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ONE CLASS OF SMOOTH BOUNDED SOLUTIONS TO THE CAUCHY PROBLEM FOR A THREE-DIMENSIONAL FILTRATION MODEL WITH DARCY'S LAW

The first results on the use of the apparatus of four-dimensional mathematics for solving the three-dimensional model of the Navier-Stokes equations by the analytical method were obtained by the Kazakh mathematician Professor M.M. Abenov. After the author of this article with other researchers proved the theorem on the existence of a solution to the Cauchy problem for a three-dimensional model of filtration theory.

This paper is devoted to the study of a three-dimensional model of the filtration theory in one of the spaces of four-dimensional numbers. The purpose of this article is to obtain an analytical solution of the three-dimensional Cauchy problem for the mathematical model of linear filtration model by the method of four-dimensional regular functions.

In this study, a class of infinitely differentiable and bounded functions of the initial conditions of the Cauchy problem, satisfying the Cauchy-Riemann condition, with five degrees of freedom for a specific four-dimensional function is found, and also a class of infinitely differentiable and bounded solutions of this problem is found that satisfy the linear Darcy law.

Keywords: continuity equation, Darcy's law, four-dimensional function, Cauchy-Riemann condition.

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ДАРСИ ЗАҢЫ БАР ҮШ ӨЛШЕМДІ ФИЛЬТРАЦИЯ МОДЕЛІНІҢ КОШИ ЕСЕБІНІҢ ТЕГІС ШЕКТЕЛГЕН ШЕШІМДЕРІНІҢ БІР КЛАСЫ

Төртөлшемді математика аппаратын Навье-Стокс теңдеулерінің үш өлшемді моделін аналитикалық әдіспен шешу үшін қолданудың алғашқы нәтижелерін қазақстандық математик профессор М.М. Әбенов қол жеткізді. Кейиниректе осы мақаланың авторы басқа ізденушілермен қатар фильтрация теориясының үш өлшемді моделі үшін Коши мәселесінің шешімі бар туралы теореманы дәлелдеген.

Бұл жұмыс төрт өлшемді сандар кеңістіктерінің бірінде фильтрация теориясының үш өлшемді моделін зерттеуге арналған. Аталып отырған мақаланың мақсаты төрт өлшемді тұрақты функциялар әдісімен сызықтық фильтрацияның математикалық моделі үшін үш өлшемді Коши есебінің аналитикалық шешімін алу болып табылады.

Бұл жұмыста нақты төрт өлшемді функция үшін бес еркіндік дәрежесі бар Коши-Риман шартын қанағаттандыратын Коши есебінің бастапқы шарттарының шексіз дифференциалданатын және шектелген функцияларының класы табылды, сонымен қатар шексіз дифференциалданатын және шектелген шешімдерінің класы табылды. Табылған шешімдер класы Дарси заңын қанағаттандырады.

Түйін сөздер: үздіксіздік теңдеуі, Дарси заңы, төрт өлшемді функция, Коши-Риман шарты.

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ОДИН КЛАСС ГЛАДКИХ ОГРАНИЧЕННЫХ РЕШЕНИЙ ЗАДАЧИ КОШИ ДЛЯ ТРЕХМЕРНОЙ МОДЕЛИ ФИЛЬТРАЦИИ С ЗАКОНОМ ДАРСИ

Первые результаты по применению аппарата четырехмерной математики для решения трехмерной модели уравнений Навье-Стокса аналитическим методом были получены казахстанским математиком профессором М.М. Абеновым. После авторам настоящей статьи и другими исследователями была доказана теорема о существовании решения задачи Коши для трехмерной модели теории фильтрации.

Настоящая работа посвящена исследованию трехмерной модели теории фильтрации в одном из пространств четырехмерных чисел. Целью настоящей статьи является получение аналитического решения трехмерной задачи Коши для математической модели линейной фильтрации методом четырехмерных регулярных функций.

В данной работе найден класс бесконечно дифференцируемых и ограниченных функций начальных условий задачи Коши, удовлетворяющие условию Коши-Римана, с пятью степенями свободы для конкретной четырехмерной функции, а также найден класс бесконечно дифференцируемых и ограниченных решений этой задачи, которые удовлетворяют линейному закону Дарси.

Ключевые слова: уравнение неразрывности, закон Дарси, четырехмерная функция, условие Коши-Римана.

1 Introduction

The theory of filtration is one of the main areas of scientific research due to its economic importance in connection with the extraction of oil and gas products, the development of subsoil, where the main extraction technologies are managed by the laws of filtration theory. The main law of the filtration theory, which describes the movement in a porous medium, is the linear Darcy law [1-14].

A large number of scientific works are devoted to the study of three-dimensional models of fluid motion. Basically, such problems of hydrodynamics and filtration are solved by numerical methods, since the solution by the analytical method is difficult. To avoid this difficulty, professor M.M. Abenov [15-16] proposed a new method for solving the continuity equation in four-dimensional space. In the work [15] M.M. Abenov provides an analytical solution of the Navier-Stokes equations using four-dimensional regular functions.

To obtain an analytical solution of the three-dimensional model of the filtration theory, in the article [19] a new approach to solving the problem in the space of four-dimensional numbers M5 is studied. The norm and properties of the space M5 were studied in [17-18]. In this paper, we study the Cauchy problem for a three-dimensional non-stationary mathematical model of filtration with the linear Darcy law.

2 Problem statement

Consider the motion of a compressible fluid in a porous medium. The continuity equation has the following form

$$m\frac{\partial\rho}{\partial t}+div\left(\rho\overrightarrow{v}\right)=0 \qquad (1)$$

where $m = m(x, y, z)$ – known medium porosity, $\rho(x, y, z, t)$ – liquid density, $\overrightarrow{v}(x,t) = (v_1(x,y,z,t), v_2(x,y,z,t), v_3(x,u,z,t))$ – filtration velocity, $x = (x, y, z) \in R^3$ – spatial coordinates, $t \in R_+$ - time.

Let's write Darcy's law in the form

$$-\nabla P=\frac{\mu}{k}\overrightarrow{v}, \qquad (2)$$

where μ – liquid viscosity, $k = k(x, y, z)$ – permeability coefficient depending only on the properties of the porous medium, $P(x, y, z, t)$ – fluid pressure. The system of equations (1) - (2) describes the non-stationary motion of a fluid in a porous medium in a three-dimensional space R^3. From physical considerations, it is obvious that the solutions of this system should be bounded functions in the entire space.

The initial conditions for the system (1) - (2) are set in the form:

$$\rho\left(x,y,z,0\right)=\rho_0(x,y,z), 0<\rho_{min}\leq\rho_0\left(x,y,z\right)\leq\rho_{max}<\infty, \qquad (3)$$

$$v_i\left(x,y,z,0\right)=\varphi_i(x,y,z), (x,y,z)\in R^3,\quad i=1,2,3., \qquad (4)$$

where the functions on the right-hand side are smooth and bounded functions in the entire space. In the paper [19], the existence theorem for a solution to the Cauchy problem (1) – (4) was proved under certain conditions on the initial data, and was obtained explicit analytical formulas for the solution.

In this paper, it is interested to find a specific type of initial conditions under which the solution of the considered problem is smooth and bounded in the entire space R^3 for any values of $t > 0$.

3 Methodology

Let us rewrite (1) as follows [15]

$$m\frac{\partial\rho}{\partial t}+\overrightarrow{v}\cdot grad\ \rho+\rho\ div\overrightarrow{v}=0$$

Let us introduce the function $\delta(x, y, z, t)$ and define it as follows

$$m\frac{\partial\rho}{\partial t}+\overrightarrow{v}\cdot grad\ \rho=\frac{\rho}{c}\frac{\partial\delta}{\partial t} \qquad (5)$$

where c – some characteristic filtration velocity. It is easy to see that $\delta(x, y, z, t)$ has the dimension of velocity. Dividing both sides of equation (5) by $\rho(x, y, z, t)$ we obtain the equation

$$m\frac{\partial\theta}{\partial t}+\overrightarrow{v}\cdot grad\theta=\frac{1}{c}\frac{\partial\delta}{\partial t}, \qquad (6)$$

where $\delta\left(x,y,z,t\right)=ln\rho(x,y,z,t)$.

Rewrite (1) taking into account (5)

$$\frac{\partial v_1}{\partial x}+\frac{\partial v_2}{\partial y}+\frac{\partial v_3}{\partial z}+\frac{1}{c}\frac{\partial \delta}{\partial t}=0 \tag{7}$$

For equation (7), we set the initial Cauchy data in the form

$$v_1|_{t=0}=\varphi_1\left(x,y,z\right);v_2|_{t=0}=\varphi_2\left(x,y,z\right);$$
$$v_3|_{t=0}=\varphi_3\left(x,y,z\right);\ \ \delta|_{t=0}=\delta_0\left(x,y,z\right). \tag{8}$$

where $\delta_0(x,y,z)$ can be determined through $\rho_0(x,y,z)$ solving the equation (6).

Similarly [19], the solution of problem (7) – (8) is sought in the form

$$\begin{aligned} v_1&=q_1w_1(p_1x,p_2y,p_3z,p_4ct)\\ v_2&=q_2w_2(p_1x,p_2y,p_3z,p_4ct)\\ v_3&=q_3w_3(p_1x,p_2y,p_3z,p_4ct)\\ \delta&=q_4w_4(p_1x,p_2y,p_3z,p_4ct)\end{aligned} \tag{9}$$

Using the Cauchy-Riemann conditions from (9) can be obtained formulas relating the components of the 4-vectors of velocity:

$$\begin{array}{c} \frac{1}{p_1q_1}\frac{\partial v_1}{\partial x}=\frac{1}{p_2q_2}\frac{\partial v_2}{\partial y}=\frac{1}{p_3q_3}\frac{\partial v_3}{\partial z}=\frac{1}{p_4q_4}\frac{\partial \delta}{\partial t}\\ \frac{1}{p_1q_2}\frac{\partial v_2}{\partial x}=-\frac{1}{p_2q_1}\frac{\partial v_1}{\partial y}=\frac{1}{p_3q_4}\frac{\partial \delta}{\partial z}=-\frac{1}{p_4q_3}\frac{\partial v_3}{\partial t}\\ \frac{1}{p_1q_3}\frac{\partial v_3}{\partial x}=\frac{1}{p_2q_4}\frac{\partial \delta}{\partial y}=-\frac{1}{p_3q_1}\frac{\partial v_1}{\partial z}=-\frac{1}{p_4q_2}\frac{\partial v_2}{\partial t}\\ \frac{1}{p_1q_4}\frac{\partial \delta}{\partial x}=-\frac{1}{p_2q_3}\frac{\partial v_3}{\partial y}=-\frac{1}{p_3q_2}\frac{\partial v_2}{\partial z}=\frac{1}{p_4q_1}\frac{\partial v_1}{\partial t} \end{array}$$

From these equalities can obtain the following problems and their solutions for the functions $v_1\left(x,y,z,t\right),\ v_2\left(x,y,z,t\right),\ v_3\left(x,y,z,t\right),\ \delta\left(x,y,z,t\right)$ [19]

$$\left\{\begin{array}{c} \frac{\partial^2 v_1}{\partial t^2}=\frac{{p_4}^2c^2}{{p_1}^2}\frac{\partial^2 v_1}{\partial x^2}\\ v_1|_{t=0}=\varphi_1\left(x,y,z\right)\\ \frac{\partial v_1}{\partial t}|_{t=0}=\frac{p_4q_1c}{p_1q_4}\frac{\partial \delta_0}{\partial x} \end{array}\right.$$

$$v_1\left(x,y,z,t\right)=\frac{1}{2}\left(\varphi_1\left(x+\frac{p_4c}{p_1}t,y,z\right)+\varphi_1\left(x-\frac{p_4c}{p_1}t,y,z\right)\right)$$

$$+\frac{1}{2}\frac{q_1}{q_4}\left(\delta_0\left(x+\frac{p_4c}{p_1}t,y,z\right)-\delta_0\left(x-\frac{p_4c}{p_1}t,y,z\right)\right) \tag{10}$$

$$\left\{\begin{array}{c} \frac{\partial^2 v_2}{\partial t^2}=\frac{{p_4}^2c^2}{{p_1}^2}\frac{\partial^2 v_2}{\partial x^2}\\ v_2|_{t=0}=\varphi_2\left(x,y,z\right)\\ \frac{\partial v_2}{\partial t}|_{t=0}=-\frac{p_4q_2c}{p_1q_3}\frac{\partial \varphi_3}{\partial x} \end{array}\right.$$

$$v_2(x,y,z,t) = \frac{1}{2}\left(\varphi_2\left(x+\frac{p_4c}{p_1}t,y,z\right)+\varphi_2\left(x-\frac{p_4c}{p_1}t,y,z\right)\right)$$

$$-\frac{1}{2}\frac{q_2}{q_3}\left(\varphi_3\left(x+\frac{p_4c}{p_1}t,y,z\right)-\varphi_3\left(x-\frac{p_4c}{p_1}t,y,z\right)\right) \quad (11)$$

$$\begin{cases} \frac{\partial^2 v_3}{\partial t^2} = \frac{{p_4}^2c^2}{{p_1}^2}\frac{\partial^2 v_3}{\partial x^2} \\ v_3\big|_{t=0} = \varphi_3(x,y,z) \\ \frac{\partial v_3}{\partial t}\big|_{t=0} = -\frac{p_4q_3c}{p_1q_2}\frac{\partial \varphi_2}{\partial x} \end{cases}$$

$$v_3(x,y,z,t) = \frac{1}{2}\left(\varphi_3\left(x+\frac{p_4c}{p_1}t,y,z\right)+\varphi_3\left(x-\frac{p_4c}{p_1}t,y,z\right)\right)$$

$$-\frac{1}{2}\frac{q_3}{q_2}\left(\varphi_2\left(x+\frac{p_4c}{p_1}t,y,z\right)-\varphi_2\left(x-\frac{p_4c}{p_1}t,y,z\right)\right) \quad (12)$$

$$\begin{cases} \frac{\partial^2 \delta}{\partial t^2} = \frac{{p_4}^2c^2}{{p_1}^2}\frac{\partial^2 \delta}{\partial x^2} \\ \delta\big|_{t=0} = \delta_0(x,y,z) \\ \frac{\partial \delta}{\partial t}\big|_{t=0} = \frac{p_4q_4c}{p_1q_1}\frac{\partial \varphi_1}{\partial x} \end{cases}$$

$$\delta(x,y,z,t) = \frac{1}{2}\left(\left(\delta_0\left(x+\frac{p_4c}{p_1}t,y,z\right)+\delta_0\left(x-\frac{p_4c}{p_1}t,y,z\right)\right)\right.$$

$$\left.+\frac{q_4}{q_1}\left(\varphi_1\left(x+\frac{p_4c}{p_1}t,y,z\right)-\varphi_1\left(x-\frac{p_4c}{p_1}t,y,z\right)\right)\right)$$

Further find $\rho(x,y,z,t)$ from [15]

$$\rho(x,y,z,t) = \rho_0[C_1,C_2,C_3]\times\exp\left[\frac{1}{2cm}\left(\left(\delta_0\left(x+\frac{p_4c}{p_1}t,y,z\right)+\delta_0\left(x-\frac{p_4c}{p_1}t,y,z\right)\right)\right.\right.$$

$$\left.\left.+\frac{q_4}{q_1}\left(\varphi_1\left(x+\frac{p_4c}{p_1}t,y,z\right)-\varphi_1\left(x-\frac{p_4c}{p_1}t,y,z\right)\right)\right)\right] \quad (13)$$

Thus, using the methodology for solving the four-dimensional equation of the solenoidal field, can find the solution of the continuity equation (1) in the form (10)-(12) of the velocity component, (13) the density.

Let us turn to the solution of the three-dimensional Darcy equation (2). Write as follows [19]

$$P(x,y,z,t) = -\frac{\mu}{k}\int_0^x v_1(\xi,y,z,t)\,d\xi-\frac{\mu}{k}\int_0^y v_2(0,\eta,z,t)d\eta-\frac{\mu}{k}\int_0^z v_3(0,0,\zeta,t)d\zeta+C(t) \quad (14)$$

where $C(t) = p_\infty + \frac{\mu}{k}\int_0^\infty v_3(0,0,\zeta,t)\,d\zeta$.

4 Results

Consider a three-dimensional filtration model (1)-(2). Let us assume that the viscosity of the liquid and the coefficient of porosity and permeability are constant values. As initial conditions, take the components of the four-dimensional function $exp\left(-X^2\right)$, which are infinitely differentiable and bounded functions in the entire half-space:

$$\begin{aligned}
&\varphi_1(x,y,z) = \frac{1}{4}q_1\left[exp\left(-\left(p_1\left(x-\frac{1}{\sqrt{s_{12}}}y+\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)+exp\left(-\left(p_1\left(x+\frac{1}{\sqrt{s_{12}}}y-\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)\right.\\
&\left.+exp\left(-\left(p_1\left(x-\frac{1}{\sqrt{s_{12}}}y-\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)+exp\left(-\left(p_1\left(x+\frac{1}{\sqrt{s_{12}}}y+\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)\right],\\
&\varphi_2(x,y,z) = -\frac{1}{4}\frac{q_1}{\sqrt{s_{12}}}\left[exp\left(-\left(p_1\left(x-\frac{1}{\sqrt{s_{12}}}y+\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)-exp\left(-\left(p_1\left(x+\frac{1}{\sqrt{s_{12}}}y-\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)\right.\\
&\left.+exp\left(-\left(p_1\left(x-\frac{1}{\sqrt{s_{12}}}y-\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)-exp\left(-\left(p_1\left(x+\frac{1}{\sqrt{s_{12}}}y+\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)\right],\\
&\varphi_3(x,y,z) = -\frac{1}{4}\frac{q_1}{\sqrt{s_{13}}}\left[-exp\left(-\left(p_1\left(x-\frac{1}{\sqrt{s_{12}}}y+\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)+exp\left(-\left(p_1\left(x+\frac{1}{\sqrt{s_{12}}}y-\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)\right.\\
&\left.+exp\left(-\left(p_1\left(x-\frac{1}{\sqrt{s_{12}}}y-\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)-exp\left(-\left(p_1\left(x+\frac{1}{\sqrt{s_{12}}}y+\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)\right],\\
&\delta_0(x,y,z) = -\frac{1}{4}\frac{\sqrt{s_{13}}q_1}{\sqrt{s_{12}}s_{14}}\left[exp\left(-\left(p_1\left(x-\frac{1}{\sqrt{s_{12}}}y+\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)+exp\left(-\left(p_1\left(x+\frac{1}{\sqrt{s_{12}}}y-\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)\right.\\
&\left.-exp\left(-\left(p_1\left(x-\frac{1}{\sqrt{s_{12}}}y-\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)-exp\left(-\left(p_1\left(x+\frac{1}{\sqrt{s_{12}}}y+\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)\right]
\end{aligned} \tag{15}$$

and satisfy the conditions

$$\begin{aligned}
\frac{\partial\varphi_1(x,y,z)}{\partial x} &= s_{12}\frac{\partial\varphi_2(x,y,z)}{\partial y} = s_{13}\frac{\partial\varphi_3(x,y,z)}{\partial z}\\
\frac{\partial\varphi_2(x,y,z)}{\partial x} &= \frac{\partial\varphi_1(x,y,z)}{\partial y} = s_{14}\frac{\partial\delta_0(x,y,z)}{\partial z}\\
\frac{\partial\varphi_3(x,y,z)}{\partial x} &= \frac{s_{12}s_{14}}{s_{13}}\frac{\partial\delta_0(x,y,z)}{\partial y} = \frac{\partial\varphi_1(x,y,z)}{\partial z}\\
\frac{\partial\delta_0(x,y,z)}{\partial x} &= \frac{s_{13}}{s_{14}}\frac{\partial\varphi_3(x,y,z)}{\partial y} = \frac{s_{13}}{s_{14}}\frac{\partial\varphi_2(x,y,z)}{\partial z}
\end{aligned}$$

at $(x,y,z) \in R^3$, where s_{12}, s_{13}, s_{14} are given positive real constants. Then the Cauchy problem (1), (3)-(4) has the following solution:

$$\begin{aligned}
&v_1(x,y,z,t) = \frac{1}{4}q_1\left[exp\left(-\left(p_1\left(x+\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct-\frac{1}{\sqrt{s_{12}}}y+\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)\right.\\
&+exp\left(-\left(p_1\left(x+\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct+\frac{1}{\sqrt{s_{12}}}y-\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)
\end{aligned} \tag{16}$$

$$+exp\left(-\left(p_1\left(x-\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct-\frac{1}{\sqrt{s_{12}}}y-\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)$$

$$+exp\left(-\left(p_1\left(x-\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct+\frac{1}{\sqrt{s_{12}}}y+\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)\Bigg]$$

$$v_2(x,y,z,t)=-\frac{1}{4}\frac{q_1}{\sqrt{s_{12}}}\Bigg[exp\left(-\left(p_1\left(x+\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct-\frac{1}{\sqrt{s_{12}}}y+\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)$$

$$-exp\left(-\left(p_1\left(x+\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct+\frac{1}{\sqrt{s_{12}}}y-\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right) \qquad (17)$$

$$+exp\left(-\left(p_1\left(x-\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct-\frac{1}{\sqrt{s_{12}}}y-\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)$$

$$-exp\left(-\left(p_1\left(x-\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct+\frac{1}{\sqrt{s_{12}}}y+\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)\Bigg]$$

$$v_3(x,y,z,t)=-\frac{1}{4}\frac{q_1}{\sqrt{s_{13}}}\Bigg[-exp\left(-\left(p_1\left(x+\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct-\frac{1}{\sqrt{s_{12}}}y+\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)$$

$$+exp\left(-\left(p_1\left(x+\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct+\frac{1}{\sqrt{s_{12}}}y-\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right) \qquad (18)$$

$$+exp\left(-\left(p_1\left(x-\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct-\frac{1}{\sqrt{s_{12}}}y-\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)$$

$$-exp\left(-\left(p_1\left(x-\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct+\frac{1}{\sqrt{s_{12}}}y+\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)\Bigg]$$

$$\delta(x,y,z,t)=-\frac{1}{4}\frac{\sqrt{s_{13}}q_1}{\sqrt{s_{12}}s_{14}}\Bigg[exp\left(-\left(p_1\left(x+\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct-\frac{1}{\sqrt{s_{12}}}y+\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)$$

$$+exp\left(-\left(p_1\left(x+\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct+\frac{1}{\sqrt{s_{12}}}y-\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)$$

$$-exp\left(-\left(p_1\left(x-\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct-\frac{1}{\sqrt{s_{12}}}y-\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)$$

$$-exp\left(-\left(p_1\left(x-\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct+\frac{1}{\sqrt{s_{12}}}y+\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)\Bigg]$$

Then find $\rho(x,y,z,t)$

$$\rho(x,y,z,t)=\rho_0\left[C_1,C_2,C_3\right]\times\exp\left[-\frac{1}{4cm}\frac{\sqrt{s_{13}}q_1}{\sqrt{s_{12}}s_{14}}\left(exp\left(-\left(p_1\left(x+\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct-\frac{1}{\sqrt{s_{12}}}y+\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)\right.\right.$$
$$+exp\left(-\left(p_1\left(x+\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct+\frac{1}{\sqrt{s_{12}}}y-\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right) \tag{19}$$
$$-exp\left(-\left(p_1\left(x-\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct-\frac{1}{\sqrt{s_{12}}}y-\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)$$
$$\left.\left.-exp\left(-\left(p_1\left(x-\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct+\frac{1}{\sqrt{s_{12}}}y+\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)\right)\right]$$

Let's move on to solving the three-dimensional Darcy equation. Find the required pressure by substituting (16)-(18) into the equation for pressure (14):

$$P(x,y,z,t)=-\frac{\mu}{k}\int_0^x\frac{1}{4}q_1\left[exp\left(-\left(p_1\left(\xi+\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct-\frac{1}{\sqrt{s_{12}}}y+\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)\right.$$
$$+exp\left(-\left(p_1\left(\xi+\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct+\frac{1}{\sqrt{s_{12}}}y-\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)$$
$$+exp\left(-\left(p_1\left(\xi-\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct-\frac{1}{\sqrt{s_{12}}}y-\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)$$
$$\left.+exp\left(-\left(p_1\left(\xi-\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct+\frac{1}{\sqrt{s_{12}}}y+\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)\right]d\xi$$
$$-\frac{\mu}{k}\int_0^y\left(-\frac{1}{4}\frac{q_1}{\sqrt{s_{12}}}\left[exp\left(-\left(p_1\left(\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct-\frac{1}{\sqrt{s_{12}}}\eta+\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)\right.\right.$$
$$-exp\left(-\left(p_1\left(\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct+\frac{1}{\sqrt{s_{12}}}\eta-\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)$$
$$+exp\left(-\left(p_1\left(-\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct-\frac{1}{\sqrt{s_{12}}}\eta-\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)$$
$$\left.\left.-exp\left(-\left(p_1\left(-\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct+\frac{1}{\sqrt{s_{12}}}\eta+\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)\right]\right)d\eta$$
$$-\frac{\mu}{k}\int_0^z\left(-\frac{1}{4}\frac{q_1}{\sqrt{s_{13}}}\left[-exp\left(-\left(p_1\left(\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct+\frac{1}{\sqrt{s_{13}}}\zeta\right)\right)^2\right)\right.\right. \tag{20}$$
$$+exp\left(-\left(p_1\left(\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct-\frac{1}{\sqrt{s_{13}}}\zeta\right)\right)^2\right)$$
$$+exp\left(-\left(p_1\left(-\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct-\frac{1}{\sqrt{s_{13}}}\zeta\right)\right)^2\right)$$
$$\left.\left.-exp\left(-\left(p_1\left(-\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct+\frac{1}{\sqrt{s_{13}}}\zeta\right)\right)^2\right)\right]\right)d\zeta+C(ct)$$

Verify the fulfillment of the pressure equation in the following way. In the direction x, the integral is solved in an obvious way

$$\frac{\partial P}{\partial x}=-\frac{1}{4}\frac{\mu}{k}q_1\left[exp\left(-\left(p_1\left(x+\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct-\frac{1}{\sqrt{s_{12}}}y+\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)\right.$$

$$+exp\left(-\left(p_1\left(x+\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct+\frac{1}{\sqrt{s_{12}}}y-\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)$$

$$+exp\left(-\left(p_1\left(x-\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct-\frac{1}{\sqrt{s_{12}}}y-\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)$$

$$\left.+exp\left(-\left(p_1\left(x-\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct+\frac{1}{\sqrt{s_{12}}}y+\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)\right]=-\frac{\mu}{k}v_1$$

In the direction y, take the differential $\frac{\partial}{\partial y}$ under the integral, and calculate as follows

$$\frac{\partial P}{\partial y}=-\frac{\mu}{k}\int_0^x\frac{\partial}{\partial y}\left(\frac{1}{4}q_1\left[exp\left(-\left(p_1\left(\xi+\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct-\frac{1}{\sqrt{s_{12}}}y+\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)\right.\right.$$

$$+exp\left(-\left(p_1\left(\xi+\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct+\frac{1}{\sqrt{s_{12}}}y-\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)$$

$$+exp\left(-\left(p_1\left(\xi-\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct-\frac{1}{\sqrt{s_{12}}}y-\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)$$

$$\left.\left.+exp\left(-\left(p_1\left(\xi-\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct+\frac{1}{\sqrt{s_{12}}}y+\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)\right]\right)d\xi$$

$$-\frac{\mu}{k}\left(-\frac{1}{4}\frac{q_1}{\sqrt{s_{12}}}\left[exp\left(-\left(p_1\left(\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct-\frac{1}{\sqrt{s_{12}}}y+\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)\right.\right.$$

$$\left.\left.-exp\left(-\left(p_1\left(\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct+\frac{1}{\sqrt{s_{12}}}y-\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)\right]\right)$$

$$-\frac{\mu}{k}\left(-\frac{1}{4}\frac{q_1}{\sqrt{s_{12}}}\left[exp\left(-\left(p_1\left(-\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct-\frac{1}{\sqrt{s_{12}}}y-\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)\right.\right.$$

$$\left.\left.-exp\left(-\left(p_1\left(-\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct+\frac{1}{\sqrt{s_{12}}}y+\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)\right]\right)$$

$$=-\frac{\mu}{k}\left(-\frac{1}{4}\frac{q_1}{\sqrt{s_{12}}}\right)\int_0^x exp\left(-\left(p_1\left(\xi+\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct-\frac{1}{\sqrt{s_{12}}}y+\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)$$

$$d\left(-\left(p_1\left(\xi+\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct-\frac{1}{\sqrt{s_{12}}}y+\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)$$

$$-\frac{\mu}{k}\left(\frac{1}{4}\frac{q_1}{\sqrt{s_{12}}}\right)\int_0^x exp\left(-\left(p_1\left(\xi-\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct-\frac{1}{\sqrt{s_{12}}}y-\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)$$

$$d\left(-\left(p_1\left(\xi-\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct-\frac{1}{\sqrt{s_{12}}}y-\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)$$

$$-\frac{\mu}{k}\left(-\frac{1}{4}\frac{q_1}{\sqrt{s_{12}}}\right)\left(\int_0^x exp\left(-\left(p_1\left(\xi-\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct-\frac{1}{\sqrt{s_{12}}}y-\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)\right.$$

$$\left.d\left(-\left(p_1\left(\xi-\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct-\frac{1}{\sqrt{s_{12}}}y-\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)\right)$$

$$-\frac{\mu}{k}\left(\frac{1}{4}\frac{q_1}{\sqrt{s_{12}}}\right)\left(\int_0^x exp\left(-\left(p_1\left(\xi-\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct+\frac{1}{\sqrt{s_{12}}}y+\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)\right.$$

$$\left.d\left(-\left(p_1\left(\xi-\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct+\frac{1}{\sqrt{s_{12}}}y+\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)\right)$$

$$-\frac{\mu}{k}\left(-\frac{1}{4}\frac{q_1}{\sqrt{s_{12}}}\left[exp\left(-\left(p_1\left(\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct-\frac{1}{\sqrt{s_{12}}}y+\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)\right.\right.$$

$$\left.\left.-exp\left(-\left(p_1\left(\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct+\frac{1}{\sqrt{s_{12}}}y-\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)\right]\right)$$

$$+\frac{\mu}{k}\left(-\frac{1}{4}\frac{q_1}{\sqrt{s_{12}}}\left[exp\left(-\left(p_1\left(-\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct-\frac{1}{\sqrt{s_{12}}}y-\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)\right.\right.$$

$$\left.\left.-exp\left(-\left(p_1\left(-\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct+\frac{1}{\sqrt{s_{12}}}y+\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)\right]\right)$$

$$=-\frac{\mu}{k}\left(-\frac{1}{4}\frac{q_1}{\sqrt{s_{12}}}\left[exp\left(-\left(p_1\left(x+\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct-\frac{1}{\sqrt{s_{12}}}y+\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)\right.\right.$$

$$-exp\left(-\left(p_1\left(x+\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct+\frac{1}{\sqrt{s_{12}}}y-\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)$$

$$+exp\left(-\left(p_1\left(x-\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct-\frac{1}{\sqrt{s_{12}}}y-\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)$$

$$\left.\left.-exp\left(-\left(p_1\left(x-\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct+\frac{1}{\sqrt{s_{12}}}y+\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)\right]\right)=-\frac{\mu}{k}v_2.$$

In the direction z, take the differential $\frac{\partial}{\partial z}$ under the integral, and calculate as follows

$$\frac{\partial P}{\partial z}=-\frac{\mu}{k}\left(\frac{1}{4}\frac{q_1}{\sqrt{s_{13}}}\right)\int_0^x exp\left(-\left(p_1\left(\xi+\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct-\frac{1}{\sqrt{s_{12}}}y+\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)$$

$$d\left(-\left(p_1\left(\xi+\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct-\frac{1}{\sqrt{s_{12}}}y+\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)$$

$$-\frac{\mu}{k}\left(-\frac{1}{4}\frac{q_1}{\sqrt{s_{13}}}\right)\int_0^x exp\left(-\left(p_1\left(\xi+\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct+\frac{1}{\sqrt{s_{12}}}y-\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)$$

$$d\left(-\left(p_1\left(\xi+\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct+\frac{1}{\sqrt{s_{12}}}y-\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)$$

$$-\frac{\mu}{k}\left(-\frac{1}{4}\frac{q_1}{\sqrt{s_{13}}}\right)\int_0^x exp\left(-\left(p_1\left(\xi-\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct-\frac{1}{\sqrt{s_{12}}}y-\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)$$

$$d\left(-\left(p_1\left(\xi-\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct-\frac{1}{\sqrt{s_{12}}}y-\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)$$

$$-\frac{\mu}{k}\left(\frac{1}{4}\frac{q_1}{\sqrt{s_{13}}}\right)\int_0^x exp\left(-\left(p_1\left(\xi-\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct+\frac{1}{\sqrt{s_{12}}}y+\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)$$

$$d\left(-\left(p_1\left(\xi-\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct+\frac{1}{\sqrt{s_{12}}}y+\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)$$

$$-\frac{\mu}{k}\left(\frac{1}{4}\frac{q_1}{\sqrt{s_{13}}}\right)\int_0^y exp\left(-\left(p_1\left(\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct-\frac{1}{\sqrt{s_{12}}}y+\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)$$

$$d\left(-\left(\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct-\frac{1}{\sqrt{s_{12}}}y+\frac{1}{\sqrt{s_{13}}}z\right)^2\right)$$

$$-\frac{\mu}{k}\left(-\frac{1}{4}\frac{q_1}{\sqrt{s_{13}}}\right)\int_0^y exp\left(-\left(p_1\left(\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct+\frac{1}{\sqrt{s_{12}}}y-\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)$$

$$d\left(-\left(p_1\left(\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct+\frac{1}{\sqrt{s_{12}}}y-\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)$$

$$-\frac{\mu}{k}\left(-\frac{1}{4}\frac{q_1}{\sqrt{s_{13}}}\right)\int_0^y exp\left(-\left(p_1\left(-\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct-\frac{1}{\sqrt{s_{12}}}y-\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)$$

$$d\left(-\left(p_1\left(-\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct-\frac{1}{\sqrt{s_{12}}}y-\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)$$

$$-\frac{\mu}{k}\left(\frac{1}{4}\frac{q_1}{\sqrt{s_{13}}}\right)\int_0^y exp\left(-\left(p_1\left(-\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct+\frac{1}{\sqrt{s_{12}}}y+\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)$$

$$d\left(-\left(p_1\left(-\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct+\frac{1}{\sqrt{s_{12}}}y+\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)$$

$$-\frac{1}{4}\frac{\mu}{k}\frac{q_1}{\sqrt{s_{13}}}exp\left(-\left(p_1\left(\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct+\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)$$

$$+\frac{1}{4}\frac{\mu}{k}\frac{q_1}{\sqrt{s_{13}}}exp\left(-\left(p_1\left(\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct-\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)$$

$$+\frac{1}{4}\frac{\mu}{k}\frac{q_1}{\sqrt{s_{13}}}exp\left(-\left(p_1\left(-\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct-\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)$$

$$-\frac{1}{4}\frac{\mu}{k}\frac{q_1}{\sqrt{s_{13}}}exp\left(-\left(p_1\left(-\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct+\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)$$

$$=-\frac{1}{4}\frac{q_1}{\sqrt{s_{13}}}\left[-exp\left(-\left(p_1\left(x+\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct-\frac{1}{\sqrt{s_{12}}}y+\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)\right.$$

$$+exp\left(-\left(p_1\left(x+\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct+\frac{1}{\sqrt{s_{12}}}y-\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)$$

$$+exp\left(-\left(p_1\left(x-\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct-\frac{1}{\sqrt{s_{12}}}y-\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)$$

$$\left.-exp\left(-\left(p_1\left(x-\frac{\sqrt{s_{12}}}{\sqrt{s_{13}}}s_{14}\left(1+\frac{1}{s_{12}}+\frac{1}{s_{13}}\right)ct+\frac{1}{\sqrt{s_{12}}}y+\frac{1}{\sqrt{s_{13}}}z\right)\right)^2\right)\right]=-\frac{\mu}{k}v_3.$$

Indeed, the equality is satisfied, from (20) have explicitly obtained equalities in directions

$$\frac{\partial P}{\partial x} = -\frac{\mu}{k}v_1,\ \frac{\partial P}{\partial y} = -\frac{\mu}{k}v_2, \frac{\partial P}{\partial z} = -\frac{\mu}{k}v_3.$$

Thus, obtained a class of infinitely differentiable and bounded in the entire half-space solutions of the three-dimensional model of filtration theory with five degrees of freedom $q_1,\ p_1,\ s_{12},\ s_{13},\ s_{14}$ of system (1)-(2) with initial conditions (3), (4) in the form (16)-(20).

5 Conclusion

In this paper, was studied a three-dimensional model of the filtration theory with the linear Darcy law. A class of infinitely differentiable and bounded in the entire half-space initial conditions and solutions of a three-dimensional model of filtration theory with five degrees of freedom $q_1,\ p_1,\ s_{12},\ s_{13},\ s_{14}$ is found. The method for solving the problem of three-dimensional filtration model is the application of the theory of functions of four-dimensional variables M5.

References

[1] Donald, A. Nield, Adrian Bejan. "Convection in Porous Media." USA: Springer; Third edition (2006).

[2] Frank, A. Coutelieris, J.M.P.Q Dlgado. "Transport Processes in Porous Media." USA: Springer (2012).

[3] Oliver, Coussy. "Mechanics and Physics of Porous Solids." UK: Wiley (2010).

[4] IHAR (International Association for Hydraulic Research). "Fundamentals of Transport Phenomena in Porous Media." Israel: Elsevier (1972).

[5] Kambiz, Vafai. "Handbook of Porous Media." USA: Taylor & Francis Group; Second Edition (2005).

[6] Entov, V.N. "Filtration theory." Sorosov educational journal, no. 2. (1998): 121-128.

[7] Polubarinova-Kochina, P.Ya. "Теория движения грунтовых вод." M.: Nauka (1977).

8. Dullien, F.A.L. "Porous Media: Fluid Transport and Pore Structure." USA: Academic Press; Second Edition (1991).

[8] Whintaker, S. "The Forchheimer equation: A theoretical development." Transport in Porous Media , no. 25 (1996): 27–61.

[9] Derek, B. Ingham, Ioan Pop. "Transport Phenomena in Porous Media." USA: Great Britain (2005).

[10] Tamer, O.S., Toropov, E.S., Shevnina, T.E., Vorobieva, T.I. "Research of Reservoir Rock Properties in Violation of Darcy's Linear Law. Transport and Storage of Hydrocarbons." IOP Conf. Series: Materials Science and Engineering 154 (2016). https://doi:10.1088/1757-899X/154/1/012006

[11] Hassanizadeh, S.M., Gray, W.G. "High Velocity Flow in Porous Media." Transport in Porous Media 2 (1987): 521-531.

[12] Bear, J. "Dynamics of fluids in porous media." Elsevier, New York (1972).

[13] Alekseev, B.V. "Analytical solution of the Leibenson equation and filtration theory." Fine chemical technologies 11, no. 1 (2016): 34-39.15.

[14] Abenov M.M. Chetirehmernaya matematika. Metody i prilozheniya. Nauchnaya monographia [Four-dimensional mathematics: Methods and applications. Scientific monograph]. Almaty.: Publishing House Kazakh University, 2019. - 176.

[15] Abenov M.M., Gabbassov M.B. Anyzotropnie chetirehmernie prostranstva ili novie kvaternioni [Anisotropic four-dimensional spaces or new quaternions]. Preprint, Nur-Sultan. 2020.

[16] Rakhymova A.T., Gabbassov M.B., Shapen K.M., "On one space of four-dimensional numbers," *Journal of Mathematics, Mechanics and Computer Science (Vol 4)* (2020): 199-225.

[17] Rakhymova A.T., Gabbassov M.B., Shapen K.M., "Functions in one space of four-dimensional numbers," *Journal of Mathematics, Mechanics and Computer Science (Vol 2)* (2021): 139-154.

[18] Rakhymova A.T., Gabbassov M.B., Ahmedov A.A. Analytical Solution of the Cauchy Problem for a Nonstationary Three-dimensional Model of the Filtration Theory. Journal of Advanced Research in Fluid Mechanics and Thermal Sciences. Vol. 87 No. 1: November (2021): 118 – 133.