is the unit matrix, $\chi^{\epsilon}(x)$ is the characteristic function of the pore space, $\chi^{\epsilon}(x)\ \mu_0$ is the dimensionless viscosity of the fluid, λ_0 is the dimensionless Lam constant.

$$\nabla \cdot \omega \ = \ 0 \tag{23}$$

averaged equations of angular momentum

$$\nabla \cdot \tilde{P} + \rho F = 0. \tag{24}$$

Where

$$P = n_1 : D(x, \frac{\partial \omega}{\partial t}) + n_2 : D(x,\omega) + \int_0^t n_3(t - \tau : D(x, \omega(x,\tau))d\tau - \rho I$$

and the averaged transport equation

$$\frac{d\rho}{dt} \equiv \frac{\partial \rho}{\partial t} + \frac{\partial \omega}{\partial t} \cdot \nabla \rho = 0, \quad \rho = m\rho_f + (1-m)\rho_s \tag{25}$$

supplemented by boundary

$$\omega(x,t) = 0, \quad x \in S, \quad t \in (0,T).$$

$$\frac{\partial \rho}{\partial n} = 0, \quad x \in S, \quad t \in (0,T).$$

and initial conditions

$$\omega(x,0) = 0, \quad x \in \Omega.$$

$$\rho(x,0) = m\rho_0(x), \quad x \in \Omega.$$

where n_1 is a fourth rank tensor, is symmetric and positive definite, n is the unit outward normal vector to the S boundary. The system of equations (23) - (25), supplemented with boundary and initial conditions, is nothing more than Musket's averaged model of the joint motion of fluid and pore space.

5 Conclusions

When studying the Rayleigh-Taylor instability in hydrodynamics, the following stages are traced: linear, asymptotic, intermediate, regular, and turbulent. The most investigated is the Rayleigh-Taylor instability for the case where there is a flat interface. The linear stage is well studied in the works of Rayleigh, Taylor and Lewis, the regular asymptotic stage is studied in the works of Birkhoff. But the analytical apparatus of mathematics for the analysis of the Rayleigh-Taylor instability is not enough. Experimental studies are very laborious and can be obtained only numerically. [9, 10] Numerical approaches are based on the use of velocity-pressure variables and current-vortex velocity or velocity-vortex velocity variables. Also, the formulation of the problem makes it easy to extend the numerical methods for calculating plane flows to the three-dimensional case. But the continuity equation for an incompressible fluid contains velocity components, so there is no direct relationship with pressure. In the course of computer simulation, it was revealed that the motion of fluids, which is described by the system of equations (17) - (19), depends on the following parameters: the ratios $\delta = \rho_f^+/\rho_f^-$, where ρ_f^+ *and* ρ_f^-, are the densities of the upper and lower liquids, respectively, μ^+, μ^- the viscosity of the liquids, λ_0 the Lamй elastic coefficient, and the pore size ϵ, that is, the Rayleigh-Taylor instability is observed, as in the case when the walls of the region are a solid. Numerical calculations were carried out for various values of λ, δ, and a constant value of the viscosity of liquids and for the same ϵ, $\delta = 1.25$ unit cell size:
-for $\epsilon = 2 \cdot 10^{-5}$, $\lambda \to 0$ there is a change in the interface between the liquids;
-for $\epsilon = 2 \cdot 10^{-5}$, $\lambda = 0.5$, $\delta \to \infty$ there is a change in the interface between the liquids;
-for $\epsilon = 2 \cdot 10^{-5}$, $\lambda = 0.5$, $\delta \to 1$ no change in the interface between the liquids is observed.

The state of the issue of joint motion of liquids in the porous space was investigated. The rationale was given for the choice of an averaged filtration model instead of a microscopic one. New microscopic mathematical models of the motion of viscous incompressible liquids of various viscosities in the pore space are derived.

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