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NONLINEAR VIBRATIONS OF THE "ROTOR – JOURNAL BEARINGS" SYSTEM

The equations of motion of a rotor system mounted on journal bearings with a non-linear characteristic are solved by high-precision analytical methods. A new technique has been developed for solving nonlinear differential equations of motion of rotor systems mounted on journal bearings, taking into account nonlinearity of reaction forces of the lubricating layer.Algebraic systems of equations were obtained that allow us to determine amplitudes of nonlinear oscillations of the rotor and supports, and construct the amplitude-frequency characteristics of the system for varying parameters of the rotor, supports and fluid depending on the angular velocity of the rotor. The conditions and frequency intervals for the presence of self-oscillations of the rotor and supports were determined. The amplitude-frequency characteristics of the nonlinear oscillations of the rotor system are obtained, taking into account nonlinearity of characteristics of journal bearings.The optimal parameters depending on the size of the gap and the oil film, the mass of the supports, the fluids used as a lubricating layer in the journal bearing, with rigidity and damping coefficients, at which the magnitudes of the amplitudes of self-excited oscillations have optimal values, are obtained.
Key words: Nonlinear Vibrations, Harmonic Balance Method, Journal Bearing, Sommerfeld's Hypothesis, Rotor System, Self-Excited Vibrations.

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"Ротор – сырғу мойынтіректері" жүйесінің бейсызық тербелістері

Жоғары дәлдікті аналитикалық әдістермен сипаттамасы бейсызық болып табылатын сырғу мойынтіректерінде орнатылған роторлық жүйенің қозғалыс теңдеулері шешілді.Майлау қабаты реакция күштерінің бейсызықтығын ескере отырып, сырғу мойынтіректерінде орнатылған роторлық жүйелер қозғалысының бейсызық дифференциалдық теңдеулерін шешудің жаңа әдістемесі жасалды. Ротор мен тіректердің бейсызық тербелістерінің амплитудасын анықтауға және ротордың бұрыштық жылдамдығына қатысты кезіндегі ротордың, тіректердің және сұйықтықтың параметрлерін вариациялау кезінде жүйенің амплитудалық-жиілік сипаттамаларын құруға мүмкіндік беретін алгебралық теңдеулер жүйесі алынды. Сырғу мойынтіректерінің сызықты емес сипаттамаларын ескере отырып роторлық жүйенің бейсызықты тербелістерінің амплитудалық-жиіліктік сипаттамалары тұрғызылды. Жүйенің өздігінен қозатын тербелістер амплитудасының мәні оптимальді мәнге ие болатындай саңылаудың қалыңдығы мен майлауқабаты, тіректердің массасы, сырғумойынтірегінде майлау қабаты ретінде қолданылатын сұйықтықпен, қатаңдықжәне демпферлік коэффициенттермен байланысты оптимальді параметрлер анықталды.
Түйін сөздер: Бейсызық тербелістер, гармоникалық баланс әдісі, сырғу мойынтірегі, Зоммерфельд гипотезасы, роторлық жүйе, өздігінен қозатын тербелістер.

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Нелинейные колебания системы "Ротор - подшипники скольжения"

Решены уравнения движения роторной системы, установленных на подшипниках скольжения с нелинейной характеристикой высокоточными аналитическими методами. Разработана новая методика решений нелинейных дифференциальных уравнений движения роторных систем, установленных на подшипниках скольжения, с учетом нелинейности сил реакций смазочного слоя. Были получены алгебраические системы уравнений, позволяющие определить амплитуды нелинейных колебаний ротора и опор, и построить амплитудно-частотные характеристики системы при варьировании параметров ротора, опор и жидкости в зависимости от угловой скорости ротора. Были определены условия и интервалы частот наличия автоколебаний ротора и опор. Построены амплитудно-частотные характеристики нелинейных колебаний роторной системы, с учетом нелинейности характеристик подшипников скольжения. Определены оптимальные параметры связанные с толщиной зазора и масленой пленки, массой опор, жидкости использующиеся в качестве смазочного слоя в подшипнике скольжения, с коэффициентами жесткости и демпфирования, при которых величины амплитуд самовозбуждающихся колебаний имеют оптимальные значения.

Ключевые слова: Нелинейные колебания, метод гармонического баланса, подшипник скольжения, гипотеза Зоммерфельда, роторная система, самовозбуждающиеся колебания.

1 Introduction

Journal bearings have a number of significant advantages over rolling bearings. They are resistant to a wide range of loads and dynamic disturbances, capable of operating at higher rotational speeds, have a long service life and low cost, and are easy to operate.

Due to specific properties of hydrodynamic forces caused by the presence of a lubricating layer during rotation of the rotor in journal bearings, self-excited oscillations (self-oscillations) with large amplitudes can arise in a wide range of rotation speeds. Therefore, it is often necessary to develop suppression measures in industry and production and study the behavior of this type of oscillation depending on various physical and geometric parameters of the system.

2 Literature review

At present, journal bearings, used in many rotary machines as key elements and serving to transfer rotational energy, are complex elements for dynamic analysis since under certain geometric and operating parameters they can cause, as mentioned above, self-excited [1-3], parametric [3, 4] and chaotic oscillations [4, 5]. As at operating frequencies of the system similar to the model considered in this paper, self-excited oscillations often occur, the paper studies the conditions for occurrence and further behavior of these oscillations.

One of the first researchers who studied the phenomenon of self-excitation and the reasons for its occurrence was Newkirk in 1924 [6]. Together with Taylor, he conducted the first experimental study of this phenomenon and explained the causes of self-excited oscillations [7]. When studying self-oscillations, in many cases the problem is reduced to studying precessional motion of the system. Approximate solutions, assuming that the load on the stud is sufficiently small, were first obtained by Hagg [8] and Yukio Hori [9]. Works on the analysis of the precessional movement of the stud in the oil-filled bearing were also carried out by Kesten [10].

Conditions for stability of the equilibrium position of the rotor system mounted on journal bearings, as well as the nature of unsteady motion in an unstable position, were studied by Someya [11]. Experimental studies of these phenomena were also carried out by such authors as Hagg, Boecker, Schnittger and Hori [8, 9, 12, 13]. Different results were obtained concerning the influence of oil viscosity and the size of backlash in the bearing. Some authors such as Schnittger have noted the benefits of low viscosity as it contributes to stud stability. Other authors such as Boecker, Schnittger and Pinkus [14] noted that high viscosity is more conducive to stability. According to the third group of authors, such as Hummel [15] and Hagg, both of the above cases are equivalent. Different points of view are also observed when studying the effect of bearing width on system dynamics. However, researchers agree that the unbalance of the rotor has no effect on the occurrence and intensity of self-excited oscillations. Some authors obtained different frequency of self-excited oscillations [16-19]. For most authors, the frequency of self-excited oscillations coincided with the natural frequency of the rotor, in some cases, for example, Pinkus, it increased with increasing speed, while Schnittger experimentally obtained results in which the frequency curve first decreased and then began to increase [13, 14].

Experimental studies of self-excited oscillations as a whole showed not only the complexity of this problem, but also revealed a number of specific features of this phenomenon. The most important of the identified effects is "inertia" (dragging), i.e. self-excited oscillations, after arising at a certain frequency, continue to exist even when the rotor speed decreases below the frequencies of occurrence of self-excited oscillations [20, 21-23]. Another feature is the possibility of occurrence of self-excited oscillations under the action of a short-term pulse, for example, a blow to the rotor, at speeds that are lower than the characteristic speeds at which self-excited oscillations arise [24, 25].

3 Statement of the problem and equations of motion

Consider a vertical solid rotor of mass m symmetrically mounted on a flexible shaft with respect to supports. The shaft is mounted on elastic supports. The rotor system rotates on journal bearings of mass m_0 with an angular velocity ω (Figure 1). Equivalent rigidity of the elastic field of supports is c; δ is the size of the clearance in the bearing; t is the oil temperature in the bearing; μ is the oil viscosity in the bearing; d is the diameter of the bearing spike; L is the length of the bearing; D is the bearing diameter; l is the length of the shaft; k_1, k_2 are damping coefficients; e is the rotor unbalance.

To derive the equations of motion, we introduce the fixed coordinate system Oxy. Let in this system x_1, y_1 be coordinates of O_1 (the center of the elastic support), x_2, y_2 be coordinates of O_2 (the center of the bearing spike), x_3, y_3 be coordinates of O_3 (the center of gravity of the rotor), φ be the polar angle of the line of centers.

Taking into account that

$$x_3 = x_2 + e\cos\omega t, \quad y_3 = y_2 + e\sin\omega t, \tag{1}$$

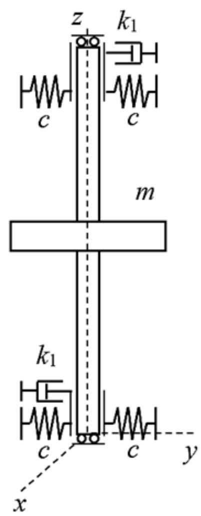

Figure 1: Rotor system rotating on journal bearings

we obtain the differential equations of motion of the system

$$\begin{aligned} m_0\ddot{x}_1 + k_1\dot{x}_1 + cx_1 - 2\left(P_e\cos\varphi + P_\varphi\sin\varphi\right) = 0, \\ m_0\ddot{y}_1 + k_1\dot{y}_1 + cy_1 - 2\left(P_e\sin\varphi - P_\varphi\cos\varphi\right) = 0, \\ m\ddot{x}_2 + k_2\dot{x}_2 + 2\left(P_e\cos\varphi + P_\varphi\sin\varphi\right) = me\omega^2\cos\omega t, \\ m\ddot{y}_2 + k_2\dot{y}_2 + 2\left(P_e\sin\varphi - P_\varphi\cos\varphi\right) = me\omega^2\sin\omega t. \end{aligned} \tag{2}$$

where P_e and P_φ are determined from the Sommerfeld hypothesis, according to which no restrictions are imposed on the length of the lubricating layer between the bearing and the stud and are determined as [26]

$$P_e = \frac{12\pi\mu LR^3\dot{\chi}}{\delta^2\left(1-\chi^2\right)^{3/2}}, \quad P_\varphi = \frac{12\pi\mu LR^3\chi\left(\omega - 2\dot{\varphi}\right)}{\delta^2\left(2+\chi^2\right)\sqrt{1-\chi^2}}.$$

The first two equations of system (2) are equations of motion of the support under the action of elastic forces cx_1, cy_1, damping forces $k_1\dot{x}_1$, $k_1\dot{y}_1$, and reaction forces of the lubricating layer P_e and P_φ, directed in the opposite direction to the forces of the same name shown in Figure 2.

The second two equations of system (2) determine the equations of motion of the rotor under the action of the reaction forces of the lubricating layer P_e and P_φ, and the external damping forces $k_2\dot{x}_2$, $k_2\dot{y}_2$. In order for the equations of system (2) in combination with the equations of hydrodynamic forces to form a closed system, it is necessary to express the eccentricity of the stud center e and the polar angle φ through the coordinates of the center of the elastic support x_1, y_1 and the coordinates of the center of the stud x_2, y_2. Figure 2 shows that

$$x_2 - x_1 = e\cos\varphi, \quad y_2 - y_1 = e\sin\varphi. \tag{3}$$

Then

$$e = \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2}, \tag{4}$$

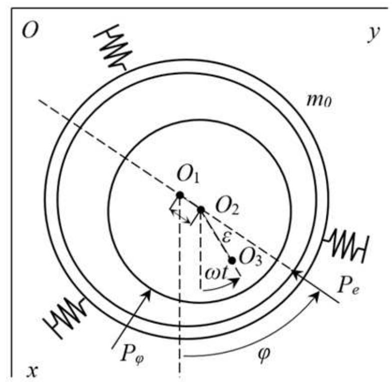

Figure 2: Reaction forces of journal bearings

$$\dot{e} = \frac{(x_2 - x_1)(\dot{x}_2 - \dot{x}_1) + (y_2 - y_1)(\dot{y}_2 - \dot{y}_1)}{e}, \tag{5}$$

$$\sin\varphi = \frac{(y_2 - y_1)}{e}, \quad \cos\varphi = \frac{(x_2 - x_1)}{e}, \tag{6}$$

$$\dot{\varphi} = \frac{(x_2 - x_1)(\dot{y}_2 - \dot{y}_1) - (y_2 - y_1)(\dot{x}_2 - \dot{x}_1)}{e^2}. \tag{7}$$

The system of equations (2) and equations (4)-(7) together with expressions for the reaction forces of the lubricating layer, the form of which depends on the accepted hypothesis, forms a closed system of nonlinear equations, the integration of which in general is not possible. To obtain an approximate solution of the equations of motion (2), we introduce complex variables of the form

$$z_1 = x_1 + iy_1, \quad z_2 = x_2 + iy_2, \quad z_3 = e(\cos\varphi + i\sin\varphi). \tag{8}$$

Then equations (2) and reaction forces can be rewritten as

$$\begin{gathered} m\ddot{z}_2 + c(z_2 - z_3) + k_1(\dot{z}_2 - \dot{z}_3) = 0, \\ c(z_2 - z_3) + k_1(\dot{z}_2 - \dot{z}_3) = 2(P_e - iP_\varphi)e^{i\varphi}, \\ P_e = \frac{6\mu LR^3}{\delta^2}\frac{2\chi^2(\omega - 2\Omega)}{(2+\chi^2)(1-\chi^2)}, \quad P_\varphi = \frac{6\mu LR^3}{\delta^2}\frac{\pi\chi(\omega - 2\Omega)}{(2+\chi^2)\sqrt{(1-\chi^2)}}. \end{gathered} \tag{9}$$

Let the system be "weakly" nonlinear, then its solution can be sought as

$$z_2 = \delta a e^{i(\Omega t - \gamma)}, \quad z_3 = \delta\chi e^{i\Omega t}. \tag{10}$$

Thus, substituting solutions in the form (10) into the equations of motion of system (9) and equating the terms in front of the same harmonics, we obtain a system of algebraic equations for the rotor amplitudes in the form

$$\begin{aligned} &-a\alpha^2\cos\gamma + a\cos\gamma - \chi + Da\alpha\sin\gamma = 0, \\ &a\alpha^2\sin\gamma - a\sin\gamma - D\alpha\chi + Da\alpha\cos\gamma = 0, \end{aligned} \tag{11}$$

where

$$\varphi = \Omega t, \quad A = a\delta, \quad D = k_1/m\Omega, \quad \alpha = 1/\sqrt{1-D^2}.$$

From system (11) we find that

$$a = \frac{\chi^2\left(1+D^2\alpha^2\right)}{\left(1-\alpha^2\right)^2 + D^2\alpha^2}, \quad \gamma = \frac{a}{\chi}\frac{D\alpha^3}{1+D^2\alpha^2}. \tag{12}$$

Thus, by varying parameters of the dimensionless damping D, dimensionless frequency of self-excited oscillations α, etc., we obtain amplitude-frequency characteristic for a rotor system mounted on journal bearings, taking into account nonlinearity of the reaction forces of the lubricating layer of journal bearings (Figures 3-13).

4 Results and discussion

The calculations were carried out for a rotor system rotating at a speed of 0 to 20000 rpm. It should be noted that five main parameters vary during the calculation, namely, the viscosity of the fluid in the lubricating layer, the mass of the supports, the damping coefficient, the rigidity coefficient of the equivalent field of elasticity and the size of the gap in the bearing, since these parameters are fundamental in the study of the behavior of self-excited vibrations. The analysis of vibrations was carried out on the basis of the analytical solution of the system of equations (11), with the following initial data: rotor mass $m = 5$ kg, support mass $m_0 = 0.15$ kg, clearance in the bearing $\delta = 0.06$ mm, oil temperature in the bearing $t = 50°$ C, bearing oil viscosity $\mu = 22.39$ mPa.s (turbine oil), bearing stud diameter $d = 20$ mm, bearing length $L = 20$ mm, bearing diameter $D = 20{+}2\delta$ mm, shaft length $l = 650$ mm; the equivalent rigidity of the elastic field of the support $c = 29$ kg/s^2, damping coefficients $k_1 = 42$ kg/s, $k_2 = 6.59$ kg/s.

Figure 3 shows the amplitude-frequency characteristics of the system with a gap of $\delta =$ 0.06 mm. It can be seen from the figure that with a rigid fastening (red curve), the system performance is limited by the rotation speed, which is approximately equal to twice the critical speed of the rotor. Starting from 6000 rpm, intense self-oscillations arise in the system in a wide frequency range. With the elastic mounting (blue curve), the vibration level is many times lower. The rotor, mounted on elastic supports, does not have a self-oscillation zone, and the system acquires the ability for stable operation at speeds of 20,000 rpm and higher, i.e. at speeds twenty times the first critical speed. When the rotor starts up after an easy and calm transition through two critical rotation speeds, the first self-centering zone is detected, in which operation with small vibration amplitudes is possible.

The second, even wider self-centering zone is located in the range from 6,000 to 20,000 rpm. Finally, it can be seen from the figure that the range of possible speeds of stable rotation of the rotor due to rotor mounting on elastic supports has increased three times compared to the rigid mounting of bearings, and this is especially important, the upper limit of the speed of rotation of the rotor has no fundamental boundaries. At the same time, it is observed that rotor mounting on elastic supports leads to a decrease in the level of vibrations not only in the areas of self-centering, but also during transition through resonant modes. In this case, the lower the rigidity of the supports, the less the vibration overloads.

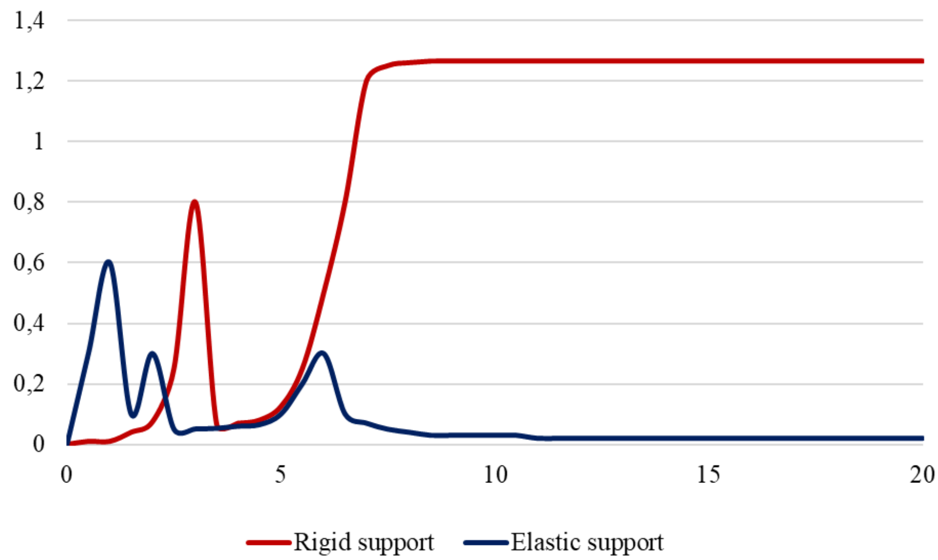

Figure 3: Rotor amplitudes with elastic and rigid mounting in the case when $d = 20$ mm, $l = 650$ mm, $c = 29$ kg/s^2, $\delta = 0.06$ mm, $t = 50°$ C, $k_1 = 42$ kg/s, $k_2 = 6.59$ kg/s, $\mu = 22.39$ mPa.s (turbine oil)

Figures 4 and 5 show the amplitude-frequency characteristics of the rotor and support, depending on the type of oil in the sleeve bearing, when $t = 50°$ C, $\delta = 0.06$ mm, pressure 1 atm. In the first case (red curve), when $\mu = 14.99$ mPa.s (anhydrous glycerol), the amplitudes of both the rotor and the support are maximum. Further, as the viscosity of the liquid increases, the amplitudes decrease and have minimum values at maximum values of viscosity (black curve), i.e. $\mu = 40$ mPa.s (fuel oil). In this case, the optimal values correspond to the case when turbine oil is used, i.e. when $\mu = 22.39$ mPa.s, as further increase in viscosity may lead to violation of the thermal regime in the journal bearing.

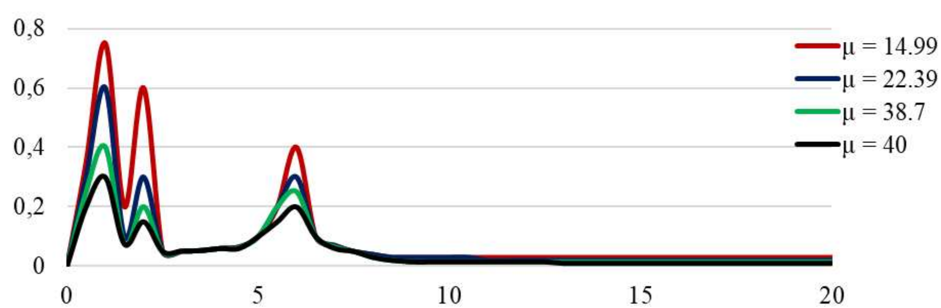

Figure 4: Rotor amplitudes at different values of fluid viscosity in the bearing when $m = 5$ kg, $m_0 = 0.15$ kg, $d = 20$ mm, $l = 650$ mm, $c = 29$ kg/s^2, $\delta = 0.06$ mm, $t = 50°$ C, $k_1 = 42$ kg/s, $k_2 = 6.59$ kg/s, $\mu = 22.39$ mPa.s (turbine oil)

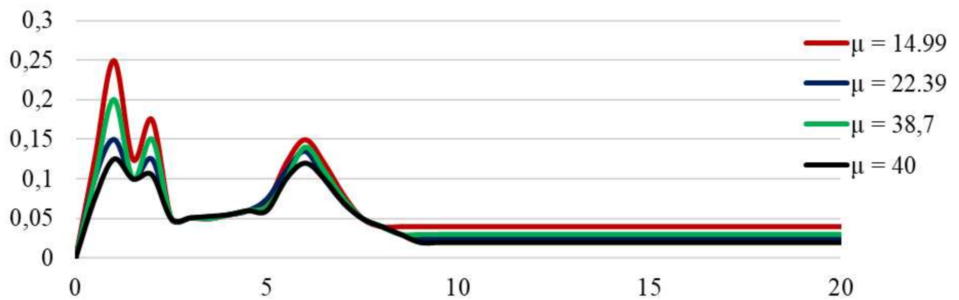

Figure 5: Support amplitudes at different values of fluid viscosity in the bearing when $m = 5$ kg, $m_0 = 0.15$ kg, $d = 20$ mm, $l = 650$ mm, $c = 29$ kg/s^2, $\delta = 0.06$ mm, $t = 50°$ C, $k_1 = 42$ kg/s, $k_2 = 6.59$ kg/s, $\mu = 22.39$ mPa.s (turbine oil)

Figures 6 and 7 show the amplitude-frequency characteristics of the rotor and support depending on the weight of the support. In both cases, the amplitudes of the rotor and support are damped with an increase in the mass of the support, since the support, with a sufficiently large mass, serves as an anti-weight and acts as a vibration damper, i.e. there is an anti-resonance phenomenon, for example, when $m_0 = 1$ kg (black curve). It should be noted that with an increase in the mass of the support, critical frequencies are shifted towards smaller angular velocities, whereas strong displacements of self-centering areas are not observed. With a decrease in the mass of the support, resonance frequencies are shifted towards large angular velocities, and amplitudes also increase, the first section of self-centering is also narrowed, for example, the case when $m_0 = 0.15$ kg (red curve).

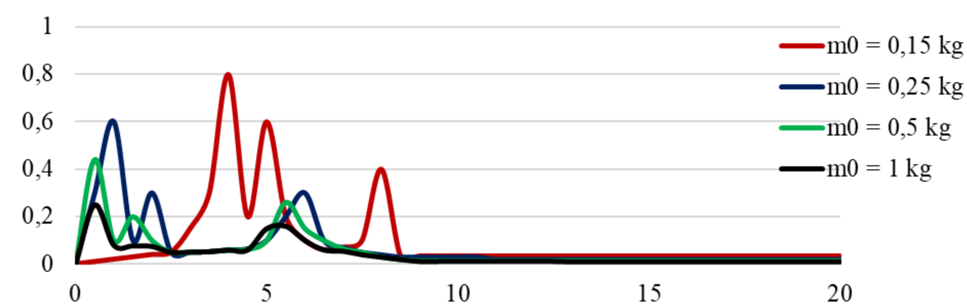

Figure 6: Rotor amplitudes at different values of the support mass in the case when $m = 5$ kg, $d = 20$ mm, $l = 650$ mm, $c = 29$ kg/s^2, $\delta = 0.06$ mm, $t = 50°$ C, $k_1 = 42$ kg/s, $k_2 = 6.59$ kg/s, $\mu = 22.39$ mPa.s (turbine oil)

Figures 8 and 9 show the amplitude-frequency characteristics of the rotor and support depending on the damping coefficient, for gaps $\delta = 0.06$ mm. Here, the amplitudes sharply decrease when passing through resonances. Moreover, the damping effect of the elastic supports is most effective when passing through the first and second critical speeds of the rotor. The influence of damping of supports on the third critical speed is less significant.

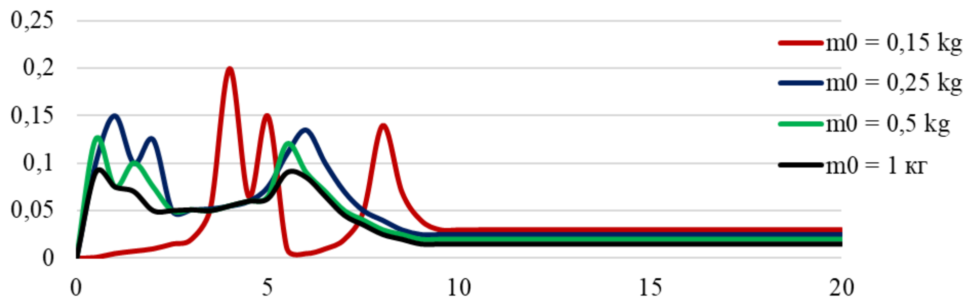

Figure 7: Support amplitudes for different values of the support mass in the case when $m = 5$ kg, $d = 20$ mm, $l = 650$ mm, $c = 29$ kg/s^2, $\delta = 0.06$ mm, $t = 50°$ C, $k_1 = 42$ kg/s, $k_2 = 6.59$ kg/s, $\mu = 22.39$ mPa.s (turbine oil)

An increase in the vibration amplitudes in the self-centering zones is not observed. Smooth operation of the system with low vibration amplitudes is observed in these zones.

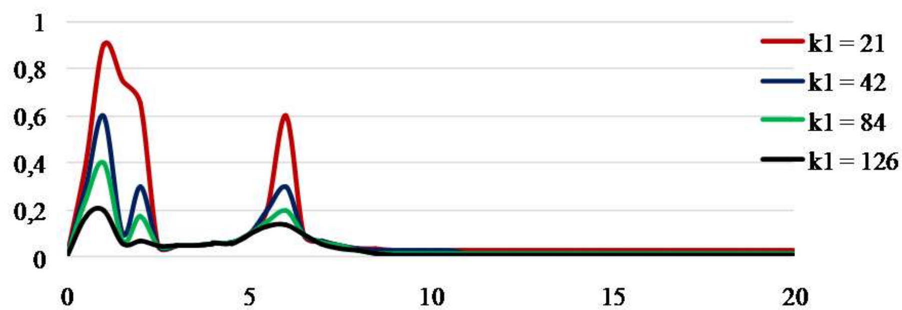

Figure 8: Rotor amplitudes at different values of the damping coefficient k_1 in the case when $m = 5$ kg, $m_0 = 0.15$ kg, $d = 20$ mm, $l = 650$ mm, $c = 29$ kg/s^2, $\delta = 0.06$ mm, $t = 50°$ C, $k_1 = 42$ kg/s, $k_2 = 6.59$ kg/s, $\mu = 22.39$ mPa.s (turbine oil)

At different values of rigidity of the equivalent field of the supports, there is also a shift in the vibration amplitudes along the frequency axis and change in their magnitudes (Figures 10 and 11). For example, with an increase in rigidity, the amplitudes of both the rotor and the supports increase. Also, with an increase in the coefficient, the peaks of the amplitudes are shifted towards higher angular velocities. In general, an increase in rigidity, as was shown initially (Figure 3), does not have a positive effect on the behavior of the system, while with an increase in compliance, the opposite picture is observed.

Figures 12 and 13 show the amplitude-frequency characteristics of the rotor and support, depending on the width of the gap in the journal bearing. As can be seen from the figures, an increase in the width of the gap adversely affects the operation of the system. An increase in the gap width leads to an increase in the amplitude of both the rotor and the support. With a decrease in the gap width, the opposite effect is observed, i.e. the minimum values of δ correspond to the minimum values of the amplitudes. But since, in practice, a small gap

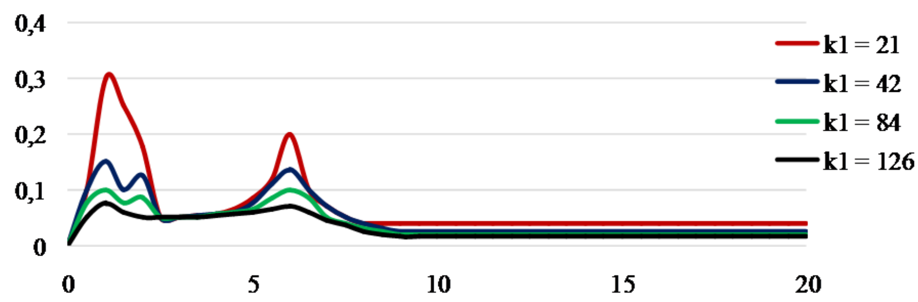

Figure 9: Support amplitudes at different values of the damping coefficient k_1 in the case when $m = 5$ kg, $m_0 = 0.15$ kg, $d = 20$ mm, $l = 650$ mm, $c = 29$ kg/s^2, $\delta = 0.06$ mm, $t = 50°$ C, $k_1 = 42$ kg/s, $k_2 = 6.59$ kg/s, $\mu = 22.39$ mPa.s (turbine oil)

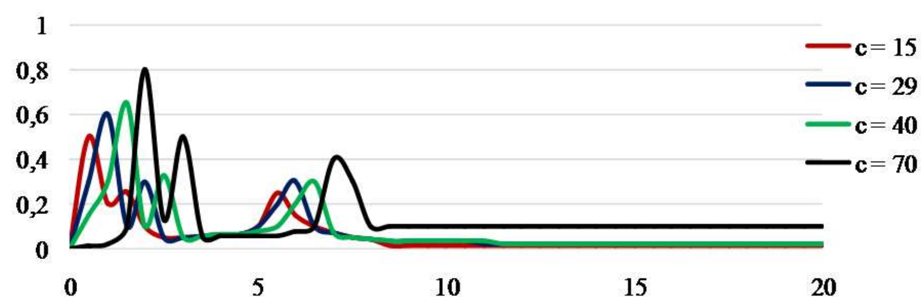

Figure 10: Rotor amplitudes at different values of the rigidity coefficient c in the case when $m = 5$ kg, $m_0 = 0.15$ kg, $d = 20$ mm, $l = 650$ mm, $c = 29$ kg/s^2, $\delta = 0.06$ mm, $t = 50°$ C, $k_1 = 42$ kg/s, $k_2 = 6.59$ kg/s, $\mu = 22.39$ mPa.s (turbine oil)

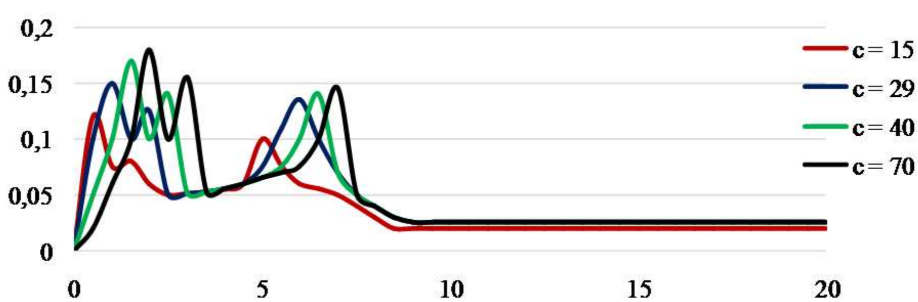

Figure 11: Support amplitudes at different values of the rigidity coefficient c in the case when $m = 5$ kg, $m_0 = 0.15$ kg, $d = 20$ mm, $l = 650$ mm, $c = 29$ kg/s^2, $\delta = 0.06$ mm, $t = 50°$ C, $k_1 = 42$ kg/s, $k_2 = 6.59$ kg/s, $\mu = 22.39$ mPa.s (turbine oil)

width entails violation of the thermal regime due to heating [27], the best option in this case is the gap value $\delta = 0.06$ mm.

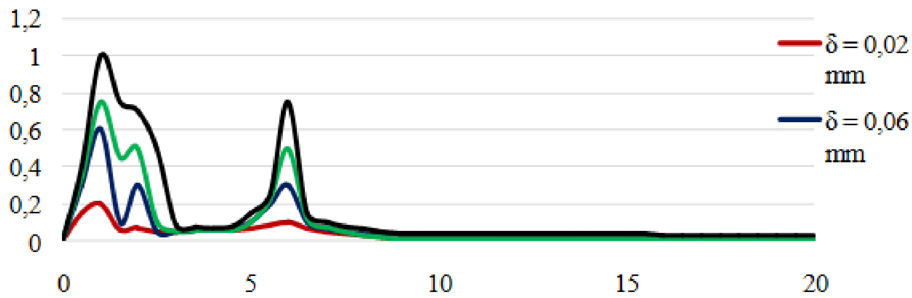

Figure 12: Rotor amplitudes for different values of the gap thickness δ in the case when $m = 5$ kg, $m_0 = 0.15$ kg, $d = 20$ mm, $l = 650$ mm, $c = 29$ kg/s^2, $\delta = 0.06$ mm, $t = 50°$ C, $k_1 = 42$ kg/s, $k_2 = 6.59$ kg/s, $\mu = 22.39$ mPa.s (turbine oil)

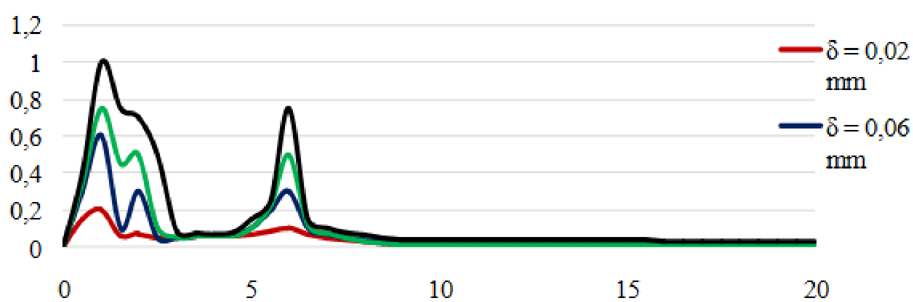

Figure 13: Rotor amplitudes for different values of the gap thickness δ in the case when $m = 5$ kg, $m_0 = 0.15$ kg, $d = 20$ mm, $l = 650$ mm, $c = 29$ kg/s^2, $\delta = 0.06$ mm, $t = 50°$ C, $k_1 = 42$ kg/s, $k_2 = 6.59$ kg/s, $\mu = 22.39$ mPa.s (turbine oil)

In the first resonant zone, the vibrations of the disk and supports occur in phase, i.e. the type of the waveform is cylindrical precession. In the second zone, vibrations of the supports occur in antiphase with respect to each other; in this case, in the region of the disk, vibrations have a node. Thus, in the second zone, the mode of vibrations is a skew-symmetric precession. In the third resonant zone, the vibrations of the supports with respect to each other occur in phase, and near the disk – in antiphase. Thus, the third form of vibrations is a two-node symmetrical form, the type of which resembles the first form of vibrations of an unsupported shaft. It should be noted that the location and types of the first and second modes of vibrations are determined mainly by the compliance of the supports,whereas the third form is caused by bending vibrations of the rotor shaft. Thus, these studies show that the zones of increased vibrations are narrow resonant zones due to dynamic and static imbalances of the rotor.

5 Conclusion

Installation of rotors in elastic supports leads to complete suppression of self-oscillations that occurred during rigid mounting of journal bearings, and oscillations of the system over the entire speed range become purely forced. The damping efficiency of elastic supports is very high and increases with decreasing rigidity. Self-centering of the system in non-resonant zones leads to significant reduction in the magnitude of vibrations and vibration overloads of the system. Installation of the rotor in elastic supports "linearizes"the dynamic system "rotor – supports" . It should also be noted that the main parameter that determines the type of oscillations is the size of the gap of the journal bearing, since with its increase the amplitudes will increase, and at its limiting values, self-excited oscillations will turn into a chaotic type of oscillations, which will negatively affect the stability of the system even at high speeds. According to the theory of self-centering [28], where it is shown that overloads in self-centering areas are determined only by the magnitude of the unbalance and the rigidity of the supports, it can be concluded that vibration overloads of the system will practically not increase even with a significant value of the rotor unbalance. Therefore, with sufficient compliance of the supports, even with large imbalances, one can expect stable operation of the machine with a moderate level of vibration overloads in a wide range of speeds.

6 Acknowledgement

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