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STUDYING DYNAMICS OF A CANTILEVER BAR WITH VARIABLE BENDING STIFFNESS

In this paper, there are studied the dynamic processes (free and forced oscillations) of isotropic cantilever plates in the form of an isosceles (wedge-shaped) triangle. In the study, the finite difference method has been applied using a regular one-dimensional (linear) grid. The finite-difference equations developed by the authors for point-distributed masses along the length of the wedge are presented, taking into account the linearly variable bending stiffness. On this basis, the results of studies in the form of amplitude-frequency characteristics (frequencies, dynamic forces and deflections) in the resonant and near-resonant regions have been obtained. The content of theoretical provisions and applied results can be widely used in the scientific and engineering fields and in the field of mechanics of structures.

Key words:triangular plate, numerical method, grid method, dynamic deflections and forces, variable bending stiffness, bar analogy, frequency spectrum, amplitude-frequency characteristics.

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АЙНЫМАЛЫ ИІЛУ ҚАТАҢДЫҒЫ БАР КОНСОЛЬДІ ӨЗЕКТІҢ ДИНАМИКАСЫН ЗЕРТТЕУ

Бұл жұмыста изотропты консольдық пластиналардың динамикалық процестерін (еркін және мәжбүрлі тербелістер) тең қабырғалы (сына тәрізді) үшбұрыш түрінде зерттейміз. Зерт-теу үшін кәдімгі бір өлшемді (сызықтық) торды қолдану арқылы ақырлы айырымдар әдісі қолданылды. Сызықты-айнымалы иілу қатаңдығын ескере отырып, сына ұзындығы бойын-ша нүктелік-таралған массалар үшін әзірлеген соңғы айырмашылық теңдеулері ұсынылған. Олардың негізінде резонанстық және жақын резонанстық аймақтардағы амплитудалық-жиілік сипаттамалары (жиілік, динамикалық күштер және ауытқулар) түріндегі зертте-улердің нәтижелері алынды. Теориялық ережелер мен қолданбалы нәтижелердің мазмұны ғылыми және инженерлік салаларда және құрылымдар мен конструкциялар механикасы саласында кеңінен қолданылады.

Түйін сөздер: Үшбұрышты пластина, сандық әдіс, тор әдісі, динамикалық ауытқулар мен күштер, айнымалы иілу қатаңдығы, өзектік ұқсастық, жиілік спектрі, амплитудалық-жиілік сипаттамалары.

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ИССЛЕДОВАНИЕ ДИНАМИКИ КОНСОЛЬНОГО СТЕРЖНЯ ПЕРЕМЕННОЙ ИЗГИБНОЙ ЖЕСТКОСТИ

В данной работе исследуются динамические процессы (свободные и вынужденные колебания) изотропных консольных пластин в форме равнобедренного (клиновидного) треугольника. Для исследования применен метод конечных разностей с использованием регулярной одномерной (линейной) сетки. Приведены разработанные авторами конечно-разностные уравнения для точечно-распределенных масс по длине клина с учетом линейно-переменной изгибной жесткости. На их основе получены результаты исследований в виде амплитудно-частотных характеристик (частоты, динамические усилия и прогибы) в резонансной и -около резонансной областях. Содержание теоретических положений и прикладных результатов найдет широкое применение в научной и инженерной сферах и в области механики конструкций и сооружений.

Ключевые слова: треугольная пластина, численный метод, метод сеток, динамические прогибы и усилия, переменная изгибная жесткость, стержневая аналогия, спектр частот, амплитудно-частотные характеристики.

1 Introduction

Thin triangular plates (two-dimensional mechanical systems) are widely used in various branches of technology in the form of load-bearing elements of various designs.

The theory of calculations of triangular plates was previously widely studied by scientists [1-3]. At the same time, the calculation of cantilever triangular plates still creates some technical problems due to the presence of edges free from fastening, as well as due to the fact that a zone of zero bending stiffness is formed at the apex of the triangle, which leads to mathematical uncertainties.

Works [4, 5] study the bending of elastic isotropic isosceles triangular plates with the following boundary conditions: hinged or rigid edges; they were calculated using the finite element method with the use of approximating functions. As an example, a calculation is given for the bending of a plate in the form of a right-angled triangle with hinged support along the contour.

Studies [6, 7] consider the static calculation of triangular plates with hinged edges by the Ritz method for bending, give the corresponding formulas for the analytical determining of the coordinate functions coefficients used to calculate the bending deflections, and perform the calculation for bending under the action of a concentrated force.

The considered triangular plates of regular and irregular shape with different boundary conditions are widely used in the form of elements of buildings and structures, machines and mechanisms. In [8-10], several exact solutions are given for triangular plates with homogeneous boundary conditions (either hinged support along the entire contour, or rigid pinching). In other cases, variational methods are used [11, 12], such as the finite difference method [13], as well as the finite element method [14]. Methods of solving dynamic problems for triangular plates are presented in the reference manual [15]. In works [16, 17], a method of solving triangular plates in single trigonometric series is considered only in cases of continuous support along the contour. Paper [18] outlines the theoretical foundations of an original engineering method of solving problems of the technical theory of plates: the method of interpolation by the shape factor. The method is based on the use of isoperimetric properties and patterns of change in the shape factor of the area under geometric transformations, which also allows calculating triangular plates. In [19, 20], the possibility of applying the method of interpolation by the shape factor to the calculation of triangular plates from an orthotropic material is considered.

In [21], a geometrically nonlinear problem of bending a cantilever bar is considered; an analytical solution is applied for the case of the Cosserat-Timoshenko bar (taking into account bending, shear, and tension rigidities), in contrast to the previously known Kirchhoff models. The given results can be used in verification tests of various software systems.

In the other works, there are considered the problems of various structures dynamics, such as plates, bar systems, including the problems of dynamic stability in eccentrically compressed metal rods under the action of stationary and moving dynamic loads [22, 23], composite bars [24], bar dynamics in case of longitudinal impact, taking into account the curvature of the bar and the eccentricity of the load.

2 Theoretical provisions and methods of calculation

Cantilever bars with bending stiffness that varies linearly (in the form of a triangle), the so-called "wedge-shaped"bars, are widely used in various branches of technology: mechanical engineering, construction, aircraft shipbuilding, transport, energy, etc. In this regard, their calculations for strength, stiffness, stability, especially under various dynamic effects, are of current importance to this day. Here there is considered a cantilever bar with variable bending stiffness under dynamic load $P(t)$ (Fig. 1a).
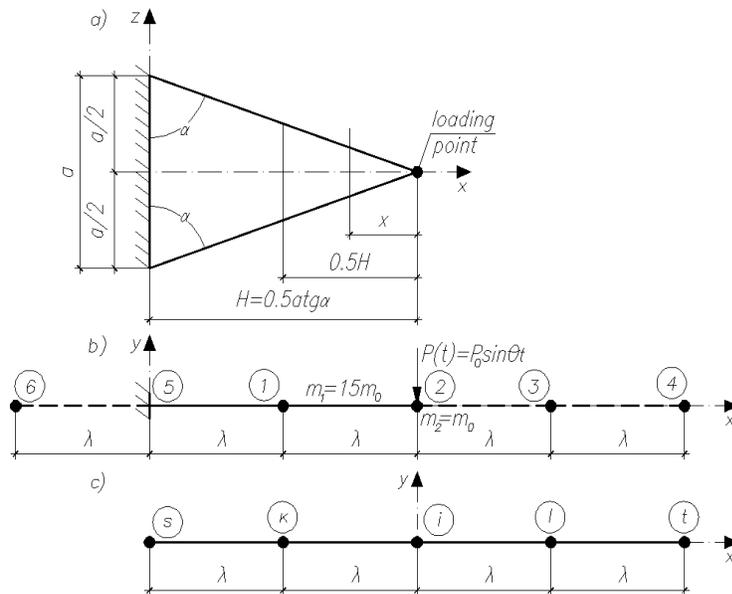
image 1: Towards the calculation of the cantilever bar of variable bending stiffness a) - given pattern; b) - calculation pattern of the finite difference method; c) - linear grid pattern

$$m_2 = m_0 = \frac{0,015625(a^2 tg\alpha)h\gamma}{g}; m_1 = \frac{0,2344(a^2 tg\alpha)h\gamma}{g} = 15m_0$$

$$\begin{cases} EJ_0 = \frac{Eah^3}{12} = 0,0833ah^3 \\ EJ_5 = \frac{0+0,75EJ_0}{12} = 0,075EJ_0 \\ EJ_1 = \frac{(0,75+0,25)EJ_0}{2} = 0,5EJ_0 \\ EJ_2 = \frac{(0,25+0)EJ_0}{2} = 0,125EJ_0 \end{cases}$$

In a particular case, the dynamic load $P(x)$ can be vibrational, i.e. $P(t) = P\sin\theta t$.

Here: E is the modulus of elasticity of the material, h is the thickness (rod height); γ is the volumetric weight of the material; g is acceleration of gravity; P_0 is the amplitude value of the dynamic load, t is time factor, θ is the circular frequency of the disturbing dynamic force.

The aim of the study is to solve a dynamic problem: to determine the frequency spectrum and forms of natural (free) oscillations without taking into account deformation, to identify amplitude-frequency characteristics, to determine the dynamic forces of inertia, displacements, internal forces, to study resonance phenomena.

The research method is the numerical method of finite differences (FDM) using a regular "linear" grid [25-27]. To illustrate the main theoretical provisions and applied results, a small "density" of the grid ($n = 2$) was adopted (Fig. 1b). If it is necessary to increase the accuracy of the final results obtained, in the future, the researcher can increase the "density" of the grid and apply the appropriate electronic computer technology.

The initial differential equation of dynamic processes of bars with variable bending stiffness has the form [28]:

$$[EJ(x)y''(x)]'' - \omega^2 m(x)y(x) = q(t), \tag{1}$$

where $y = y(x)$ is the transverse (bending) movement of the beam axis; $q(t)$ is the given dynamic force (load); ω is the circular frequency of the beam free oscillations; $m(x)$ is the value-variable mass along the beam length.

The considered system (beam) dynamics will be studied in two stages: 1) free oscillations; 2) forced oscillations.

3 Free oscillations (without taking into account damping)

In this case the right part of equation (1) transforms into zero, since $q(t) \equiv 0$.

To apply the FDM, there is used a "linear" grid that divides the length of the beam "H" into two equal parts ($n = 2$), with a grid step $\lambda = 0,5H$.

Then equation (1) for the i-th grid node (Fig. 1c) in finite differences (taking into account the bar variable bending stiffness) will take the form (for free oscillations):

$$d_k y_s - 2(d_k + d_i)y_k + (d_k + 4d_i + d_\ell - \chi_i)y_i - 2(d_i + d_\ell)y_\ell + d_\ell y_t = 0, \tag{2}$$

where $d_i = J_i/J_0, d_. = J_./J_0, d_\ell = J_\ell/J_0$, $J_0 = 0,083at^3$ is the linear (initial) value of the beam inertia moment (at node "5";

$$\chi_i = \chi_0\frac{m_i}{m_0}; \chi_0 = \frac{m_0\omega^2}{EJ_0}\lambda^4 \tag{3}$$

is the frequency parameter for free oscillations.

There is obtained finite-difference equation (2) for the analysis nodes of the grid ($i = 1, 2$) (Fig. 1b), and a result a system of linear algebraic equations (SLAE) of the 2nd order. By solving it, there are determined the χ_i and χ_0 values. Then, from formula (3) there is calculated the ω_i value:

$$\omega_i = \frac{\sqrt{\chi_i}}{\lambda^2} \cdot \sqrt{\frac{E J_0}{m_i}} = \frac{n^2 \sqrt{\chi_i}}{\ell^2} \cdot \sqrt{\frac{E J_0}{m_i}} = \frac{\sqrt{\chi_0} n^2}{\ell^2} \cdot \sqrt{\frac{E J_0}{m_0}}. \tag{4}$$

When writing equation (2), the deflections of the contour nodes of the "linear"grid will also be "captured nodes (3, 4, 6); their values will be excluded from the boundary conditions at the ends of the bar (at nodes 5, 2):

$$y_6 = y_1; \quad y_3 = 2y_2 - y_1; \quad y_4 = 4y_2 - 2y_1. \tag{5}$$

Taking into account expressions (5), there is obtained corresponding resolving equations of the FDM:

a) node 1 ($d_5 = J_5/J_0 = 0, 375 J_0; d_1 = J_1/J_0 = 0, 5; d_2 = J_2/J_0 = 0, 125 J_0; \chi_1 = 15\chi_0$);

$$(0, 375) y_1 + (1 + 4 \cdot 0, 5 + 0, 125 - 15\chi_0) y_1 - 2 (0, 5 + 0, 125_0) y_2 + 0, 125 (2y_2 - y_1) = 0;$$

$$(3, 375 - 15\chi_0) y_1 - y_2 = 0. \tag{6}$$

b) node 2 ($d_1 = 0, 5; d_2 = 0, 125; d_3 = 0, 0; \chi_2 = \chi_0$);

$$-2 (0, 5 + 0, 125) y_1 + (0, 5 + 4 \cdot 0, 125 + 0, 0 - \chi_0) y_2 - 2 \cdot 0, 125 (2y_2 - y_1) = 0;$$

$$-0, 75 y_1 + (0, 5 - \chi_0) y_2 = 0. \tag{7}$$

Two equations (6) and (7) are brought into a single SLAE:

$$\begin{cases} (3, 375 - 15\chi_0) y_1 - y_2 = 0; \\ -0, 75 y_1 + (0, 5 - \chi_0) y_2 = 0. \end{cases} \tag{8}$$

According to (8), there is made a characteristic equation to determine the spectrum of free oscillations frequencies (in the form of the 2nd order determinant):

$$D = \begin{vmatrix} (3, 375 - 15\chi_0) & -1, 0 \\ -0, 75 & (0, 5 - \chi_0) \end{vmatrix} = 0. \tag{9}$$

By opening determinant (9) there are determined $\chi_{0,1}; \ \chi_{0,2}$:

$$\chi_{0,1} = 0, 606; \quad \chi_{0,2} = 0, 119. \tag{10}$$

Based on (10) and according to formula (4), there are calculated:

$$\omega_1 = \frac{1,38}{\ell^2}\sqrt{\frac{EJ_0}{m_0}}; \quad \omega_2 = \frac{3,114}{\ell^2}\sqrt{\frac{EJ_0}{m_0}}. \tag{11}$$

In work [28] there is given the result $\omega_1^* = \frac{1,58}{\ell^2}\sqrt{\frac{EJ_0}{m_0}}$ (the deflection from the authors' result makes 12.7%). This error can be reduced by increasing the grid density (by taking $n > 2$).

According to (10) and system of equations (8), there are determined the ratios between the amplitude displacements with the principal forms of the free oscillations:

a) the 1st principal form $\chi_{0,1} = 0,606$

$$-5,715 - \rho_{21} = 0; \rho_{21} = \frac{y_{21}}{y_{11}} = -5,715.$$

5) the 2nd principal form $\chi_{0,2} = 0,119$

$$1,59 - \rho_{22} = 0; \rho_{22} = \frac{y_{22}}{y_{12}} = 1,59.$$

Fig. 2 shows the principal forms of the free oscillations in the form of "standing" waves.
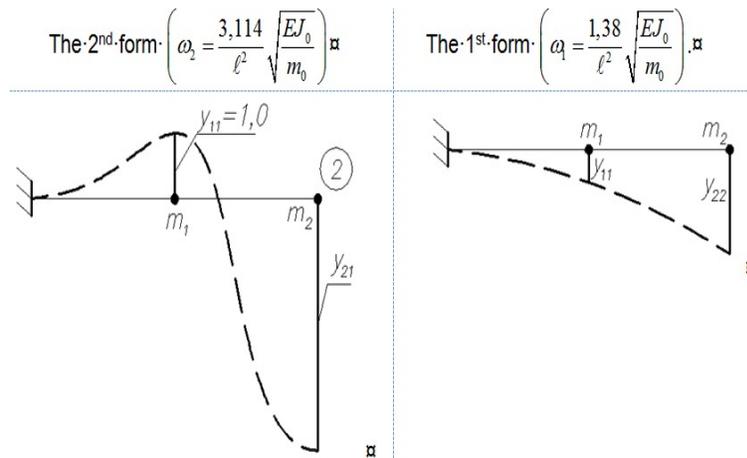
image 2: Principal forms of free oscillations

4 Forced oscillations under the action of the harmonic vibration load

The differential equation of forced oscillations of bars with variable bending stiffness has form (1), while it is necessary to take ($\omega = \theta$) (θ is the circular frequency of forced vibrations (see [28]). The concentrated load $P(t)$ can be written in terms of the equivalent load $q(t)$ (Fig. 3).

According to Figure 3a,b there are the following:
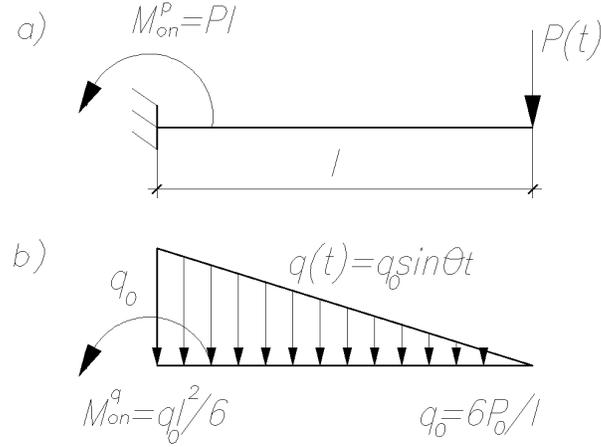
image 3: Substitution of load types

$$\left.\begin{array}{l} M_{op}^p = P(t) \cdot \ell \\ M_{op}^p = q_0 \ell^2/6 \end{array}\right\}. \tag{12}$$

By equating the left and the right parts of (12), there is obtained: $P(t)\ell = q_0\ell^2/6$; from here

$$q_0 = \frac{6P(t)}{\ell}; \tag{13}$$

while:

$$P(t) = P_0 \sin\theta t, \tag{14}$$

where $_0$ is the amplitude of external load.

Equation (1) is written down in the finite differences based on expression (2) with taking into account that

$$\chi_0 = \frac{m_i\theta^2}{m_i E J_0}\lambda^4 :$$

$$d_k y_s - 2(d_k + d_i)y_k + (d_k + 4d_i + d_\ell - \chi_i)y_i - 2(d_i + d_\ell)y_\ell + d_\ell y_t = \left(\frac{q_0\alpha\lambda^4}{EJ_0}\right), \tag{15}$$

where

$$\chi_i = \frac{m_i\theta^2\lambda^4}{EJ_0}; \quad \alpha_i = \frac{q_i}{q_0}; \quad \chi_0 = \frac{m_0\omega^2\lambda^4}{EJ_0}; \quad \chi_i = \chi_0\frac{m_i}{m_0};$$

$$q_0 = \frac{6P}{\ell}; \quad \theta = \beta\omega_1,$$

here $\beta = 0,5; \ 1,0; \ 1,5; \ 2,0$ is the coefficient for studying the change of amplitude-frequency characteristics.

$$\theta = \beta\frac{1,38}{\ell^2}\sqrt{\frac{EJ_0}{m_0}}; \quad \chi_i = \gamma_i m_0\beta = \theta/\omega_1; \quad \gamma_i = m_i/m_0;$$

$$\chi_i = \frac{1,9044\gamma_i\beta^2}{n^4}. \tag{16}$$

5 Results and discussion

According to Figure 1b: $\gamma_1 = 15; \gamma_2 = 1,0; \ \alpha_1 = 0,5q_0; \alpha_2 = 0,125q_0; \ (n = 2)$ is the grid density.

According to (16), with $n = 2$:

$$\chi_i = 0,11903\gamma_i\beta^2, i = 2. \tag{17}$$

There are obtained finite-difference equations (15) (taking into account expression (17) for the analysis nodes of the grid $i = 1, 2$ (Fig. 1b):

a) node 1 ($d_5 = 0,375; \ d_1 = 0,5; \ d_2 = 0,125; \ \gamma_1 = 15; \ \alpha_1 = 0,5$);

$$0,375y_1 + (1 + 4\cdot0,5 + 0,125 - 1,78545\cdot\beta^2)y_1 - 2(0,5 + 0,125)y_2 + 0,125(2y_2 - y_1) = 0,5q_0\lambda^4$$

or

$$y_1(3,375 - 1,78545\cdot\beta^2) + y_2(-1,0) = \frac{0,5q_0\lambda^4}{EJ_0}. \tag{18}$$

b) node 2 ($d_1 = 0,5; \ d_2 = 0,125; \ d_3 = 0,0; \ \gamma_1 = 1,0; \ \alpha_1 = 0,125$)

$$-2(0,5 + 0,125y_1) + (0,5 + 4\cdot0,125 + 0,0 - 0,11903\cdot\beta^2)y_2 -$$
$$-2\cdot0,125(0,5 + 0,125)y_2(2y_2 - y_1) = \frac{0,125q_0\lambda^4}{EJ_0}$$

$$y_1(-0,75) + y_2(0,5 - 0,11903\cdot\beta^2) = \frac{0,125q_0\lambda^4}{EJ_0}. \tag{19}$$

When bringing equations (18), (19) into a single SLAE and accepting $\lambda = \ell/2$ as a grid step, there is obtained along the X axis:

$$\begin{cases} (3,375 - 1,78545 \cdot \beta^2)y_1 - y_2 = \frac{0,03125q_0\lambda^4}{EJ_0} \\ -0,75y_1 + (0,5 - 0,11903\beta^2) = \frac{0,0078125q_0\lambda^4}{EJ_0} \end{cases} \quad (20)$$

System (20) is solved with the values of $(\beta = 0,5; \ 1,0; \ 1,5; \ 2,0)$:
Then there are presented the results of calculations for the option of $\beta = 0,5$.

$$\begin{cases} (2,93y_1 - y_2 = \frac{0,03125q_0\lambda^4}{EJ_0} \\ -0,75y_1 + 0,47y_2 = \frac{0,0078125q_0\lambda^4}{EJ_0} \end{cases} \quad (21)$$

$$\theta_1 = 0,5\omega_1 = \frac{0,69}{\ell^2}\sqrt{\frac{EJ_0}{m_0}}.$$

a) there are determined displacements from the solution of equation (21):

$$y_1 = \frac{0,036q_0\ell^4}{EJ_0}; y_2 = \frac{0,074q_0\ell^4}{EJ_0}.$$

b) there are calculated inertia forces according to [29]:

$$J_i^* = \frac{y_i}{m\theta^2}, (\theta_1 = 0,5\omega_1). \quad (22)$$

According to formula (20), there are the following:

$$J_1^* = \frac{y_1 m_0}{1,5m_1(1,5\omega_1)^2} = \frac{0,0036}{0,4761 \cdot 15} = 5 \cdot 10^4 q;$$

$$J_2^* = \frac{y_2 m_0}{m_1(0,5\omega_1)^2} = \frac{0,0074}{0,4761 \cdot 1} = 155,43 \cdot 10^4 q, \omega_1 = \frac{1,38}{\ell^2}\sqrt{\frac{EJ_0}{m_0}}.$$

There are calculated the ordinates of the dynamic moment curve (M_g) by the finite difference method (according to [26]):

$$M_i = EJ_i\frac{d^2x}{dx_i} = \frac{y_k - 2y_i + y_\ell}{\lambda^2} = EJ_i. \quad (23)$$

According to (23):
$M_2 = 0$ (the cantilever end);

$$M_1 = q\ell^2(y_2 - 2y_1)0,5 = 2q\ell^2(0,074 - 2 \cdot 0,036) = (40 \cdot 10^{-4})q\ell^2;$$
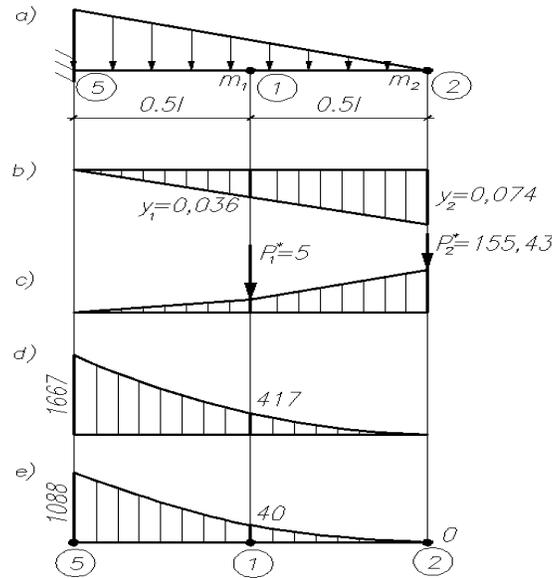
image 4: Towards calculating a cantilever triangular bar a) – calculation pattern; b), c) – deflection curve $\left(y_i \cdot \frac{EJ_0}{ql^4}\right)$ and inertia forces $\left(\frac{J_i^* \cdot 10^4}{q}\right)$; d) – static bending moment curve $(M_i^{q0} \cdot 10^4 (ql^2))$; e) – dynamic moment curve $\left(M_d \cdot \frac{10^4}{ql^2}\right)$

$$M_5 = q\ell^2 40,375(2y_1) = (0,108)q\ell^2.$$

According to Fig. 3: $M_{\max} = M_5 = 720 \cdot 10^{-4}q\ell^2 = 720 \cdot 10^{-4} \cdot 6p_0\ell = 0,432p_0\ell$

Table 1 shows the results of calculating amplitude-frequency characteristics of the considered structure (Fig. 1a) with changing the $\beta = \theta/\omega_1$ ratios.

Table 1 – Results of calculations depending on the β parameter changes

$\beta = \theta_i/\omega_1$	θ_i	Dynamic displacements		Inertia forces		Dynamic moments	
		y_1	y_2	J_1^*	J_2^*	M_1	M_5
0,5	$\frac{0,69}{\ell^2}\sqrt{\frac{EJ_0}{m_0}}$	$0,036\frac{q\ell^4}{EJ_0}$	$0,074\frac{q\ell^4}{EJ_0}$	$0,5 \cdot 10^{-4}q$	$155 \cdot 10^{-4}q$	$40 \cdot 10^{-4}q\ell^2$	$-1080 \cdot 10^{-4}q\ell^2$
1,0	$\frac{1,38}{\ell^2}\sqrt{\frac{EJ_0}{m_0}}$	$-0,136\frac{q\ell^4}{EJ_0}$	$-0,248\frac{q\ell^4}{EJ_0}$	$-47,6 \cdot 10^{-4}q$	$-1302 \cdot 10^{-4}q$	$480 \cdot 10^{-4}q\ell^2$	$-4080 \cdot 10^{-4}q\ell^2$
1,5	$\frac{2,07}{\ell^2}\sqrt{\frac{EJ_0}{m_0}}$	$-0,0106\frac{q\ell^4}{D}$	$-0,0141\frac{q\ell^4}{EJ_0}$	$-1,65 \cdot 10^{-4}q$	$32,9 \cdot 10^{-4}q$	$700 \cdot 10^{-4}q\ell^2$	$-318 \cdot 10^{-4}q\ell^2$
2,0	$\frac{2,76}{\ell^2}\sqrt{\frac{EJ_0}{m_0}}$	$-0,01407\frac{q\ell^4}{D}$	$-0,0169\frac{q\ell^4}{EJ_0}$	$-1,22 \cdot 10^{-4}q$	$-22 \cdot 10^{-4}q$	$112 \cdot 10^{-4}q\ell^2$	$-421 \cdot 10^{-4}q\ell^2$

According to the data of the Table, there are built graphic dependences: $y_i = f(\beta), (i = 1, 2)$ (Fig. 5); $M_i = f(\beta), (i = 1, 5)$ (Fig. 6).
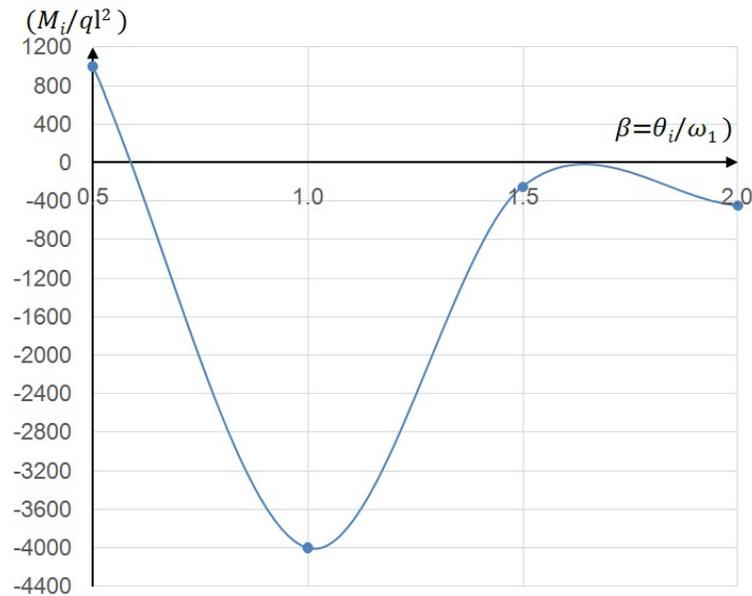
image 5: Deflection at the beam nodes dependence on changing disturbing force θ_i
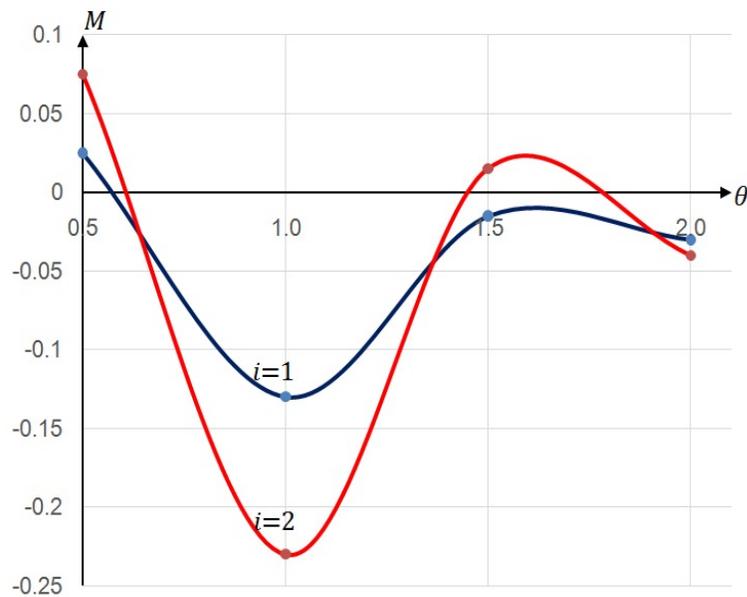
image 6: Dynamic moment M_i dependence on changing frequency θ_i

According to the Table, it can be seen that in the case of resonance, dynamic displacements and bending moments approach their maxima.

Thus, the use of a bar analogy (a cantilever bar of variable bending stiffness allowed applying the finite difference method based on a linear grid, which greatly simplified studying a cantilever triangular plate bypassing the problem of free edges and the presence of an acute angle at the end of the cantilever; at the same time, the results obtained meet the requirements

engineering precision.

6 Conclusions

1. In this work, dynamic processes (free and forced vibrations) of a cantilever beam with variable bending stiffness (wedge-shaped cantilever beam) have been studied.

2. As a research method, the finite difference method (FDM) has been selected using a regular linear grid; this method makes it possible to obtain the necessary results in the final form (in numerical values of a discrete type).

3. To illustrate the above theoretical provisions and the method of numerical calculation, a grid with low density has been used (the number of divisions along the length of the rod has been $n = 2$); in this case, in order to obtain more accurate results in the future, a grid with a large density can be used, i.e. $n > 2$, followed by the use of electronic computers. The following results have been obtained: the frequency spectrum of natural vibrations of the beam and their main forms with two conditional concentrated masses $m_1 m_2$; dynamic displacements, inertial forces and ordinates of the dynamic diagram of moments at the frequency of the disturbing force $\theta_1 = 0, 5\omega_1$ (ω_1 is the fundamental tone of natural oscillations).

4. The following research work has been carried out: there have been obtained analytical and graphical dependences of dynamic displacements, inertial forces, ordinates of the dynamic diagram of moments depending on the frequency ratios $(\beta = \theta_i/\omega_1), i = 0, 5; 1, 0; 1, 5; 2, 0)$; at this, the corresponding resonant characteristics (at $\beta^* = 1, 0$) have been revealed (Figs. 5, 6). In the course of the study, the following has been established: a) at $\theta_1 = 0, 5\omega_1$, the ordinates of the dynamic moment diagram (Fig. 4e) are slightly less than the static diagram (Fig. 4d); b) the graphs $M_i = f(\beta_i) y_i = f(\beta_i)$, (Figs. 5, 6) clearly show resonance phenomena (at $\beta^* = 1, 0$).

5. The theoretical provisions and applied results presented in this paper can be used both in studies in the field of mechanics of a deformable solid body and in designing new progressive buildings and structures, machines, elements of aircraft, ships, etc.

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