4-бөлім **Раздел 4** **Section 4**

Қолданбалы математика **Прикладная математика** **Applied Mathematics**

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DEVELOPMENT OF METHODS AND ALGORITHMS FOR ESTIMATING THE TEMPERATURE DISTRIBUTION IN THE BODY OF A RECTANGULAR PARALLELEPIPED SHAPE UNDER THE INFLUENCE OF HEAT FLOW AND THE PRESENCE OF HEAT EXCHANGE

The article describes methods and computational algorithms for estimating the temperature distribution law in the body of a rectangular parallelepiped shape under the influence of heat flow and the presence of heat exchange. It is believed that one of the faces is amenable to heat flow, and the other faces are insulated or are under the influence of the environment. To use the variational approach, the total energy functional is calculated, taking into account the boundary conditions. Minimizing the functional and equating it to zero, we obtain a system of linear equations, the solution of which gives the temperature of a rectangular parallelepiped at the nodal points. Further, substituting these nodal temperature values into the approximating function, we obtain the law of temperature distribution in the body in the form of a rectangular parallelepiped. The temperature distribution law is obtained by dividing a rectangular parallelepiped into one, two and three elements. To speed up the process of calculating the temperature distribution law, an algorithm has been developed that allows you to create a program code that increases the calculation efficiency by an order of magnitude. This is achieved by the fact that the created code contains only a system of linear equations, unlike the main program, which forms a general functional of full energies, calculates derivatives of this functional and obtains a system of equations.

Key words: variational approach, thermal conductivity, heat flow, rectangular parallelepiped, heat exchange, temperature.

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Жылу ағынының әсері және жылу алмасудың болуы кезінде тікбұрышты параллелепипед пішініндегі денеде температураның таралуын бағалау үшін әдістер мен алгоритмдерді әзірлеу

Мақалада жылу ағынының әсер етуі және жылу алмасудың болуы кезінде тікбұрышты параллелепипед формасындағы денеде температураның таралу Заңын бағалау үшін әдістер мен есептеу алгоритмдері сипатталған. Жылу ағыны бір бетке түседі, ал басқа беттер жылу оқшауланған немесе қоршаған ортаның әсерінен болады деп саналады. Вариациялық тәсілді қолдану үшін шекаралық жағдайларды ескере отырып, толық энергияның функционалдығы есептеледі. Функционалдылықты азайтып, оны нөлге теңестіре отырып, біз сызықтық теңдеулер жүйесін аламыз, оның шешімі түйіндік нүктелердегі тікбұрышты параллелепипедтің температурасын береді. Әрі қарай, температураның түйіндік мәндерін жуықтау функциясына алмастыра отырып, біз тікбұрышты параллелепипед түрінде денеде температураның таралу Заңын аламыз. Температураның таралу заңы тікбұрышты параллелепипедті бір, екі

және үш элементке бөлу арқылы алынады. Температураның таралу Заңын есептеу процесін тездету үшін алгоритм әзірленді, ол есептеу тиімділігін ретті арттыратын бағдарлама кодын жасауға мүмкіндік береді. Бұған құрылған кодта негізгі бағдарламадан айырмашылығы тек сызықтық теңдеулер жүйесі бар, ол толық энергияның жалпы функционалдығын құрайды, осы функцияның туындыларын есептейді және теңдеулер жүйесін алады.
Түйін сөздер: Вариациялық тәсіл, жылу өткізгіштік, жылу ағыны, тікбұрышты параллелепипед, жылу алмасу, температура.

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Разработка методов и алгоритмов для оценки распределения температуры в теле формы прямоугольного параллелепипеда при воздействии теплового потока и наличия теплообмена

В статье описаны методы и вычислительные алгоритмы для оценки закона распределения температуры в теле формы прямоугольного параллелепипеда при воздействии теплового потока и наличия теплообмена. Считается, что на одно из грани поддается тепловой поток, а другие грани теплоизолированные или находятся под воздействием окружающей среды. Для использования вариационного подхода вычисляется функционал полной энергии, учитывающий граничные условия. Минимизируя функционал и приравнивая его к нулю получаем систему линейных уравнений, решение которой дает температуру прямоугольного параллелепипеда в узловых точках. Далее, подставляя эти узловые значения температур в аппроксимирующую функцию, получим закон распределения температуры в теле форме прямоугольного параллелепипеда. Закона распределения температуры получена при разбиения прямоугольного параллелепипеда на один, два и три элемента. Для ускорения процесса вычисления закона распределения температуры разработан алгоритм, позволяющий создать код программы, который увеличивают эффективность вычисления на порядок. Это достигается тем, что создаваемый код содержит только систему линейных уравнений в отличии от основной программы которая формирует общую функционал полной энергий, вычисляет производные от этого функционала и получает систему уравнений.

Ключевые слова: вариационный подход, теплопроводность, тепловой поток, прямоугольный параллелепипед, теплообмен, температура.

1 Introduction

In [1], general problems on the use of the finite element method for determining the thermomechanical characteristics of various solids are considered. In [2], the temperature distribution law is compared for a rectangular parallelepiped and a rod with a similar size, other things being equal. In [3], a computational algorithm and a method for determining the temperature field along the length of a rod with a limited length and variable cross-section are proposed. In [4], an energy method is considered for determining the law of temperature distribution, three components of deformation and stress, provided that both ends of a rod of variable cross-section are rigidly fixed. In [5], the stationary solution of thermal conductivity problems with low convergence is shown for a rectangle with specified zero temperatures, with the exception of one surface with an abrupt temperature change. In [6], exact nonstationary solutions of thermal conductivity in two-dimensional rectangles heated at the boundary are considered. In [7], the stationary problem of thermal conductivity through a local fractional derivative is investigated. Thermal conductivity, convective heat exchange, radiation heat exchange, thermal and hydromechanical calculations of heat exchange devices, as well as heat and mass transfer during phase and chemical transformations are systematically considered [8]. In [9], the basics of calculating heat transfer through a layer of matter are described. In [10], a local fractional Euler method is proposed to consider the stationary problem of thermal conductivity. In [12], the equation of thermal conductivity of an eccentric spherical ring with an inner surface maintained at a constant temperature and an outer surface subject to convection is analytically solved. In [13], a simple and accurate model is proposed for predicting the dimensionless parameter of the shape factor. In [14], a three-dimensional equation of unsteady thermal conductivity was investigated by approximating the spatial derivatives of the second order by a five-point central difference in cylindrical coordinates. In [15], a three-dimensional unsteady thermal conductivity equation is considered, taking into account the presence of convective terms and phase transitions. In [16], one computational approach to the calculation of the heat equation is presented, which differs in the case of three-dimensional oblique unstructured grids by the compactness of the grid pattern and the unconditional stability of the numerical algorithm. The work [17] is part of a broader study on the assessment of heat transfer through the elements of building enclosing structures by the most accurate assessment of the effect of thermal bridges of the most common building structures. Currently, the discontinuous Galerkin method with discontinuous basis functions is widely used in [18], which is characterized by a high order of accuracy of the resulting solution. In [19], the transition region between two solids whose state differs from the state of the contacting media is investigated. In [20], a technique for optimizing the control of the thermal conductivity process in a solid is considered. [21] describes an approximate method for calculating the temperature field in solids heated by convection and radiation simultaneously, when the thermophysical properties of a substance depend on temperature. In [23], the main problems of the modern theory of heat transfer are outlined, including many that go beyond the standard courses. In [24], solutions to problems of unsteady thermal conductivity are considered by several methods. Mathematical steps leading to the calculation of the temperature field in multidimensional multilayer bodies are described in [25] and numerical results for two-layer bodies are presented. In [26], the temperature distribution under the influence of a heat flow from one side of a parallelepiped and during heat exchange with the opposite side is considered using an approach where the solution of the problem is expressed as a solution of one-dimensional problems and multiple Fourier series or their generalization are used. In [27], a representative stationary problem of thermal conductivity in rectangular bodies with uniformly distributed heat generation is analytically investigated.

2 Problem statement

Consider a solid body in the form of a rectangular parallelepiped (Figure 1). The origin of the coordinates is located in the lower left corner of a rectangular parallelepiped, as shown in the figure. The nodal points are numbered starting from the lower-left near corner (node 0). The dimensions of a rectangular parallelepiped along the x, y and z axes are denoted by a, b and c, respectively. Convective heat exchange occurs on the face (0, 1, 2, 3), and a heat flow is applied to the face (4, 5, 6, 7).

The task is to find the law of temperature distribution at any point of a solid body in the form of a rectangular parallelepiped.

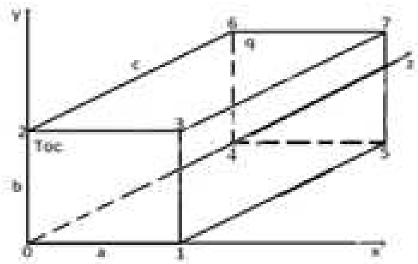

Figure 1: Diagram of the solid under study in the form of a rectangular parallelepiped

Mathematically, the stationary problem is reduced to solving the heat equation:

$$K_{xx}\left(\frac{\partial^2 T}{\partial x^2}\right)+K_{yy}\left(\frac{\partial^2 T}{\partial y^2}\right)+K_{zz}\left(\frac{\partial^2 T}{\partial z^2}\right)=0, \tag{1}$$

under restrictions:

– of the second kind

$$\left(K_{xx}\frac{dT}{dx}+K_{yy}\frac{dT}{dy}+K_{zz}\frac{dT}{dz}\right)\Big|_{S_1}+q=0 \quad \text{at } S_1 \tag{2}$$

– the third kind

$$\left(K_{xx}\frac{dT}{dx}+K_{yy}\frac{dT}{dy}+K_{zz}\frac{dT}{dz}\right)\Big|_{S_2}+h(T-T_{\text{OC}})=0 \quad \text{at } S_2 \tag{3}$$

where

T – is the temperature, ℃;

q – heat flow, $\frac{\text{кВт}}{m^2}$;

K_{xx}, K_{yy} and K_{zz} – thermal conductivity coefficients in the directions x, y and z, $\frac{\text{кВт}}{\text{м℃}}$;

h – heat transfer coefficient, $\frac{\text{кВт}}{\text{м}^2\text{℃}}$;

S_1 – the surface on which the heat flow enters, m^2;

S_2 – the surface where heat exchange occurs, m^2;

T_{OC} – ambient temperature, ℃.

Equation (2) is the boundary condition for heat flow, and equation (3) is for convective heat transfer.

The task is to find a solution to equation (1) under constraints (2) and (3) using a variational approach.

3 Material and methods

3.1 Research methodology

A variational approach is used to solve the problem [1]. According to this approach, the solution of the problem under consideration is equivalent to minimizing the temperature functional at the nodal points:

$$J = \int\limits_V \frac{1}{2}[K_{xx}\left(\frac{\partial T}{\partial x}\right)^2 + K_{yy}\left(\frac{\partial T}{\partial y}\right)^2 + K_{zz}\left(\frac{\partial T}{\partial z}\right)^2]dv + \\ + \int\limits_{S_1} qdS + \int\limits_{S_2} \frac{h}{2}(T - T_{\text{OC}})^2 dS = J_1 + J_2 + J_3 + J_4 + J_5, \tag{4}$$

where V is the volume of the body under consideration.

For a rectangular parallelepiped (Fig. 1), formula (4) has the form:

$$J = \frac{1}{2}\int\limits_0^a\int\limits_0^b\int\limits_0^c [K_{xx}\left(\frac{\partial T}{\partial x}\right)^2 + K_{yy}\left(\frac{\partial T}{\partial y}\right)^2 + K_{zz}\left(\frac{\partial T}{\partial z}\right)^2] + \\ + \int\limits_0^a\int\limits_0^b qTdxdy\Big|_{z=c} + \frac{h}{2}\int\limits_0^a\int\limits_0^b (T - T_{\text{OC}})^2 dxdy\Big|_{z=0} \tag{5}$$

When the side faces of a rectangular parallelepiped are not thermally insulated, the following terms are added to the functional J, taking into account heat transfer:

$$J_5 = \frac{h}{2}\int\limits_0^a\int\limits_0^c (T - T_{\text{OC}})^2 dxdz\Big|_{y=0},$$

$$J_6 = \frac{h}{2}\int\limits_0^a\int\limits_0^c (T - T_{\text{OC}})^2 dxdz\Big|_{y=b},$$

$$J_7 = \frac{h}{2}\int\limits_0^a\int\limits_0^c (T - T_{\text{OC}})^2 dydz\Big|_{x=a},$$

$$J_8 = \frac{h}{2}\int\limits_0^a\int\limits_0^c (T - T_{\text{OC}})^2 dydz\Big|_{x=0},$$

where J_5, J_6, J_7, J_8 – characterize the heat exchange on the faces (0, 1, 4, 5), (2, 3, 6, 7), (0, 2, 4, 6), (1, 3, 5, 7) a rectangular parallelepiped, respectively.

In this case, the total functional is $J = \sum\limits_{i=1}^{8} J_i$.

To minimize the functional J, the temperature $T(x, y, z)$ is approximated by a third-order polynomial:

$$T(x, y, z) = \lambda_0 + \lambda_1 + \lambda_2 + \lambda_3 z + \lambda_4 xy + \lambda_5 xz + \lambda_6 yz + \lambda_7 xyz. \tag{6}$$

Suppose that the temperature values are set at the nodal points of a rectangular parallelepiped [4]:

$$T(x_0, y_0, z_0) = T_0;\ T(x_1, y_1, z_1) = T_1;\ T(x_2, y_2, z_2) = T_2;\ T(x_3, y_3, z_3) = T_3;$$

$$T(x_4, y_4, z_4) = T_4;\ T(x_5, y_5, z_5) = T_5;\ T(x_6, y_6, z_6) = T_6;\ T(x_7, y_7, z_7) = T_7.$$

After substituting These values into formula (7) and bringing similar terms, we get [4]:

$$\begin{gathered} T(, y, z) = \varphi_0(x, y, z) * T_0 + \varphi_1(x, y, z) * T_1 + \varphi_2(x, y, z) * T_2 + \\ +\varphi_3(x, y, z) * T_3 + \varphi_4(x, y, z) * T_4 + \varphi_5(x, y, z) * T_5 + \varphi_6(x, y, z) * T_6 + \varphi_7(x, y, z) * T_7; \end{gathered} \tag{7}$$

$-a \le x \le a$; $-b \le x \le b$; $-c \le x \le c$, where

$$\begin{gathered} \varphi_0(x, y, z) = 1 - \frac{z}{c} - \frac{y}{b} + \frac{yz}{bc} - \frac{x}{a} + \frac{xz}{ac} + \frac{xy}{ab} - \frac{xyz}{abc}; \\ \varphi_1(x, y, z) = \frac{x}{a} - \frac{xz}{ac} - \frac{xy}{ab} + \frac{xyz}{abc}; \\ \varphi_2(x, y, z) = \frac{y}{b} - \frac{yz}{bc} - \frac{xy}{ab} + \frac{xyz}{abc}; \\ \varphi_3(x, y, z) = \frac{xy}{ab} - \frac{xyz}{abc}; \\ \varphi_4(x, y, z) = \frac{z}{c} - \frac{yz}{bc} - \frac{xz}{ac} + \frac{xyz}{abc}; \\ \varphi_5(x, y, z) = \frac{xz}{ac} - \frac{xyz}{abc}; \\ \varphi_6(x, y, z) = \frac{yz}{bc} - \frac{xyz}{bc^2}; \\ \varphi_7(x, y, z) = \frac{xyz}{bc^2}; \end{gathered} \tag{8}$$

Differentiating (8) by the variables x, y and z we get:

$$\begin{aligned} \frac{\partial T}{\partial x} &= \sum_{i=1}^{7} \frac{\partial \varphi_i}{\partial x} T_i, \\ \frac{\partial T}{\partial y} &= \sum_{i=1}^{7} \frac{\partial \varphi_i}{\partial y} T_i, \\ \frac{\partial T}{\partial z} &= \sum_{i=1}^{7} \frac{\partial \varphi_i}{\partial z} T_i, \end{aligned} \tag{9}$$

Expression J (5) after substitution of T and $\left(\frac{\partial T}{\partial x}, \frac{\partial T}{\partial y}, \frac{\partial T}{\partial z}\right)$ from (9) is calculated using the Python sympy module, the expression of which is not given here because of the bulkiness.

After differentiating the functional J by variables $\overline{T_0, T_7}$, and we equate it to zero. As a result, we obtain a system of linear equations with respect to variables $\overline{T_0, T_7}$, which, for example, for the thermally insulated case, when we consider a cube with the length of the sides a, has the form (10):

$$\begin{Bmatrix} d & 0 & 0 & f & 0 & f & f & f \\ 0 & d & f & 0 & f & 0 & f & f \\ 0 & f & d & 0 & f & f & 0 & f \\ f & 0 & 0 & d & f & f & f & 0 \\ 0 & f & f & f & d & 0 & 0 & f \\ f & 0 & f & f & 0 & d & f & 0 \\ f & f & 0 & f & 0 & f & d & 0 \\ f & f & f & 0 & f & 0 & 0 & d \end{Bmatrix} \begin{pmatrix} T_0 \\ T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \end{pmatrix} +$$

$$+ \begin{Bmatrix} g & \frac{g}{2} & \frac{g}{2} & \frac{g}{4} & 0 & 0 & 0 & 0 \\ \frac{g}{2} & g & \frac{g}{4} & \frac{g}{2} & 0 & 0 & 0 & 0 \\ \frac{g}{2} & \frac{g}{4} & g & \frac{g}{2} & 0 & 0 & 0 & 0 \\ \frac{g}{4} & \frac{g}{2} & \frac{g}{2} & g & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{Bmatrix} \begin{pmatrix} T_0 \\ T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \end{pmatrix} = \begin{Bmatrix} l \\ l \\ l \\ l \\ p \\ p \\ p \\ p \end{Bmatrix} \tag{10}$$

where $d = 0.3Kxx * a$; $f = -0.083 * Kxx * a$; $g = a * *2 * h/9$; $l = T_{\text{OC}} * a * *2 * h/4$; $p = -a * *2 * q/4$.

Here, it should be noted that the developed Python program allows you to obtain a system of linear equations for any number of partitions of the sides of a rectangular parallelepiped.

The solution of the resulting system of equations makes it possible to determine the temperatures at the nodal points of a rectangular parallelepiped. Substituting these values in (9), we obtain the law of temperature distribution at any point of a rectangular parallelepiped.

All calculations were obtained using a program developed in the Python programming language.

4 Results and discussions

For the practical implementation of the proposed approach, a Python program was developed. As an example, a cube was selected (Figure 2) with the following initial data:

$a = 0.04m$, $b = 0.04m$, $c = 0.04m$, $K_{xx} = 75000\frac{\text{Вт}}{\text{м}℃}$, $T_{\text{OC}} = 40℃$, $q = -150\frac{\text{кВт}}{\text{м}^2}$, $h = 100000\frac{\text{Вт}}{\text{м}^2℃}$.

If we denote the number of partitions of a rectangular parallelepiped into elements along the x, y and z axes as mx, my and mz, respectively, then for a cube we have $mx = my = mz = m$.

Let's introduce arrays of temperatures $T1[0, 8]$, $T2[0, 27]$, $T3[0, 63]$ corresponding to the nodal points of the cube for the number of partitions $m = 1, 2, 3$ respectively. The temperature values at the nodal points of the cube for the thermally insulated case turned out to be equal

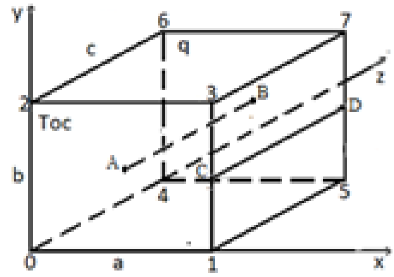

Figure 2: A solid body in the shape of a cube, consisting of one element

for $m = 1$ (fig. 2):
$T1[0;3] = 54.99$, $T1[4;7] = 62.99$;
for $m = 2$ (fig. 3):
$T2[0,8] = 55$, $T2[9,17] = 59$, $T2[18,27] = 63$;

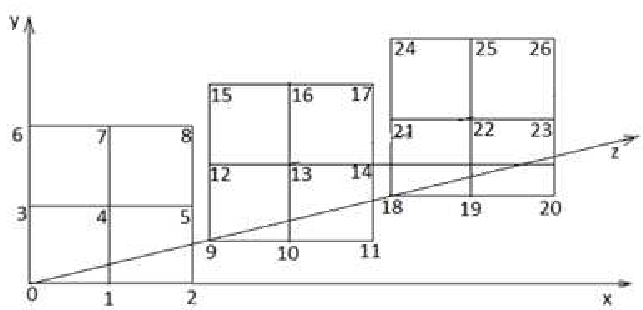

Figure 3: A solid body in the form of a cube when divided into 8 elements

for $m = 3$ (fig. 4):
$T3[0,15] = 55$, $T3[16,31] = 57.66$, $T3[32,47] = 60.33$, $T4[48,63] = 63$;

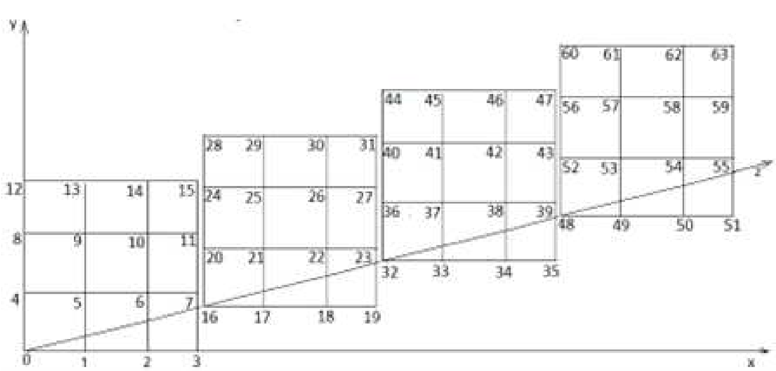

Figure 4: A solid body in the form of a cube when divided into 27 elements

The laws of temperature distribution for the thermally insulated case, when $m = 1$, $m = 2$ и $m = 3$ they turned out to be the same, corresponding to a straight line (Figure 6).

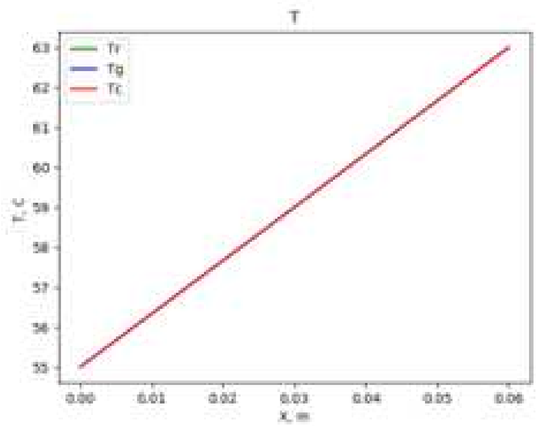

Figure 5: The law of temperature distribution in a cube for the thermally insulated case at $m = 1$, $m = 2$ and $m = 3$.

On (Fig. 5) through Tr, Tg, Tc, temperatures are indicated along the segments $(0, 4)$, (C, D), (A, B), respectively.

The law of temperature distribution on the face (1, 3, 5, 7) of the cube for the thermally insulated case is shown in (Fig. 6).

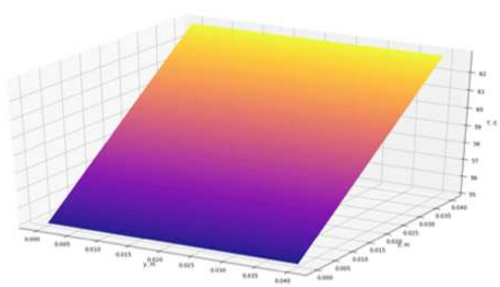

Figure 6: The law of temperature distribution in a cube in the thermally insulated case of temperature on the plane (y, z) at a fixed value $x = a$ for the thermally insulated case at $m = 1$, $m = 2$, $m = 3$

From the data $T3[0, 63]$ it can be seen that in the thermally insulated case, the temperature in the sections of the cube perpendicular to the z axis is the same.

Consider the temperatures for the edge of the cube passing through the nodes:

– (0; 4) at $m = 1$. Here the temperature in the nodes is $T1 = (41.50, 45.24)$;

– (0; 9; 18) at $m = 2$. Here the temperature in the nodes is $T2 = (41.71, 43.04, 45.19)$;

– (0; 16; 32; 48) at $m = 3$. Here the temperature in the nodes is $T3 = (41.75, 43.69, 44.14, 46.32)$.

The temperature distribution laws for the non-insulated case for segments $(0, 4)$, (A, B) and (C, D) (Fig. 7) at m=3 are shown in (Fig. 12)

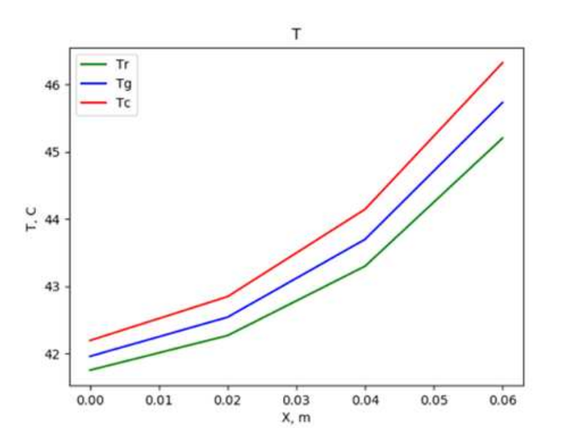

Figure 7: The law of temperature distribution in a cube for the non-insulated case at $m = 1$, $m = 2$ and $m = 3$

From (Fig. 7) it can be seen that the temperature at the middle of the cube (Tc) is greater than the temperature of the middle of the face (Tg) and its turn (Tg) is greater than (Tr). This means that the temperature values along the line AB (Fig. 7) passing through the center of the cube are greater than the temperature on the edge $(0 - 4)$ (Fig. 7) and the temperature on the line passing through the middle of the cube face (line CD of Figure 2). In turn, the temperature on the CD line (Fig. 7) is greater than the temperature on the AB line (Fig. 7). This means that the farther the line is from the center, the lower its temperature.

The law of temperature distribution in the cube for the edge (0, 4) (Fig. 7) when partitioning $m = 1$, $m = 2$ and $m = 3$ in the non-insulated case is shown in (Fig. 13).

It can be seen from (Fig. 13) that the temperature distribution law for the non-insulated case has a nonlinear character. Let's determine the maximum relative error between the partition 1 and 2. Along the z axis at $m = 1$ ($T^1 = 267.5$) and with $m = 2$ ($T^2 = 248.3$).

$$\frac{1 \_ 2}{2} 100\% = 6.5\%$$

Determine the maximum temperature deviation between partitions 2 and 3.

Determine the maximum relative error between partitions 2 and 3. Along the z axis at $m = 2$ ($T^2 = 265$) and with $m = 3$ ($T^3 = 260$).

$$\frac{2 \_ 3}{3} 100\% = 1.7\%$$

The law of temperature distribution on the face (1, 3, 5, 7) of the cube for the non-insulated case at $m = 1$ is shown in (Fig. 14).

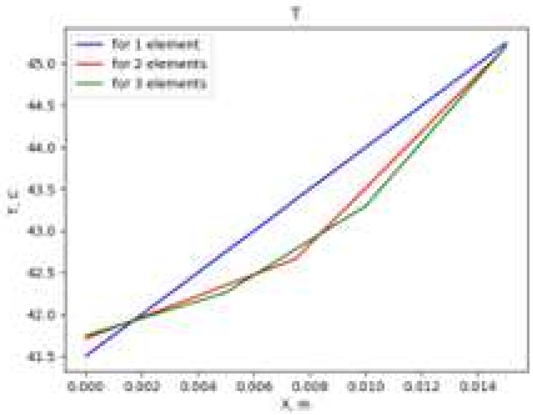

Figure 8: Temperature distribution laws for the edge (0, 4) of the cube (Fig. 7) for the non-insulated case at $m = 1$, $m = 2$, $m = 3$.

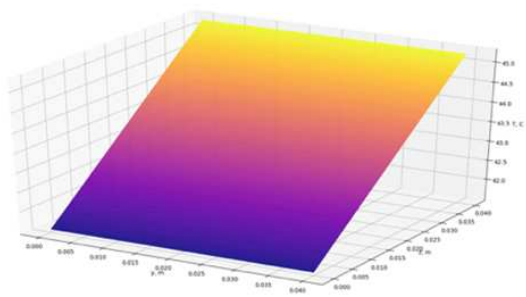

Figure 9: The law of distribution of non-temperature on the face (1, 3, 5, 7) of the cube at $m = 1$ for the non-insulated case.

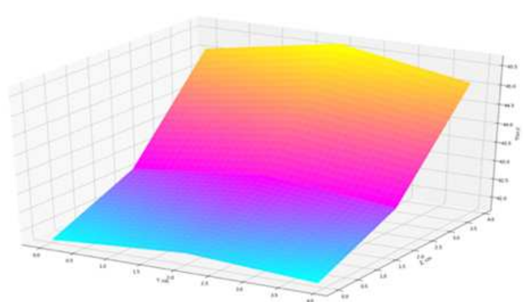

Figure 10: The law of distribution of non-temperature on the face (1, 3, 5, 7) of the cube at $m = 2$ for the non-insulated case.

The law of temperature distribution on the face (1, 3, 5, 7) of the cube for the non-insulated case at $m = 2$ is shown in (Fig. 15).

The law of temperature distribution on the edge (1, 7, 19, 25) (Fig. 8) the cube for the non-insulated case at $m = 2$ is shown in (Fig. 16).

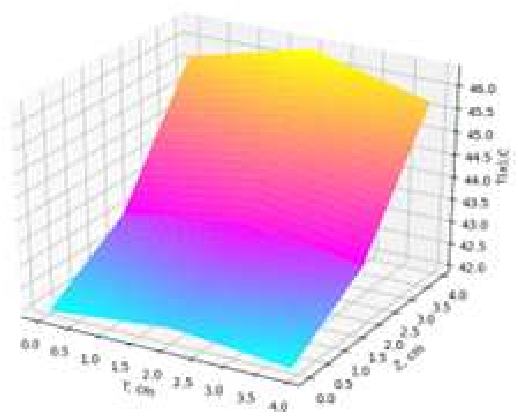

Figure 11: The law of distribution of non-temperature on the edge (1, 7, 19, 25) (Fig. 8) a cube at $m = 2$ for the non-insulated case.

5 Conclusion

In this paper, a general variational functional is obtained for determining the law of temperature distribution in the body of a rectangular parallelepiped shape, when a heat flow enters one of the faces of a rectangular parallelepiped, and heat exchange with the environment occurs on the opposite side. Nonlinear temperature approximation is used to minimize the obtained functional at discrete points. The minimization of the general functional by the temperatures set at the nodal points is carried out. At the same time, the minimization problem is reduced to solving systems of linear equations. The temperature values at the nodal points are used to estimate the temperature distribution law at any point of the body in the form of a rectangular parallelepiped.

To test the proposed approach, a comparative analysis of the laws of temperature distribution in the body of a rectangular parallelepiped shape, whose length in z is much greater than the length of the other sides, with a rod with similar geometric characteristics, is carried out. It turned out that the relative error in determining temperatures does not exceed 1.7%. This error is due to the difference in the shape and cross-sectional area of the rod and the rectangular parallelepiped perpendicular to the z axis.

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