

2-бөлім

Раздел 2

Section 2





Механика

Механика

Mechanics

IRSTI 30.19.27

DOI: <https://doi.org/10.26577/JMMCS.2023.v117.i1.05>

A.I. Iskakbayev¹ , K.S. Kutimov^{1,2*} , G.M. Yensebayeva³ ,
O. Khabidolda² 

¹Al-Farabi Kazakh National University, Almaty, Kazakhstan

²Karagandy Buketov University, Karagandy, Kazakhstan

³Korkyt-Ata Kyzylorda University, Kyzylorda, Kazakhstan

*e-mail: kiyas6@mail.ru

Modeling and research of processes for creep compliance and relaxation based on fading memory concept

Abstract. Using of rheonomic materials in engineering structures is one of the most important issues in determining strength and durability. The main task of the mechanics of a deformable solid is to simulate the processes of deformation of viscoelastic materials. Currently, there are quite well-developed theories and methods of viscoelasticity that allow to determine and describe the viscoelastic properties of materials. They differ between linear and nonlinear viscoelasticity. In the linear and nonlinear theory of viscoelasticity, such a task is reduced to finding creep and relaxation kernel. Creep and relaxation kernel are interconnected by a known integral relationship that establishes a relationship between stress, strain and time. The work is devoted to modeling of stress relaxation and strain of hereditary materials. Stress relaxation is described by a nonlinear integral equation with an Abel's kernel. New efficient method has been proposed for determining of parameters (α , δ). Bisection method is used for obtaining of parameter α . Algorithm have been developed for calculating of parameters of α and δ .

Key words: relaxation, bisection method, conditional instantaneous stress, relaxation stress.

А.И. Искакбаев¹, Қ.С. Кутимов^{1,2*}, Г.М. Еңсебаева³, Ө. Хабидолда²

¹Әл-Фараби атындағы Қазақ ұлттық университеті, Алматы қ., Қазақстан

²Е.А. Бөкетов атындағы Қарағанды университеті, Қарағанды қ., Қазақстан

³Қорқыт-Ата атындағы Қызылорда университеті, Қызылорда қ., Қазақстан

*e-mail: kiyas6@mail.ru

Жадының өшуі концепциясы негізінде жылжымалылық және релаксация процестерін модельдеу және зерттеу

Инженерлік құрылыстарда реономды материалдарды пайдалану - беріктілік пен ұзақтықты айқындаудың маңызды мәселелерінің бірі. Деформацияланатын қатты дене механикасының негізгі міндеті тұтқырға төзімді материалдарды деформациялау процестерін модельдеу болып табылады. Қазіргі уақытта тұтқыр серпімділіктің жеткілікті жақсы әзірленген теориялары мен әдістері бар, олар материалдардың тұтқырға төзімді қасиеттерін анықтауға және сипаттауға мүмкіндік береді. Оларда желілік және сызықтық емес тұтқыр серпімділігі ажыратылады. Тұтқыр серпімділіктің сызықтық және сызықтық емес торабында мұндай міндет сүргілеу және релаксация ядроларын іздеуге әкеледі. Жылжу және релаксация ядролары кернеу, деформация және уақыт арасындағы байланысты белгілейтін белгілі интегралдық қатынаспен байланысты.

Бұл жұмыс мұралы материалдардың релаксация үдерісін компьютерлік модельдеуге арналған. Кернеудің азаю үдерісі Абель ядросымен сызықты емес интегралдық теңдеуімен сипатталады. Абель өзегінің параметрлерін (α , δ) анықтаудың жаңа тиімді әдісі берілген. α параметрін анықтау үшін бисекция әдісі пайдаланылады. α және δ параметрлерін санаудың алгоритмі және сәйкес компьютерлік бағдарламасы жасалған.

Түйін сөздер: Релаксация, бисекция әдісі, шартты лездік кернеу, кернеудің азаюуы.

А.И. Искакбаев¹, К.С. Кутимов^{1,2*}, Г.М. Енсебаева³,
О. Хабидолда²

¹Казахский национальный университет имени аль-Фараби, г. Алматы, Казахстан

²Карагандинский университет имени Е.А. Букетова, г. Караганды, Казахстан

³Кызылординский университет имени Коркыт-Ата, г. Кызылорда, Казахстан

*e-mail: kiyas6@mail.ru

Моделирование и исследование процессов ползучести и релаксации на основе концепции затухающей памяти

Использование реономных материалов в инженерных сооружениях - один из важнейших вопросов определение прочности и долговечности. Основной задачей механики деформируемого твердого тела является моделирование процессов деформирования вязкоупругих материалов. В настоящее время имеются достаточно хорошо разработанные теории и методы вязкоупругости, которые позволяют определить и описать вязкоупругие свойства материалов. В них различают линейную и нелинейную вязкоупругость. В линейной и нелинейной теории вязкоупругости такая задача сводится к отысканию ядер ползучести и релаксации. Ядра ползучести и релаксации связаны между собой известным интегральным соотношением, которая устанавливает связь между напряжением, деформацией и времени.

Работа посвящена компьютерному моделированию релаксации напряжений наследственных материалов. Релаксация напряжений описывается нелинейным интегральным уравнением с ядром Абеля. Предложена новая эффективная методика определения параметров (α, δ) ядра. Для нахождения параметра α используется метод бисекции. На основе вышеуказанных теорий разработаны алгоритмы и соответствующая компьютерная программа для вычисления параметров α и δ . **Ключевые слова:** Релаксация, метод бисекции, условная мгновенная напряжения, напряжения релаксации.

1 INTRODUCTION

To determine the nature (linearity or nonlinearity) of hereditary materials deformation, a new method was proposed with the introduction of the so-called experimental rheological parameter [1, 2]:

$$k_e(t) = \frac{\varepsilon_e(t)}{\varepsilon_0^e}, \quad (1)$$

where $\varepsilon_e(t)$ is the creep strain value at time t determined experimentally; ε_0^e is conditionally instantaneous deformation at the time moment $t = 0$ determined experimentally.

According to the proposed method the values of experimental rheological parameter are calculated at different time t and stresses σ . Based on the results of the calculations, the graphs $k_e(t)$ are constructed at different stresses σ . For a physically linear material, all curves $k_e(t)$ at different stresses coincide, i.e. we have only one average curve $k_e(t)$ for all stresses. There are special curves $k_e(t)$ for a physically nonlinear material for different stresses.

Using the procedure in work [1], model (theoretical, calculated) values of a creep deformation can be determined in the following sequence:

1. Calculation of model rheological parameter values $k_m(t)$ by the formula:

$$k_m(t) = 1 + \frac{\delta}{1 - \alpha} \cdot t^{(1-\alpha)}. \quad (2)$$

2. Determination of a conditional instantaneous deformation at stresses σ :

$$\bar{\varepsilon}_0^m(\sigma) = \frac{1}{m} \sum_{i=1}^m \frac{\varepsilon_e(t_i)}{k_m(t_i)}. \quad (3)$$

3. Calculation of model values for creep deformation at stresses σ :

$$\varepsilon_m(t) = \bar{\varepsilon}_0^m(\sigma) \cdot k_m(t). \quad (4)$$

The works [3-4] determined creep processes of several hereditary materials, such as Nylon 6, Fiberglass TS8/3-250 (stretching at the angle of $\theta=0^\circ$, 45° and 90° and an asphalt concrete.

This article is a continuation of the above works, and it discusses the issues of modeling the creep and relaxation process based on the fading memory concept.

2 THE USE OF THE PROPOSED ALGORITHM AND METHODS

2.1 Polycrystalline graphite

In [5], a polycrystalline graphite was tested at the stresses of 10, 14, 18 and 22 MPa and the temperatures of $T=2000^\circ\text{C}$. The creep strain values of the polycrystalline graphite at four stresses are given in Table 1. The values of the experimental and model rheological parameter of the polycrystalline graphite obtained by formulae (1) and (2). Calculating by formula

$$\Delta k = \left| \frac{k_m(t) - k_e(t)}{k_e(t)} \right| \cdot 100\% \quad (5)$$

the maximum deviation of the calculated values for the rheological parameter of the polycrystalline graphite as per formula (5) is equal to 0.84%. Using the proposed algorithm and methods [3-4], we calculate the design values of creep strain at different stresses. By calculation according to the developed computer program, the following values of creep kernel parameters were determined: $\alpha = 0.7900$; $\delta = 0.0612$. We will determine

$$\bar{\varepsilon}_0^m = 4.02 \cdot 10^{-3} \cdot \sigma^{1.22}$$

Consequently, the creep curve for the material of the polycrystalline graphite will be described by the equation:

$$\varphi(\varepsilon) = \sigma = 90.81 \cdot \varepsilon^{0.812}$$

The difference of experimental and design values for creep strain we will obtain by the formula:

$$|\varepsilon| = |\varepsilon_e - \varepsilon_m| \quad (6)$$

The maximum difference at voltages of 10 MPa is 0.0011; 14 MPa - 0.0068; 18 MPa - 0.021; 22 MPa - 0.0259. Experimental and design values of material creep strain at stresses of 10, 14, 18 and 22 MPa are shown in Figure 1. It is obvious that the extent of coincidence for design strain with the experimental one is high.

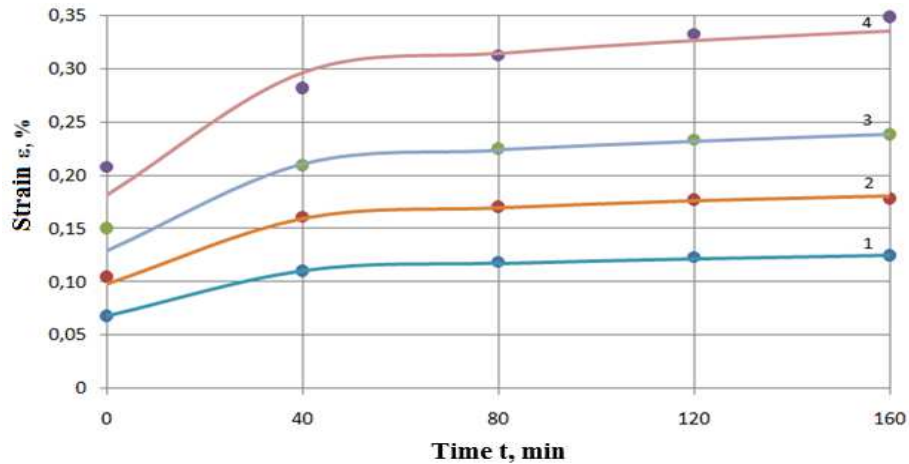


Figure 1. Creep curves of polycrystalline graphite material at different stresses: 1 – 10MPa; 2 – 14MPa; 3 – 18MPa; 4 – 22MPa.

● – experiment, – design.

2.2 Polyester polymer concrete

In the work [6], experimental studies of polyester polymer concrete were carried out at the stresses of 10, 20, 30, 40 and 50 MPa. The maximum deviation value of rheological parameter for polyester polymer concrete according to the formula (2) is equal to 0.81%. As you can see, the experimental and model rheological parameters coincide with a high accuracy. Using the proposed algorithm and methods [3-4], the following values of the creep kernel parameters are determined: $\alpha = 0.7267$; $\delta = 0.0329$.

We will determine

$$\bar{\varepsilon}_0^m = 3.779 \cdot 10^{-3} \cdot \sigma.$$

Then the creep curve for the material of polyether polymer concrete will be described by the equation

$$\varphi(\varepsilon) = \sigma = 348.9 \cdot \varepsilon$$

Experimental and design values of the creep strain of the polyester polymer concrete material at different stresses are shown in Figure 2. It is obvious that the extent of coincidence of the calculated deformations with the experimental ones is high.

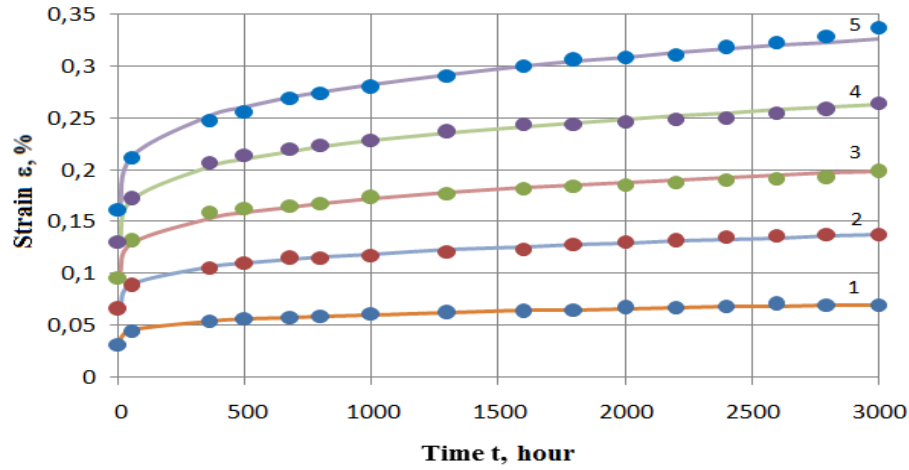


Figure 2. Creep curves of polyester polymer concrete material at different stresses: 1 – 10 MPa; 2 – 20 MPa; 3 – 30 MPa; 4 – 40 MPa; 5 – 50 MPa.

2.3 Aramid fiber Synthetical High-Strength Material

In the work [7], the aramid fiber SHSM was tested at the stresses of 330, 1000 and 1650 MPa. The experimental and model rheological parameters coincide with a high accuracy. The maximum deviation according to the formula (2) is 0.96%.

Using the proposed algorithm and methods [3-4], the following values of the creep kernel parameters are determined: $\alpha = 0.8745$; $\delta = 0.0372$.

We will determine

$$\bar{\varepsilon}_0^m = 1.1 \cdot 10^{-3} \cdot \sigma.$$

Then the creep curve for the material of aramid fiber SVM will be described by the equation

$$\varphi(\varepsilon) = \sigma = 1307.2 \cdot \varepsilon.$$

Experimental and calculated values of the creep strain for the material of aramid fiber SHSM at three stresses are shown in Figure 3.

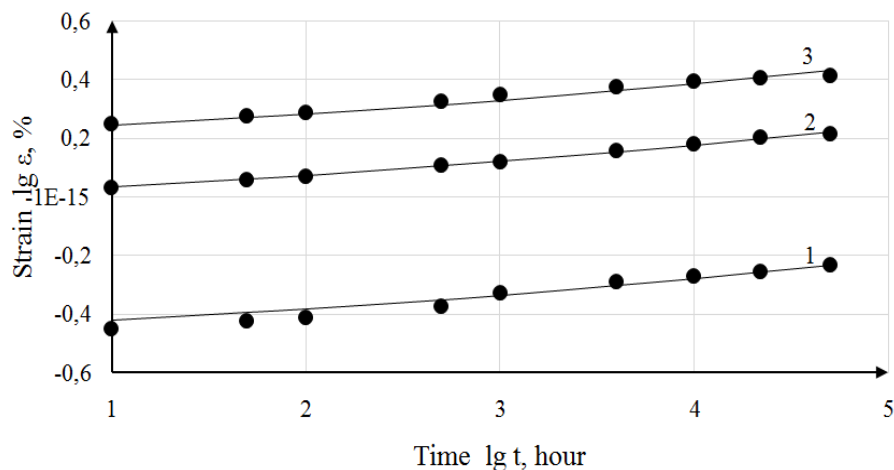


Figure 3. Creep curves of the aramid fiber material at different stresses: 1 – 330 MPa; 2 – 1000 MPa; 3 – 1650 MPa.

2.4 Resin EDT-10

In the work [7], experimental studies of resin EDT-10 were carried out at the stresses of 6.8; 13.6 and 20.4 MPa. For all stresses, the duration of the experiment was 50000 hours.

The experimental and model rheological parameters coincide with a high accuracy. The maximum deviation according to the formula (2) is 4.46%.

Using the proposed algorithm and methods [3-4], the following values of the creep kernel parameters are determined: $\alpha = 0.6105$; $\delta = 0.0219$.

We will determine

$$\bar{\varepsilon}_0^m = 3.03 \cdot 10^{-2} \cdot \sigma.$$

Then a creep curve for the material of the resin EDT-10 will be described by the equation

$$\varphi(\varepsilon) = \sigma = 33.07 \cdot \varepsilon.$$

Experimental and design values for the creep strain of the material of resin EDT-10 at three stresses are shown in Figure 4.

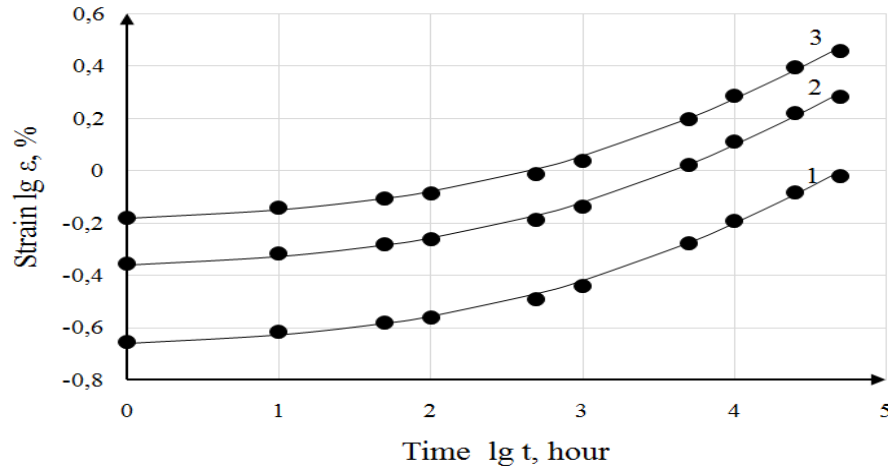


Figure 4. Creep curves of resin EDT-10 material at different stresses: 1 – 6.8 MPa; 2 – 13.6 MPa; 3 – 20.4 MPa

3 Creating Stress Relaxation Curves from Rheonomic Materials Creep Curves

If isochronous creep curves of materials are straight (see Figures 3, 6, 9, 12), then viscoelastic determining relations between stresses and deformations can be written as [8-10]:

$$\varepsilon(t, T) = \frac{1}{E}[\sigma(t, T) + \int_0^t K(t - \tau)\sigma(\tau, T)d\sigma] \quad (7)$$

where E is an elasticity modulus of a material; T is a samples test temperature, $T = const.$

Solution of the integral equation (3) will be

$$\sigma(t, T) = E[\varepsilon(t, T) - \int_0^t R(t, \tau)\varepsilon(\tau, T)d\sigma] \quad (8)$$

where $R(t, \tau)$ is the relaxation kernel.

Now we consider the kernel of type [Yu.N. Rabotnov, 1948]

$$I_\alpha(t - \tau) = \frac{1}{\Gamma(1-\alpha)}(t - \tau)^{-\alpha} \quad (9)$$

Here $\Gamma(1 - \alpha)$ is a gamma-function, $\alpha \in (0, 1)$.

The identical equation occurs [8-10]:

$$\frac{1}{1+\beta I_\alpha^*} = 1 - \beta \epsilon_\alpha^*(-\beta), \quad (10)$$

where

$$\beta = \delta \Gamma(1 - \alpha); \quad \beta > 0 \quad (11)$$

$$I_\alpha^* \cdot 1 = \int_0^t I_\alpha(t - \tau) d\tau;$$

$$\epsilon_\alpha^*(-\beta) \cdot 1 = \int_0^t \epsilon_\alpha(-\beta, t - \tau) d\tau.$$

Here $I_\alpha(t - \tau)$ is determined by the relationship (5).
 $\epsilon_\alpha(-\beta, t - \tau)$ is Rabotnov's kernel [10]:

$$\epsilon_\alpha(-\beta, t - \tau) = (t - \tau)^{-\alpha} \sum_{n=0}^{\infty} \frac{(-\beta)^n (t - \tau)^{n(1-\alpha)}}{\Gamma[(n-1)(1-\alpha)]}, \quad (12)$$

where $\Gamma(\cdot)$ is a gamma-function;
 τ is a variable of integration;
 t is time of observation.

From (6) it follows that the relaxation kernel $\beta\epsilon_\alpha(-\beta, t - \tau)$ of the integral operator $\beta\epsilon_\alpha^*(-\beta)$ corresponds to the creep kernel

$$K(t - \tau) = \frac{\beta}{\Gamma(1-\alpha)} (t - \tau)^{-\alpha} \quad (13)$$

of the integral operator βI_α^* , and the value β is determined by the relationship (7).

Now let $\varepsilon(0) = \varepsilon_0 = \text{const}$ Then from (4) we will have:

$$\sigma(t, T) = \sigma_0 \left(1 - \beta \int_0^t \epsilon_\alpha(-\beta, t - \tau) d\tau \right) = \sigma_0 (1 - \beta t^{1-\alpha} F_2). \quad (14)$$

Here $\sigma_0 = E\varepsilon_0$;

$$F_2(\alpha, x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^n}{\Gamma[(1-\alpha)(n+1)+1]} \quad (15)$$

where $x = \beta t^{1-\alpha}$ values $F_2(\alpha, x)$ tabulation [8-10].

3.1 Polycrystalline graphite

Knowing the following values of the creep kernel parameters of the material for polycrystalline graphite: $\alpha = 0.7900$; $\delta = 0.0612$. We will determine the value β by the formula (7), $\beta = 0.2668$. We will determine $\sigma(t, T)$ by the formula (10)

$$\sigma(t, T) = \sigma_0 \left(1 - 0.2668 \cdot t^{0.21} \cdot \sum_{n=0}^{\infty} \frac{(-0.2668)^n \cdot t^{0.21n}}{\Gamma(0.21 \cdot (n+1) + 1)} \right)$$

Table 1 Design values of a polycrystalline graphite relaxation stress

Time t , min	Stress σ , MPa			
	10	14	18	22
0				
40	6.0700	8.4980	10.9260	13.354
80	5.7120	7.9968	10.2816	12.5664
120	5.4980	7.6972	9.8964	12.0956
160	5.3460	7.4844	9.6228	11.7612

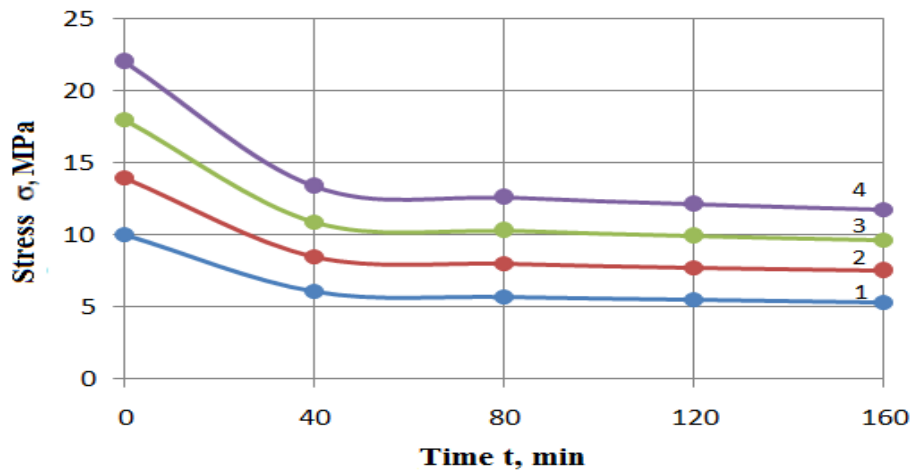


Figure 5. Stress relaxation curves of polycrystalline graphite at the strain $1 - \varepsilon_0 = 0.0681\%$; $2 - \varepsilon_0 = 0.0981\%$; $3 - \varepsilon_0 = 0.1294\%$; $4 - \varepsilon_0 = 0.1818\%$

3.2 Polyester Polymer Concrete

Knowing the following values of the creep kernel parameters of the polyester polymer concrete material: $\alpha = 0.7267$; $\delta = 0.0329$. We will determine the value of β by (7), $\beta = 0.1086$. material

Table 2. Design stress relaxation values of a polyester polymer concrete

Time t , hour	Stress σ , MPA				
	10	20	30	40	50
0					
50	7.3570	14.7140	22.0710	29.4280	36.7850
360	6.1570	12.3140	18.4710	24.6280	30.7850
500	5.9370	11.8740	17.8110	23.7480	29.6850
680	5.7270	11.4540	17.1810	22.9080	28.6350
800	5.6145	11.2290	16.8435	22.4580	28.0725
1000	5.4590	10.9180	16.3770	21.8360	27.2950
1300	5.2760	10.5520	15.8280	21.1040	26.3800
1600	5.1300	10.2600	15.3900	20.5200	25.6500
1800	5.0470	10.0940	15.1410	20.1880	25.2350
2000	4.9730	9.9460	14.9190	19.8920	24.8650
2200	4.9060	9.8120	14.7180	19.6240	24.5300
2400	4.8440	9.6880	14.5320	19.3760	24.2200
2600	4.7880	9.5760	14.3640	19.1520	23.9400
2800	4.7360	9.4720	14.2080	18.9440	23.6800
3000	4.6870	9.3740	14.0610	18.7480	23.4350

We will determine by (10)

$$\sigma(t, T) = \sigma_0 \left(1 - 0.1086 \cdot t^{0.27} \cdot \sum_{n=0}^{\infty} \frac{(-0.1086)^n \cdot t^{0.27n}}{\Gamma(0.27 \cdot (n+1) + 1)} \right)$$

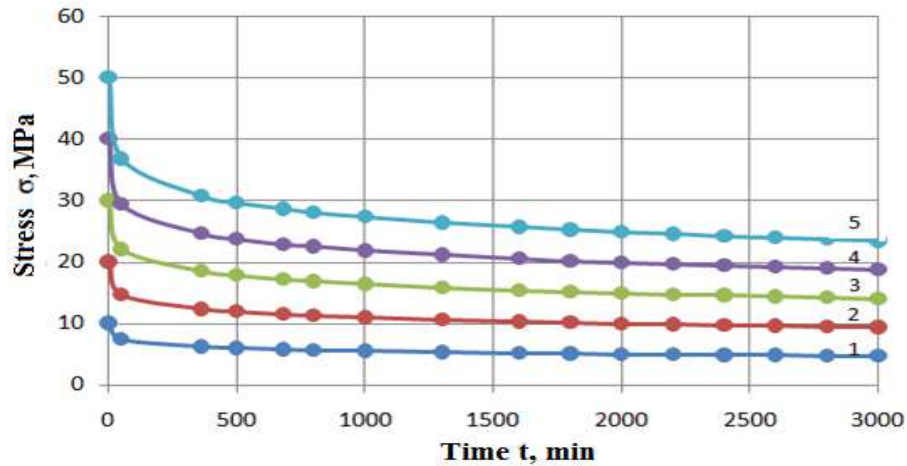


Figure 6. Stress relaxation curves of a polyester polymer concrete at the strain of
 1 – $\varepsilon_0 = 0.0338\%$; 2 – $\varepsilon_0 = 0.0662\%$; 3 – $\varepsilon_0 = 0.0958\%$; 4 – $\varepsilon_0 = 0.1269\%$; 5 – $\varepsilon_0 = 0.1573\%$

3.3 Aramid fiber SHSM

Knowing the following values of the creep kernel parameters for the aramid fiber SHSM:
 $\alpha = 0.8958$; $\delta = 0.0783$. We will determine the value of β by (7), $\beta = 0.7136$.

Table 3. Design values of stress relaxation for aramid fiber SHSM material

Time t , 10^3 hour	Stress σ , MPa		
	330	1000	1650
0	330	1000	1650
1	187.8360	569.2	939.1800
2	181.9290	551.3	909.6450
3	178.4640	540.8	892.3200
4	175.9890	533.3	879.9450
6	172.5240	522.8	862.6200
8	170.0490	515.3	850.2450
10	168.1020	509.4	840.5100
12	166.5510	504.7	832.7550
14	165.1980	500.6	825.9900
16	164.0430	497.1	820.2150
18	163.0530	494.1	815.2650
20	162.1290	491.3	810.6450
22	161.3040	488.8	806.5200
26	159.8520	484.4	799.2600
30	158.6310	480.7	793.1550
32	158.0700	479	790.3500
36	157.0470	475.9	785.2350
40	156.1560	473.2	780.7800
42	155.7270	471.9	778.6350
46	154.9350	469.5	774.6750
50	154.2420	467.4	771.2100

We will determine by (10)

$$\sigma(t, T) = \sigma_0 \left(1 - 0.7136 \cdot t^{0.104} \cdot \sum_{n=0}^{\infty} \frac{(-0.7136)^n \cdot t^{0.104n}}{\Gamma(0.104 \cdot (n+1) + 1)} \right)$$

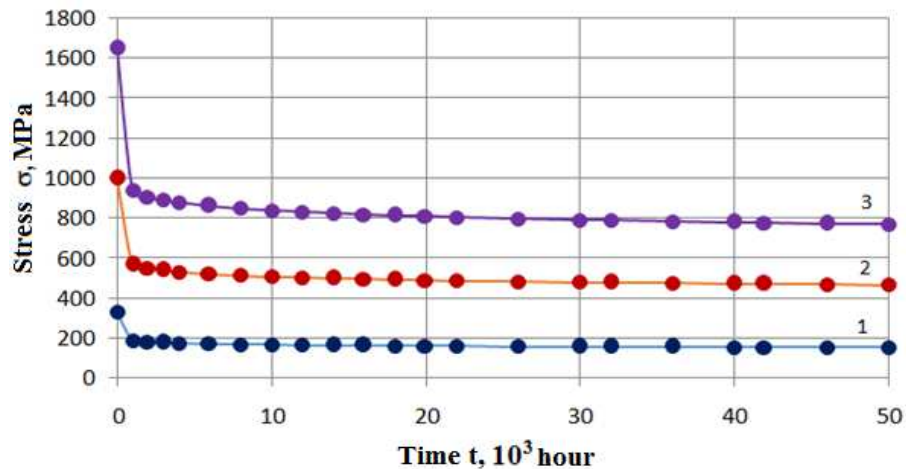


Figure 7. Aramid fiber SHSM stress relaxation curves at the strain of $1 - \varepsilon_0 = 0.2729\%$; $2 - \varepsilon_0 = 0.7705\%$; $3 - \varepsilon_0 = 1.2432\%$

3.4 Resin EDT-10

Knowing the following values of the creep kernel parameters for the resin material EDT-10: $\alpha = 0.6105$; $\delta = 0.0219$. We will determine the value of β by (7), $\beta = 0.0499$.

Table 4. Design stress relaxation values of a resin material EDT-10

Time t, hour	Stress σ , MPa		
0	6.8	13.6	20.4
1	6.4355	12.8710	19.3066
100	5.0259	10.0518	15.0776
500	4.0474	8.0947	12.1421
1000	3.5700	7.1400	10.7100
3000	2.8009	5.6018	8.4028
5000	2.4582	4.9164	7.3746
10000	2.0250	4.0501	6.0751
15000	1.7938	3.5877	5.3815
20000	1.6408	3.2817	4.9225
25000	1.5280	3.0559	4.5839
30000	1.4396	2.8791	4.3187
35000	1.3559	2.7118	4.0678
40000	1.2859	2.5718	3.8576
50000	1.2036	2.4072	3.6108

We will determine by (10)

$$\sigma(t, T) = \sigma_0 \left(1 - 0.0499 \cdot t^{0.39} \cdot \sum_{n=0}^{\infty} \frac{(-0.0499)^n \cdot t^{0.39n}}{\Gamma(0.39 \cdot (n+1) + 1)} \right)$$

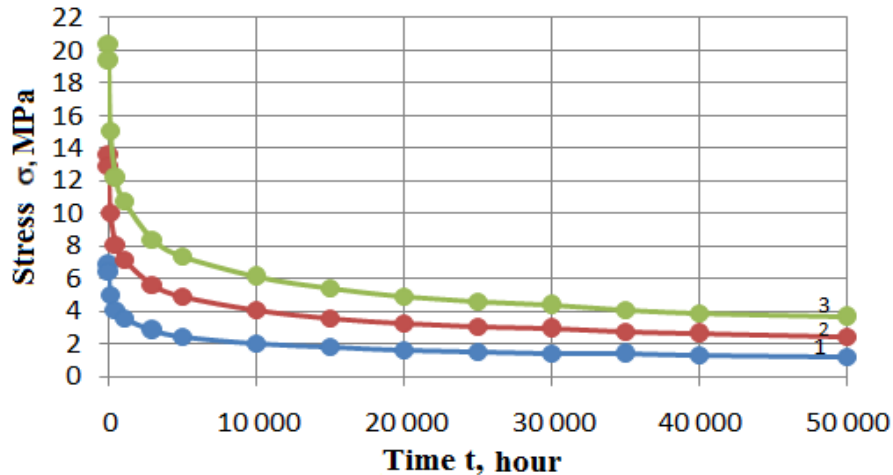


Figure 8. Stress relaxation curves of resin EDT-10 at the strain of 1 – $\varepsilon_0 = 0.2057\%$; 2- $\varepsilon_0 = 0.4113\%$; 3 – $\varepsilon_0 = 0.6170\%$

4 Conclusion

Generalizing properties of creep and stress relaxations of rheonomic materials are obtained. As mentioned above, the relaxation process cannot be considered without the creep process.

Based on this, using the results of experiments by well-known authors, stress relaxations of various rheonomic materials are calculated from the creep process. Its rheological parameters are calculated for each material. Also, according to the proposed methodology, the creep processes of these materials are calculated and modeled.

References

- [1] Cristensen R.M. "Theory of viscoelasticity: An introduction, Academic Press *New York, USA*, 1971.
- [2] Tschoegl N.N. "The phenomenological theory of linear viscoelastic behavior *An introduction, Springer-Verlag, Berlin, Germany*, 1989
- [3] Iskakbayev A., Teltayev B., Rossi C.O., Yensebayeva G. "Determination of nonlinear creep parameters for hereditary materials *Applied Sciences*, 2018, p. 1-17.
- [4] Iskakbayev A.I., Teltayev B.B., Yensebayeva G.M., Kutimov K.S. "Computer modeling of creep for hereditary materials by Abel's kernel *News of the National Academy of Sciences of the Republic of Kazakhstan. Series of Geology and technical sciences*, 6(432), 2018, p. 66-76.
- [5] Dergunov N.N., Napernik I.Kh., Rabotnov Yu.N. "Analysis of graphite behavior based on nonlinear hereditary theory *pMTF*, No.2, 1971.
- [6] Maksimov R.D., Jirgens L., Jansons J., Plume E. "Mechanical properties of polyester polymer-concrete *Mechanics of Composite Material*, Vol. 35, No.2, 1999.
- [7] Maksimov R.D., Plume E. "Long-term creep of hybrid aramid/glass-fiber-reinforced plastics *Mechanics of Composite Materials*, Vol. 37, No.4, 2001.
- [8] Rabotnov Yu.N. "Balance of the elastic medium with a aftereffect *Applied Mathematics and Mechanics*, vol. 12, issue 1, pp. 53-62, 1948.
- [9] Volterra V. "Lecons sur les fonctions de lignes *Paris. Gauthier-Villard*, 1913.
- [10] Rabotnov Yu.N. "Balance of the elastic medium with a aftereffect *Applied Mathematics and Mechanics*, vol. 12, issue 1, pp. 53-62, 1948.
- [11] Rabotnov Yu.N. "Mechanics of a deformed solid body *Moscow; Nauka*, 1988. 712 p.
- [12] Rabotnov Yu.N. "Elements of hereditary mechanics of solids *Nauka, Moscow, Russia*, 1977. (in Russ.)