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DEVELOPMENT OF A SOFTWARE MODULE FOR MODELING DRILL STRING DISPLACEMENTS

In this work, a software module for calculating drill string displacements taking into account the effect of a drilling fluid flow, external loads and the intermittent contact of the drill string with the borehole wall is developed. A generalized nonlinear mathematical model of the drill string spatial lateral vibrations underlies the module. The V.V. Novozhilov nonlinear elasticity theory and the Ostrogradsky-Hamilton variation principle were used to derive the mathematical model. To create the software module, the universal Wolfram Language with integrated computational intelligence is utilized. The functions for performing 2D visualization of the drill string spatial displacements, constructing phase portraits of the solution and conducting the comparative analysis of the obtained numerical results are included into the module. The developed module allows predicting the dynamics of the drill string before the beginning of well drilling due to the possibility of presetting the parameters of the drilling system and accounting for the environmental factors in the process of modeling for ensuring quick, safe and efficient exploration and production of natural resources.
Key words: mathematical model, nonlinear, software module, vibration, drill string.

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Бұрғылау бағанасының орнын ауыстыруын модельдеуге арналған бағдарламалық модульді әзірлеу

Бұл жұмыста жуу сұйықтығының ағынының, сыртқы жүктемелердің және бұрғылау бағанасының ұңғыма қабырғаларымен жанасу әрекетінің әсерін ескере отырып, бұрғылау бағанасының орнын ауыстыруын есептеуге арналған бағдарламалық модуль әзірленген. Модуль бұрғылау бағанасының кеңістіктік көлденең тербелістерінің жалпылама сызықты емес математикалық моделіне негізделген. Математикалық модельді құру кезінде В.В. Новожиловтың серпімділіктің сызықтық емес теориясы мен Остроградский-Гамильтонның вариациялық принципі қолданылды. Бағдарламалық модуль жасау үшін кірістірілген есептеу интеллектісі бар әмбебап Wolfram Language қолданылады. Модуль бұрғылау бағанасының кеңістіктік қозғалыстарының 2D визуализациясын орындау, шешімнің фазалық портреттерін құру және алынған сандық нәтижелердің салыстырмалы талдауын жүргізу функцияларын қамтиды. Пайдалы қазбаларды тез, қауіпсіз және тиімді барлау мен өндіруді қамтамасыз ету үшін әзірленген модуль бұрғылау жүйесінің параметрлерін алдын ала орнату мүмкіндігіне байланысты және модельдеу процесінде қоршаған орта факторларын ескере отырып, бұрғылаудың нақты басталуына дейін бұрғылау бағанасының динамикасын болжауға мүмкіндік береді.
Түйін сөздер: математикалық модель, сызықты емес, бағдарламалық модуль, тербеліс, бұрғылау бағанасы.

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Разработка программного модуля для моделирования перемещений бурильной колонны

В данной работе разрабатывается программный модуль для расчета перемещений бурильной колонны с учетом влияния потока промывочной жидкости, внешних нагрузок и контактного взаимодействия бурильной колонны со стенками скважины. В основе модуля лежит обобщенная нелинейная математическая модель пространственных поперечных колебаний бурильной колонны. При построении математической модели использованы нелинейная теория упругости В.В. Новожилова и вариационный принцип Остроградского-Гамильтона. Для создания программного модуля используется универсальный язык Wolfram Language со встроенным вычислительным интеллектом. Модуль включает функции для выполнения 2D визуализации пространственных перемещений бурильной колонны, построения фазовых портретов решения и проведения сравнительного анализа получаемых численных результатов. Разработанный модуль позволяет прогнозировать динамику бурильной колонны до непосредственного начала бурения скважин за счет возможности предварительной настройки параметров буровой системы и учету факторов окружающей среды в процессе моделирования для обеспечения быстрой, безопасной и эффективной разведки и добычи полезных ископаемых.
Ключевые слова: математическая модель, нелинейный, программный модуль, колебания, бурильная колонна.

1 Introduction

The development in the mining industry with the high current importance of using oil and gas depends on the speed, quality and safety of the work performed. To improve the quality and efficiency of well drilling requires a comprehensive, scientifically based multi-parameter study of technological processes. In particular, the studying of the influence of drilling fluids and the use of gas flow as a circulating medium during drilling is under scientific and practical interests.

In [1], the modeling of transient response of vibrations occurring in drilling by the Finite element model and the comparative analysis of the drill string vibration stress in mud and gas at different depths are carried out. It is noted that the stress changes are higher with using the gas than the drilling fluid. Ytrehus J. et al. [2] experimentally studied the impact of different drilling fluids on mechanical friction. In the work [3], a nonlinear dynamic model of the drill string system taking into account the influence of fluid-structure interaction and the effect of support stiffness was developed. The authors in [4] considered the dynamics of a drill string in a deviated well, described by a nonlinear dynamic model with four degrees of freedom, taking into account the lateral and torsional deformation of the drill string, fluid damping effects, and the stick-slip phenomenon. Ma Y. et al. [5] created a fluid–solid coupling model of the drilling system, simulating the dynamics of the drilling system with underlining the effects of a drilling fluid. In [6], the authors established an experimental setup and investigated the mass effect of the internal and external drilling fluids through the batch of experimental and numerical studies. The authors derived the additional mass coefficient for each of the considered fluids and revealed that the drilling fluid reduces the drill string vibration frequency. Wang B. et al. [7] studied the impact of an annular gas-liquid flow on the drill string dynamic characteristics. The authors derived a fluid-structure coupled mathematical model of the two-phase flow and drill string in the annulus by using the Galerkin finite element method and revealed that the drilling fluid and natural gas reduce the natural frequency of the drill string and increase the mode shape vibration amplitude. It is mentioned that the influence of the drilling fluid is greater in comparison with natural gas. The article [8] examines a drilling system consisted of drill pipes and drill collars with two different cross-section areas under gas-structure interaction, studies its vibration characteristics and the combined effects of multiple factors.

According to recent data, mining companies intend to completely transfer their resources to automating production by 2025 [9]. For this reason, there is a great demand for software, the use of which would make it possible to achieve the objectives posed to the mining industry with maximum benefit and minimum expenses. Amongst the most popular software packages oriented to the application in the mining industry, one can highlight Surfer, which is actively used by geologists, geophysicists and hydrologists for 3D visualization, contour and surface modeling; GEOVIA Surpac supporting the management of exploration projects, as well as open pit and underground operations; Datamine Discover Suite designed for visualization, analysis and mapping of 2D and 3D geoscience data; RockWorks designed to visualize surface and sub-surface data; Vulcan used for borehole modeling and allowing the assessment of mineral deposits. It is also worth noting the Petrel E&P Software Platform, which is widely used in the oil industry from exploration to the production process. However, despite an extensive study of the well drilling process, the mining industry still needs significant improvement of the drilling safety level and prevention of possible accidents requiring further in-depth study of the problems of vibrations, stability and strength of the drilling equipment using modern information technologies that allow conducting the comprehensive analysis of existing problems at higher level. In this regard, there is a need to develop multifunctional software modules that allows modeling the dynamics of drill strings applied in the mining industry, taking into account complicating environmental factors.

Therefore, this paper aims at the development of a software module for calculating the drill string displacements, based on the developed nonlinear mathematical model of the drill string spatial deformation allowing for the effects of the drilling fluid flow, external forces and the intermittent contact with the borehole wall.

2 Materials and methods

2.1 Nonlinear mathematical model

The drill string is modeled as an isotropic elastic rod of length l. Lateral displacements and rotational motion of the drill string are included through the use of two right-handed Cartesian coordinate systems: a global system $OX_1X_2X_3$ and the local one $Ox_1x_2x_3$, respectively. The drill string is affected by a compressing load $N(x_3, t)$ and a torque $M(x_3, t)$. A drilling fluid flow passes along the drill string inner tube towards the bottom as mentioned in [10]. For the contact problem, we assume that the contact of the drill string with the borehole wall occurs only at one point, and the greatest amplitude of lateral vibrations is usually observed in that section.

Then, utilizing the fundamentals of Novozhilov's nonlinear elasticity theory [11] and applying Ostrogradsky-Hamilton's variation principle, a generalized nonlinear mathematical model of the drill string spatial lateral vibrations, taking into account the drilling fluid, external loads and contact interaction with the borehole wall, is developed:

$$
\begin{aligned}
&EI_{x_2}\frac{\partial^4 u_1}{\partial x_3^4}-\rho I_{x_2}\frac{\partial^4 u_1}{\partial x_3^2\partial t^2}+\frac{\partial^2}{\partial x_3^2}\left(M(x_3,t)\frac{\partial u_2}{\partial x_3}\right)+\frac{\partial}{\partial x_3}\left(N(x_3,t)\frac{\partial u_1}{\partial x_3}\right)-\\
&-\frac{EA}{1-\nu}\frac{\partial}{\partial x_3}\left(\frac{\partial u_1}{\partial x_3}\right)^3-\frac{EA(5-6\nu)}{2(1-\nu)}\frac{\partial}{\partial x_3}\left(\frac{\partial u_1}{\partial x_3}\left(\frac{\partial u_2}{\partial x_3}\right)^2\right)+\\
&+(\rho A+\rho_f A_f)\left(\frac{\partial^2 u_1}{\partial t^2}-2\Omega\frac{\partial u_2}{\partial t}-\Omega^2 u_1\right)-\rho_f I_{x_2}\left(\frac{\partial^4 u_1}{\partial x_3^4}+2\frac{\partial^4 u_1}{\partial x_3^3\partial t}+\frac{\partial^4 u_1}{\partial x_3^2\partial t^2}\right)+\\
&+\rho_f A_f\left(V_f^2\frac{\partial^2 u_1}{\partial x_3^2}+2V_f\frac{\partial^2 u_1}{\partial x_3\partial t}-2V_f\Omega\frac{\partial u_2}{\partial x_3}\right)-\\
&+(\rho A+\rho_f A_f)\,g\left(\frac{\partial u_1}{\partial x_3}-(l-x_3)\frac{\partial^2 u_1}{\partial x_3^2}\right)-\\
&-\frac{F}{r}\left(u_1+sign\left(\Psi\right)\mu u_2\right)\delta\left(x_3-x_{3c}\right)=0,\\
&EI_{x_1}\frac{\partial^4 u_2}{\partial x_3^4}-\rho I_{x_1}\frac{\partial^4 u_2}{\partial x_3^2\partial t^2}-\frac{\partial^2}{\partial x_3^2}\left(M(x_3,t)\frac{\partial u_1}{\partial x_3}\right)+\frac{\partial}{\partial x_3}\left(N(x_3,t)\frac{\partial u_2}{\partial x_3}\right)-\\
&-\frac{EA}{1-\nu}\frac{\partial}{\partial x_3}\left(\frac{\partial u_2}{\partial x_3}\right)^3-\frac{EA(5-6\nu)}{2(1-\nu)}\frac{\partial}{\partial x_3}\left(\frac{\partial u_2}{\partial x_3}\left(\frac{\partial u_1}{\partial x_3}\right)^2\right)+\\
&+(\rho A+\rho_f A_f)\left(\frac{\partial^2 u_2}{\partial t^2}+2\Omega\frac{\partial u_1}{\partial t}-\Omega^2 u_2\right)-\rho_f I_{x_1}\left(\frac{\partial^4 u_2}{\partial x_3^4}+2\frac{\partial^4 u_2}{\partial x_3^3\partial t}+\frac{\partial^4 u_2}{\partial x_3^2\partial t^2}\right)+\\
&+\rho_f A_f\left(V_f^2\frac{\partial^2 u_2}{\partial x_3^2}+2V_f\frac{\partial^2 u_2}{\partial x_3\partial t}+2V_f\Omega\frac{\partial u_1}{\partial x_3}\right)+\\
&+(\rho A+\rho_f A_f)\,g\left(\frac{\partial u_2}{\partial x_3}-(l-x_3)\frac{\partial^2 u_2}{\partial x_3^2}\right)-\\
&-\frac{F}{r}\left(u_2-sign\left(\Psi\right)\mu u_1\right)\delta\left(x_3-x_{3c}\right)=0,
\end{aligned}
\tag{1}
$$

where E is Young's modulus, I_{x_1}, I_{x_2} axial inertia moments, ρ the drill string density, ν Poisson's ratio, A the cross-section area of the drill string, ρ_f the drilling fluid flow density, A_f the internal cross-section area of the drill string, V_f the fluid flow speed, F the impact force related to the contact interaction, r the geometric center position of the drill string cross-section, Ψ the velocity of the drill string at the contact point, μ the proportionality coefficient of the friction force that is proportional to the impact force, x_{3c} the impact location.

Boundary conditions are given by

$$
\begin{aligned}
&u_1\left(x_3,t\right)=u_2\left(x_3,t\right)=0, \qquad \left(x_3=0,x_3=l\right),\\
&EI_{x_2}\frac{\partial^2 u_1\left(x_3,t\right)}{\partial x_3^2}=EI_{x_1}\frac{\partial^2 u_2\left(x_3,t\right)}{\partial x_3^2}=0 \qquad \left(x_3=0,x_3=l\right).
\end{aligned}
\tag{2}
$$

Initial conditions are written as

$$
\begin{aligned}
&u_1\left(x_3,t\right)=u_2\left(x_3,t\right)=0, \qquad \left(t=0\right),\\
&\frac{\partial u_1\left(x_3,t\right)}{\partial t}=C_1, \quad \frac{\partial u_2\left(x_3,t\right)}{\partial t}=C_2 \qquad \left(t=0\right).
\end{aligned}
\tag{3}
$$

where C_1, C_2 are constants specifying the deviation rate of the drill string axis from its initial position in Ox_1x_3- and Ox_2x_3-planes, respectively, at the initial moment of time.

2.2 Development of a software module

The software module is designed in the form of Wolfram notebook using the universal Wolfram Language, which is a powerful interactive document that allows instant computations, supports various dynamic and high-level programming interfaces, text notes, as well as fast graphical representation and visualization of the results obtained. To carry out the necessary computations, the following libraries are included to the module:

```
Needs["DifferentialEquations`NDSolveProblems`"];
Needs["DifferentialEquations`NDSolveUtilities`"];
Needs["DifferentialEquations`InterpolatingFunctionAnatomy`"];
Needs["FunctionApproximations`"];
```

The initial data is pre-cleaned to avoid possible errors associated with previously specified values of the system parameters:

```
ClearAll["Global`*"]
```

To find a solution to the mathematical model (1)-(3), the Bubnov-Galerkin method, which is fully implemented in the software module, is applied. First, the required number of expansion modes according to the Bubnov-Galerkin method is given by

$$n = 3;$$

and the drill string displacement functions are represented as

$$\mathrm{u} = \sum_{i=1}^{n} \mathrm{fu[i][t]} \sin\left[\frac{\pi i z}{l}\right]$$
$$\mathrm{v} = \sum_{i=1}^{n} \mathrm{fv[i][t]} \sin\left[\frac{\pi i z}{l}\right]$$
$$\mathrm{uc} = \sum_{i=1}^{n} \mathrm{fu[i][t]} \sin\left[\frac{\pi i z c}{l}\right]$$
$$\mathrm{vc} = \sum_{i=1}^{n} \mathrm{fv[i][t]} \sin\left[\frac{\pi i z c}{l}\right]$$

Here u and v correspond to the displacement components $u_1(x_3, t)$ and $u_2(x_3, t)$, respectively, and uc and vc to the displacements at the point of interaction with the borehole wall.

The position of the geometric center of the drill string cross-section r and the position of the geometric center at the point of interaction with the borehole wall are determined in the program as follows:

$$\mathrm{r} = \mathrm{Sqrt}\left[\mathrm{u}^2 + \mathrm{v}^2\right]$$
$$\mathrm{rc} = \mathrm{Sqrt}\left[\mathrm{uc}^2 + \mathrm{vc}^2\right]$$

The drill string speed at the contact point Ψ and the components for the contact interaction force F are given by

$$\Psi = (\mathrm{uc} * \mathrm{D}\,[\mathrm{vc,t}] - \mathrm{vc} * \mathrm{D}\,[\mathrm{uc,t}]) + \frac{\mathrm{d1}}{2}\omega$$
$$\mathrm{F1} := \mathrm{Piecewise}[\{\{-\frac{\mathrm{Kh}}{\mathrm{rc}} * (\mathrm{rc} - \mathrm{bcl})^{1.5} * (\mathrm{uc} - \mathrm{Sign}\,[\Psi] * \mu * \mathrm{vc})\,, \mathrm{rc} >= \mathrm{bcl}\},$$
$$\{0, \mathrm{rc} < \mathrm{bcl}\}\}]$$
$$\mathrm{F2} := \mathrm{Piecewise}[\{\{-\frac{\mathrm{Kh}}{\mathrm{rc}} * (\mathrm{rc} - \mathrm{bcl})^{1.5} * (\mathrm{vc} + \mathrm{Sign}\,[\Psi] * \mu * \mathrm{uc})\,, \mathrm{rc} >= \mathrm{bcl}\},$$
$$\{0, \mathrm{rc} < \mathrm{bcl}\}\}]$$

The initial conditions are programmed as follows:

$$\mathrm{For}[\mathrm{i{=}1,i{<}n{+}1,i{+}{+}},$$
$$\mathrm{InitCondu[i]} = \mathrm{Integrate}[\{\mathrm{D[u,t]}/.\mathrm{t} \to 0\} * \sin[\frac{\pi i z}{l}], \{\mathrm{z}, 0, \mathrm{l}] ==$$
$$\mathrm{Integrate}[\mathrm{C1} * \sin[\frac{\pi i z}{l}], \{\mathrm{z}, 0, \mathrm{l}];$$
$$\mathrm{Print[InitCondu[i]]]}$$

$$\mathrm{For}[\mathrm{i{=}1,i{<}n{+}1,i{+}{+}},$$
$$\mathrm{InitCondv[i]} = \mathrm{Integrate}[\{\mathrm{D[v,t]}/.\mathrm{t} \to 0\} * \sin[\frac{\pi i z}{l}], \{\mathrm{z}, 0, \mathrm{l}] ==$$
$$\mathrm{Integrate}[\mathrm{C2} * \sin[\frac{\pi i z}{l}], \{\mathrm{z}, 0, \mathrm{l}];$$
$$\mathrm{Print[InitCondv[i]]]}$$

The expression for the axial compressive load, consisting of constant and variable parts, is introduced:

$$\mathrm{N1} = \mathrm{N0} + \mathrm{Nt}\sin[\Omega t];$$

Each of the terms of the mathematical model (1) is specified separately in the software module:

$$\mathrm{term1[1]} = (\rho * \mathrm{A} + \rho\mathrm{f} * \mathrm{Af}) * \mathrm{D[u,t,t]}$$
$$\mathrm{term1[2]} = -(\rho * \mathrm{A} + \rho\mathrm{f} * \mathrm{Af}) * \omega^2 * \mathrm{u}$$
$$\ldots$$
$$\mathrm{term1[17]} = -\frac{(5 - 6 * \nu) * \mathrm{E0{*}A}}{1 - \nu} * \mathrm{D[D[u,z]{*}D[v,z]{*}D[v,z],z]}$$
$$\mathrm{term2[1]} = (\rho * \mathrm{A} + \rho\mathrm{f} * \mathrm{Af}) * \mathrm{D[v,t,t]}$$
$$\ldots$$
$$\mathrm{term2[17]} = -\frac{(5 - 6 * \nu) * \mathrm{E0{*}A}}{1 - \nu} * \mathrm{D[D[v,z]{*}D[u,z]{*}D[u,z],z]}$$

The procedure for integrating the terms of the mathematical model is implemented in accordance with the Bubnov-Galerkin method:

```
For[i=1,i<n+1,i++,
For[j=1,j<=17,j++,
intterm1[i][j] = Integrate[term1[j] * sin[πiz/l], {z, 0, l}];
Print[“intterm1[“,i,“][“,j,“]=“, intterm1[i][j]]
]
]
```

```
For[i=1,i<n+1,i++,
For[j=1,j<=17,j++,
intterm2[i][j] = Integrate[term2[j] * sin[πiz/l], {z, 0, l}];
Print[“intterm2[“,i,“][“,j,“]=“, intterm2[i][j]]
]
]
```

To obtain a system of ordinary differential equations, the following program expressions are used:

$$\text{Do}[\text{eqn1}[\text{j}] = \sum_{i=1}^{17} \text{intterm1}[\text{j}][\text{i}] - \text{F1} * \sin[\frac{\pi j z c}{l}], \{\text{j,1,n}\}];$$

$$\text{Do}[\text{eqn2}[\text{j}] = \sum_{i=1}^{17} \text{intterm2}[\text{j}][\text{i}] - \text{F2} * \sin[\frac{\pi j z c}{l}], \{\text{j,1,n}\}];$$

It is worth noting that this software module also allows calculating and analyzing a linear mathematical model of the drill string vibrations. In addition, it is possible to study the influence of various complicating factors on the drill string oscillatory process.

The software module enables to enter the values of the main system parameters. For example, to select a material of the drill string, one needs to specify one of two values:

$$\text{material} = 1;$$
$$\text{If}\,[\text{material} = 1, \{\text{E0} = 2.110^{11}, \rho = 7800, \nu = 0.28\}\,, \{\text{E0} = 0.710^{11}, \rho = 2700, \nu = 0.34\}]$$

If the first material is chosen, then the analysis will be conducted for the steel drill string; otherwise, the vibrations of the duralumin drill string will be calculated.

To set the other parameters of the drilling system, one needs to simply enter their values. The necessary explanations are added to the module.

$$\left\{\mathrm{d1} = 0.2, \mathrm{d2} = 0.12, \mathrm{h} = \frac{\mathrm{d1} - \mathrm{d2}}{2}\right\}$$
$$\left\{\mathrm{J} = \frac{1}{64}\pi\left(\mathrm{d1}^4 - \mathrm{d2}^4\right), A = \frac{1}{4}\pi\left(\mathrm{d1}^2 - \mathrm{d2}^2\right)\right\}$$
$$\mathrm{C1} = 0.01; \mathrm{C2} = 0.01; l = 100; \omega = \frac{90.}{60.}; \Omega = \omega;$$
$$\mathrm{N0} = 1.2 * 10^4; \mathrm{Nt} = 1 * 10^4; \mathrm{M1} = 1 * 10^4;$$
$$\mathrm{Kh} = 6.78 * 10^{11}; \mu = 0.2; \mathrm{bd} = 295 * 10^{-3}; \mathrm{bcl} = \frac{\mathrm{bd} - \mathrm{d1}}{2}; \mathrm{zc} = 0.49l;$$
$$\rho\mathrm{f} = 1120; \mathrm{Af} = \frac{\pi \mathrm{d2}^2}{4}; \mathrm{Vf} = 60;$$
$$g = 9.8;$$

The time interval of the drill string oscillatory process is determined:

$$\mathrm{tk} = 150$$

To find a numerical solution to the mathematical model (1)-(3), the built-in NDSolve function with the use of a stiffness switching method is applied. If necessary, the user can choose another method for obtaining the numerical solution.

```
SolStiffSwitch1 =
NDSolve[{Table[eqn1[i]=0,{i,1,n}], Table[eqn2[i]=0,{i,1,n}],
Table[fu[i][0] == 0, {i, 1, n}], Table[InitCondu[i], {i, 1, n}],
Table[fv[i][0] == 0, {i, 1, n}], Table[InitCondv[i], {i, 1, n}],
Join[Table[fu[i], {i, 1, n}], Table[fv[i], {i, 1, n}]], {t, 0, tk},
Method → {StiffnessSwitching,
Method → {{"ExplicitRungeKutta","DifferenceOrder" → 8}, Automatic},
"NonstiffTest" → "NormBound","DiscontinuityProcessing" → False}, MaxSteps → ∞]
```

When the program completes the numerical computation, the modeling results are displayed graphically (Figure 1).

The module gives an opportunity to obtain more details on the method applied by clicking on the "+" icon (Figure 2).

The transition from the solution regarding the time components $f(t)$ to the initial displacement components is given in the module by the following expressions:

$$\mathrm{u1} = \sum_{i=1}^{n} \mathrm{fu[i][t]} \sin[\frac{\pi i z}{l}]$$
$$\mathrm{v1} = \sum_{i=1}^{n} \mathrm{fv[i][t]} \sin[\frac{\pi i z}{l}]$$

3 Results and discussions

The developed software module allows obtaining 2D visualization of the obtained numerical results with the use of the built-in Plot function. It is possible to configure all the necessary

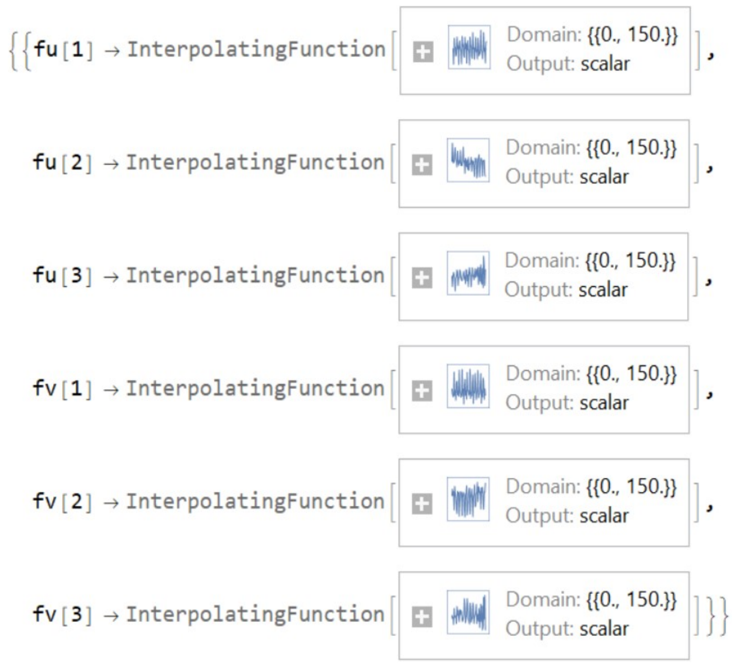

Figure 1: Compututional results

Figure 2: Details of the method applied

parameters for the output graph. The following code is utilized to plot the lateral vibrations of the drill string in the Ox_1x_3-plane:

```
Plot[u1/. SolStiffSwitch1, {t, t1, t2}, Frame → True,
FrameLabel → {Style["t", Bold], Style["u1(x3, t)", Bold]},
LabelStyle → {FontFamily → "Times New Roman", 13, GrayLevel[0]},
FrameTicksStyle → Directive[Black, Thick], ImageSize → 500.,
PlotRange → All, PlotStyle → {{AbsoluteThickness[1.5],
AbsoluteDashing[{1.5, 0}], GrayLevel[0.0]}}, AspectRatio → 1/3]
```

The Export function is used to save the graph of the drill string lateral displacements taking into account the fluid flow and the contact with the borehole. Application of this function allows selecting the desired type and resolution of the output graphic image.

Export[“u1.jpg“, %, “JPEG“, ImageResolution → 300]

Figure 3 demonstrate the results of numerical modeling of the drill string lateral vibrations in the Ox_1x_3-plane:

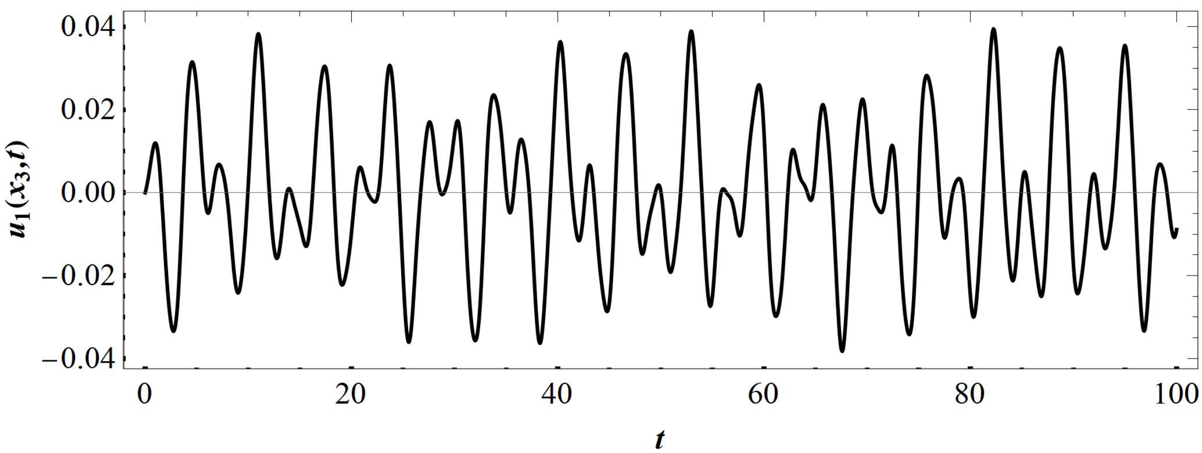

Figure 3: Drill string lateral vibrations in the Ox_1x_3-plane

The graph for the drill string vibrations in the Ox_2x_3-plane (Figure 4) is constructed and displayed in a similar way.

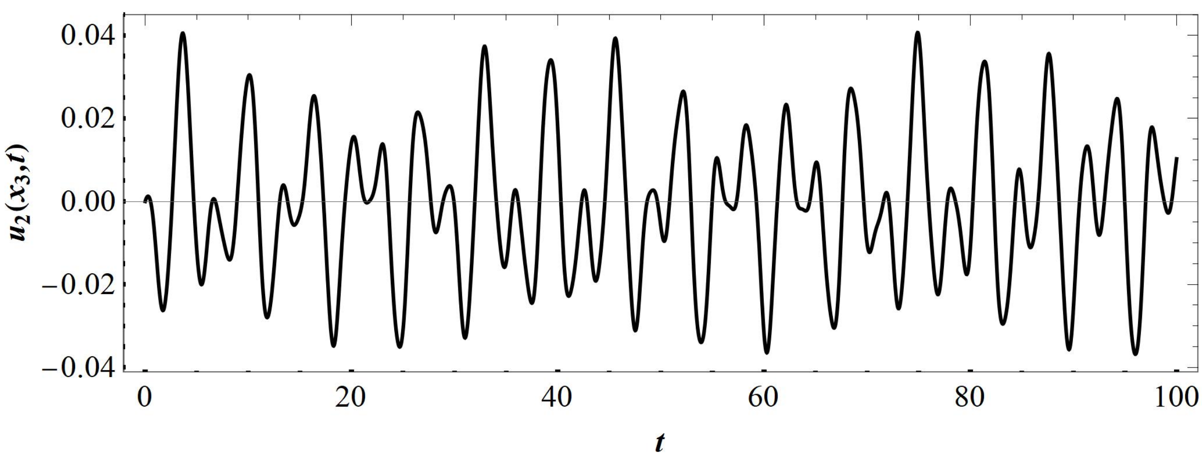

Figure 4: Drill string lateral vibrations in the Ox_2x_3-plane

Another function implemented in the software module is the construction of phase portraits of the solution to study the behavior of the studied dynamic system in the state space. The ParametricPlot function is used to construct the phase portraits.

ParametricPlot[{u1,v1}/. SolStiffSwitch1, {t, t1, t2}, Frame → True,
FrameLabel → {Style["$u_1(x_3, t)$", Bold], Style["$u_2(x_3, t)$", Bold]},
LabelStyle → {FontFamily → "Times New Roman", 14, GrayLevel[0]},
FrameTicksStyle → Directive[Black, Thick], ImageSize → 300.,
PlotRange → All, PlotStyle → {{AbsoluteThickness[1.2],
AbsoluteDashing[{1.5, 0}], GrayLevel[0.0]}}]

Figure 5 shows the phase portrait of the obtained numerical solution in terms of the displacements u_1 and u_2. As can be seen from the graph, the oscillatory process of the drill string remains stable throughout the entire period of time.

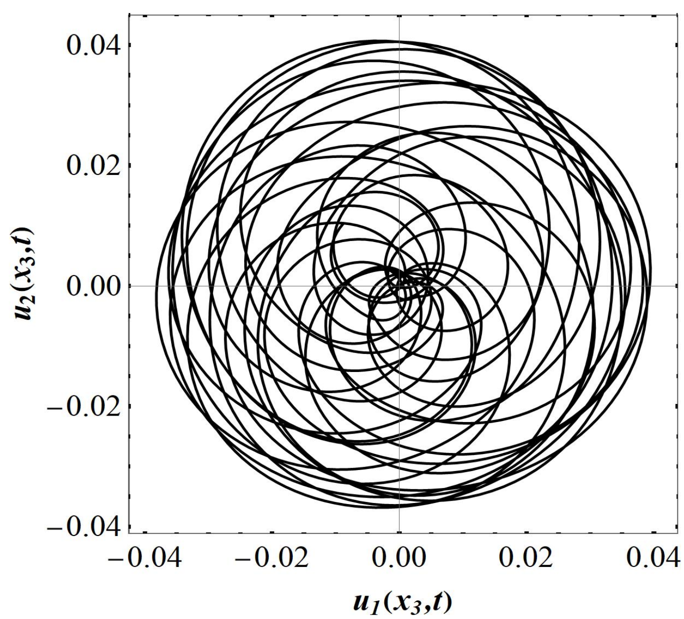

Figure 5: Phase portrait for the solution of the nonlinear model

Also, the developed software module allows conducting a comparative analysis of the modeling results obtained for various values of the system parameters and displaying several solution curves on one graph. Figure 6 illustrates the drill string vibrations in the Ox_1x_3-plane for different values of the angular speed of rotation. As shown in Figure 6, an increase in the angular speed of rotation from 1.5rad/s to 2rad/s results in the minor increase of the drill string vibration amplitude.

4 Conclusion

In this work, a software module for calculating the drill string displacements based on the generalized nonlinear mathematical model of the drill string spatial lateral vibrations, taking into account the influence of the drilling fluid flow, external loads and contact interaction with the borehole walls using the universal Wolfram Language was developed. A software implementation of the Bubnov-Galerkin method, which allows reducing systems of partial

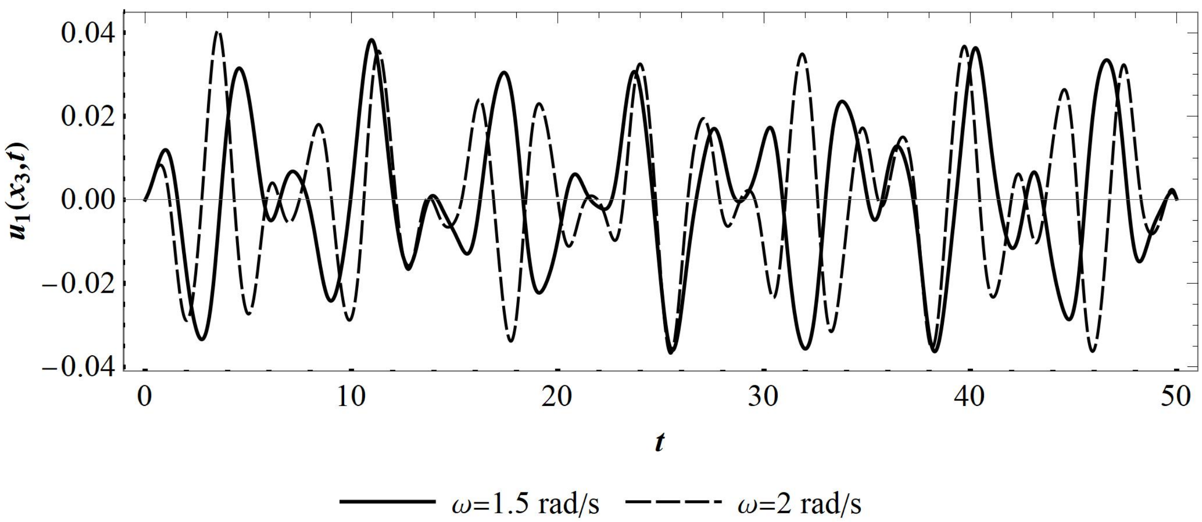

Figure 6: Comparative analysis of the modeling results at different values of the angular speed of rotation

differential equations to the systems of ordinary differential equations, was implemented. The numerical solution for the generalized nonlinear mathematical model of the drill string lateral vibrations was obtained using the stiffness switching method included into the NDSolve function. 2D visualization of the drill string displacments was carried out using the Plot function that allowed configuring all the necessary parameters for the output graphs. Phase portraits of the solution were also constructed using the ParametricPlot function to study the behavior of the dynamic system in the state space. In addition, the developed software module allowed conducting the comparative analysis of the numerical results for different values of the drilling system parameters.

The developed software module is the basis for the subsequent modules, which together will form a multifunctional package of programs for solving a wide class of problems of the drill string dynamics.

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