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IN ONE SCENARIO, THE DEVELOPMENT OF A DEFECT IN THE ATTACHMENT OF THE ROD

This article discusses the issue of the origin of a rod fastening defect. At the beginning of operation, the rod is rigidly fixed at the edges. During operation, over time, certain defects may appear at the ends of the rod. We need to find out what defects may occur? Then it is necessary to trace the further behavior of the emerging defects at the ends of the rod. This paper discusses the diagnostics of types of fastening of a structure made of interconnected rods. In this work, the state of fastening types in individual parts of the structure is determined and a number of results are obtained using mathematical analysis. Most of them assume how failures begin at the end connections of the rods, and then the scenario for their further development. Mathematical models are presented to determine the state of the rod attachments relative to the proposed scenario, and then the state in which they are in is carefully examined. Defects in fastening objects made from a system of rods are investigated using identification problems. The difference between this article and other works is that instead of the shape of the area, the size of the object, or the state of its location, defects that occur in fasteners are studied. This work is devoted to the search for types of fastening that provide the required range of vibration frequencies.
Key words: Euler-Bernoulli equation, rod, defect, Taylor formula.

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Стержень бекітуіндегі ақаудың пайда болып дамуының сценарийі туралы

Бұл мақалада стерженьдердің ақауының шығу тегі туралы мәселе қарастырылады. Жұмыстың басында стержень ұштарында қатаң бекітіледі. Уақыт өте келе стержень ұштарында белгілі бір ақаулар пайда болуы мүмкін. Бізге қандай ақаулар пайда болуы мүмкін екенін анықтау керек? Содан кейін стержень ұштарында пайда болатын ақаулардың одан әрі әрекеті туралы айтылады. Осы жұмыста өзара байланысқан стерженьдерден құралған конструкцияның бекіту түрлеріне диагностика жасау қарастырылған. Бұл жұмыста конструкцияның жеке бөлшектерінде бекіту түрлерінің ақуалы анықталды және бірқатар нәтижелер математикалық жолмен талдау арқылы алынған. Олардың көбі стерженьдердің шеттік бекітулерінде ақау қалай басталады және одан кейін олар ары қарай қандай сценаримен дамитыны ұсынылған. Ұсынылған сценарииге байланысты стерженьнің шеттік бекітуінің күйін анықтауға математикалық модельдер көрсетілген және одан кейін олар қандай күйде болатыны мұқият зерттелген. Стерженьдер жүйесінен құрастырылған обьектілердің бекітуіндегі ақауларын идентификациялау есептері бойынша зерттелінген. Осы мақаланың басқа жұмыстардан өзгешелігі – облыс формасы, обьект көлемі немесе орналасу жағдайының орнына бекітулерде пайда болатын ақаулар зерттеледі. Бұл жұмыста тербеліс жиілігінің қажетті диапазонын қамтамасыз ететін бекіту түрлерін іздеу қарастырылады.

Түйін сөздер: Эйлер-Бернулли теңдеуі, стержень, ақау, Тейлор формуласы.

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Об одном сценарии зарождения развития дефекта крепление стержня

В данной статье обсуждается вопрос зарождения дефекта крепления стержня. Вначале эксплуатации стержень по краям жестко закреплен. В процессе эксплуатации с течением времени могут появляться те или иные дефекты на концах стержня. Надо выяснить какие дефекты могут возникать? Затем надо проследить дальнейшее поведение возникающего дефектов на концах стержня. В данной работе рассматривается диагностика видов крепления конструкции из соединенных между собой стержней. В данной работе определено состояние типов крепления в отдельных частях конструкции и методом математического анализа получен ряд результатов. Большинство из них предполагают, как начинаются разрушения в концевых соединениях стержней, а затем сценарий их дальнейшего развития. Представлены математические модели для определения состояния крепления стержня относительно предложенного сценария, а затем тщательно изучено, в каком состоянии они находятся. Дефекты крепления объектов, изготовленных из системы стержней, исследуются по задачам идентификации. Отличие данной статьи от других работ состоит в том, что вместо формы области, размера объекта или состоянии его расположения изучаются дефекты, возникающие в креплениях. Данная работа посвящена поиску типов крепления, обеспечивающих необходимый диапазон частот вибрации.

Ключевые слова: Уравнение Эйлера-Бернулли, стержень, дефект, формула Тейлора.

1 Introduction

Acoustic diagnostics is the determination of the technical condition of equipment in working order based on the parameters of vibration processes. Acoustic diagnostic methods are widely used to determine the strength of various materials and the location of incipient, incipient and developing cracks. The acoustic diagnostic method is used to determine the technical condition of a structure in various environments. Acoustic diagnostic methods make it possible to study the structure itself as a whole without dismantling it. This work is devoted to the search for types of fastening that provide the required range of vibration frequencies. Such problems relate to the problems of mathematical acoustics outlined above. Even in this case, it is necessary to identify parameters that describe the state of fixation by natural frequency. More precisely, the diagnostics of the states of the edge fastenings of the rods based on the frequencies of transverse vibrations is considered.

Transverse oscillations of the rod are described by the Euler-Bernoulli equation [1], which is written in the form

$$\frac{\partial^2}{\partial x^2}\left(EJ\frac{\partial^2 w}{\partial x^2}\right)+A\rho\frac{\partial^2 w}{\partial t^2}=q(x,t) \tag{1}$$

relative to the transverse deflection $w(x,t)$.

Here are E, J, A, ρ standard physical characteristics of the material from which the rod is made. At the beginning, we consider that both ends of the rod are rigidly fixed. This means that relations

$$w(0,t)=0,\ \frac{\partial w(0,t)}{\partial x}=0,\ w(l,t)=0,\ \frac{\partial w(l,t)}{\partial x}=0$$

are fulfilled. In this case, the length of the rod is chosen equal to l. Over time, defects may appear along the rod. We believe that, first of all, defects can arise at one of the ends of the rod.

A comparative analysis of literary [1–3] sources indicates that it is easier to bend a rod than to stretch it or rotate it around the axis of the rod. The mathematically specified phenomenon is characterized by the asymptotic behavior of $w_1(x,t), w_2(x,t)$ transverse and $w_3(x,t)$ longitudinal deviations in the form of

$$\frac{1}{h^2}\big(\vec{e_1}\cdot w_1(x)+\vec{e_2}\cdot w_2(x)\big)+\frac{1}{h}\cdot\vec{e_3}\big(w_3(x)-\eta_1\frac{\partial}{\partial x}w_1(x)-\eta_2\frac{\partial}{\partial x}w_2(x)\big)+\frac{1}{h}\big(\eta_1\vec{e_2}-\eta_2\vec{e_1}\big) \quad (2)$$

Here $\vec{e_3}$ direction is along the rod, and $\vec{e_1}, \vec{e_2}$ directions are perpendicular to the rod axis [3]. In expression 2 there is also $w_4(x,t)$, which characterizes the torsion around the axis of the rod. Parameter h is also involved here, which characterizes the diameter of the cross section of the rod. Taking into account the above-mentioned effect, given by expression 2 , we can now proceed to the study of physical phenomena occurring near the fixed end of the rod.

2 Methods and materials

2.1 The scenario of the occurrence and development of defects at the point of attachment of the rod

In this point, one of the possible variants of occurrence at the end points of the rod attachment is attached. The scenario consists of four stages of emergence and development of a defect at one end of a rod. The defect at the point of attachment of the rod undergoes the following stages. At the beginning, the end of the rod is rigidly fixed, then during the operation of the rod, the conditions of rigid fixation of the rod are weakened due to the bending moment. The next stage is characterized by the fact that the actions of transverse forces cause "backlash"at the point of attachment. Each stage of the defect corresponds to its own individual frequency of transverse oscillations of the rod. The indicated natural oscillations of the rod can be measured by acoustic means. Thus, based on the measured natural frequencies of the transverse oscillations of the rod, the stage of the defect in the end fixings of the rod can be determined.

Now consider the neighborhood of rod $x=0$. That is, x is between 0 and h. Then the Taylor formula [4]

$$w(x,t)=w(0,t)+\frac{\partial w(0,t)}{\partial x}+\frac{1}{2}\frac{\partial^2 w(0,t)}{\partial x^2}x^2$$

$$\frac{\partial w(x,t)}{\partial x}\approx\frac{\partial w(0,t)}{\partial x}+\frac{\partial^2 w(0,t)}{\partial x^2}x+\frac{1}{2}\frac{\partial^3 w(0,t)}{\partial x^3}x^3$$

we use for the rigidly fixed edge $x=0$, and we get

$$\begin{cases} w(h,t)\approx\frac{1}{2}\frac{\partial^2 w(0,t)}{\partial x^2}h^2 \\ \frac{\partial w(h,t)}{\partial x}\approx\frac{\partial^2 w(0,t)}{\partial x^2}h+\frac{1}{2}\frac{\partial^3 w(0,t)}{\partial x^3}h^3 \end{cases} \quad (3)$$

We know that torque is equal to the theory of elasticity

$$M(0) = -EJ\frac{\partial^2 w(0,t)}{\partial x^2}$$

and is equal to the transverse force

$$Q(0) = EJ\frac{\partial^3 w(0,t)}{\partial x^3}$$

Therefore, from equation 3 the relations are fulfilled

$$\begin{cases} w(h,t) \approx \frac{-h^2}{2EJ}M(0) \\ \frac{\partial w(h,t)}{\partial x} \approx -\frac{h}{EJ}M(0) + \frac{h^3}{2EJ}Q(0) \end{cases}$$

for moment and transverse force [5].

Now we can predict how a defect will appear at edge $x = 0$ of the rod and according to what scenario it will develop.

Let there be at the beginning a rigid fastening of the edge $x = 0$ of the rod

$$w(0,t) = 0, \quad \frac{\partial w(0,t)}{\partial x}\Big|_{x=0} = 0. \tag{4}$$

During operation (after some time) conditions

$$w(0,t) = 0, \quad \frac{\partial w(0,t)}{\partial x}\Big|_{x=0} = \alpha_1\frac{\partial^2 w(0,h)}{\partial x^2} \tag{5}$$

are carried out taking into account relation 4. Since h — is small, the following hierarchy

$$h\frac{\partial^2 w(0,t)}{\partial x^2} \gg \frac{h^2}{2}\frac{\partial^2 w(0,t)}{\partial x^2} \gg \frac{h^3}{2}\frac{\partial^3 w(0,t)}{\partial x^3}$$

will be executed. Therefore, we will first consider $h\frac{\partial^2 w(0,t)}{\partial x^2}$, and $\frac{h^2}{2}\frac{\partial^2 w(0,t)}{\partial x^2}$, $\frac{h^3}{2}\frac{\partial^3 w(0,t)}{\partial x^3}$ are very small quantities. That is, it can be considered zero.

Here α_1 is the parameter. In this case, fastening 4 is transferred to condition 5 . This is where edge defect $x = 0$ begins to appear. In this case, it shows that the value of $h\frac{\partial^2 w(0,t)}{\partial x^2}$ and $h^3\frac{\partial^3 w(0,t)}{\partial x^3}$ is very small. Therefore, condition 5 is satisfied. The mechanical meaning of this condition 5 is to take into account the influence of angular momentum $M(0)$ on the value of the angle of inclination $\frac{\partial w(0,t)}{\partial x}$. Therefore, instead of condition 4, condition 5 should be taken into account. If previously there was a rigid mount, now it is necessary to take into account the influence of torque. If you have observed such a situation, then you can continue to use the rod. Due to the impact of torque, the rigid mount was changed to the mount under condition 4, but we continue to operate. At this stage [6], there is no need to stop using the rods, even though the angle of inclination appears. That is, it does not require repair.

But one thing should be noted: the natural frequencies of horizontal oscillations according to conditions 4 change when the natural frequencies are conditions 5. If we continue to use the rods without repair, then we will call the transition from state 4 to state 5 a level 1 defect and assume that this defect does not yet require repair. So, let us assume that conditions 5 are satisfied at edge $x = 0$. From the asymptotic relations 2, the boundary conditions 5 change to the following conditions:

$$\begin{cases} w(0,t) = \alpha_2 \frac{\partial^2 w(0,t)}{\partial x^2} \\ \frac{\partial w(0,t)}{\partial x} = \alpha_1 \frac{\partial^2 w(0,t)}{\partial x^2} \end{cases} \tag{6}$$

here $0 < \alpha_2 < \alpha_1$. In this case, we consider that a level 2 defect has occurred on edge $x = 0$ in which case the repair time will be reduced. Therefore, the risk is even higher than the previous level. Previously, the degree of destruction varied under the influence of torque. Now we need to take into account the action of the transverse force $Q(0)$. If we take these points into account [7], the rod goes through 4 stages during operation.

3 Conclusion

At stage 1 there will be a rigid fastening. At this moment, the equation of the rod is described by equation

$$w(0) = 0, \quad \frac{\partial w(0)}{\partial x}(0) = 0.$$

At this stage the rod is in a horizontal position. After the 1st stage, after rigid fastening, it moves on to bending. At this moment the equation of the rod will be

$$w(0) = 0, \quad \frac{\partial w(0)}{\partial x} = \alpha_1 \frac{\partial^2 w(0)}{\partial x^2}.$$

At this stage, the rod deviates from the horizontal position and acquires an inclined angle.

At this stage there is no need for repairs. Here the rigid fastening is maintained. Here the edge binding remains as rigid as before. Bending occurs only along the rod. That is, the rod retains its original fastening.

After the 2nd stage, edge $x = 0$ is weakened. At this stage, you can continue using the rod. We must remember that repairs must be made there in the future. That is, the edges of the rod change from a rigid attachment to a slightly looser edge. Therefore, as the ends of the rod are weakened from the rigid fastenings, play occurs. The resulting play does not completely release the rod. When there is play in the rigid fasteners, a hole appears. The equation of the rod for backlash has the form [6]

$$w(0) = \beta \frac{\partial^2 w(0)}{\partial x^2}, \quad \frac{\partial w(0)}{\partial x} = \alpha_1 \frac{\partial^2 w(0)}{\partial x^2}, \quad \alpha \gg \beta.$$

At this time we must remember the work ahead. After the 3rd stage it moves from bend to fracture. The rod at this time is described by the equation [6]

$$w(0) = \beta \frac{\partial^2 w(0)}{\partial x^2}, \quad \frac{\partial w(0)}{\partial x} = \alpha_1 \frac{\partial^2 w(0)}{\partial x^2} + \frac{\partial^3 w(0)}{\partial x^3}, \quad \alpha \gg \beta \gg \gamma.$$

At this time, a transverse force acts on the rod. As a result, the rod will break.

That is, we see here that at the end there is a transverse force. This shear force will cause the rod to break. Then from these stages we draw the following conclusions: First of all, the rod bends under the influence of a torque, under the influence of which its edges become loose, and the rod breaks under the action of a transverse force. At this time, it is necessary to urgently repair the rod. All this follows from the Taylor formula of the form of the equation of state of the rods.

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References

[1] Nazarov S.A. General scheme for averaging self-adjoint elliptic systems in multidimensional domains, including thin ones//Algebra and analysis.- 1995. - P. 681–748.

[2] Kozlova M.V. Averaging of a three-dimensional elasticity problem for a thin inhomogeneous beam // Bulletin of Moscow State University. - 1989. - P. 6-10.

[3] Kozlova S.V.,Panasenko G.P. Homogenization of a three-dimensional problem of the theory of elasticity in an inhomogeneous rod // Journal of Computational Mathematics and Mathematical Physics. – 1991. – P. 1592–1596.

[4] Maz'ya V., Nazarov S.A., Plamenevskij B.A. Asymptotic theory of elliptic boundary value problems in singularly perturbed domains - Birkhauser. - 2000.

[5] Nazarov S.A. Asymptotic analysis of thin plates and rods. Dimensionality reduction and integral estimates- Novosibirsk. - 2002.

[6] Kanguzhin B.E., Ghulam Hazrat A.R., Kaiyrbek Zh.A. Identification of the Domain of the Sturm–Liouville Operator on a Star Graph // Symmetry. - 2021. - P. 1-15.

[7] Kanguzhin B.E., Akanbay Y.N., Kaiyrbek Zh. A. On the Uniqueness of the Recovery of the Domain of the Perturbed Laplace Operator // Lobachevskii J. of Math.. - 2022. - P. 1532–1535.