2-бөлім **Раздел 2** **Section 2**

Механика **Механика** **Mechanics**

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NUMERICAL MODELING OF THE TEMPERATURE DISTRIBUTION FIELD IN A COMPLEX SHAPE STRUCTURAL ELEMENT

As you know, many parts of internal combustion engines, gas turbine power plants, steam generators of nuclear power plants and manufacturing industries experience thermal effects of various forms. At the same time, a process of thermal expansion occurs on these parts and, as a result, a thermal stress-strain state arises on them with a value that in some cases can exceed the limit value. Therefore, knowledge of the stationary field of temperature distribution in the volume of partially thermally insulated parts of a complex configuration while there is a heat flux and heat exchange in parts of its surface is an urgent task. At the same time, it is very difficult to take into account all inhomogeneous boundary conditions when solving the problem of stationary heat conduction. Therefore, a new numerical method is proposed, based on the law of conservation of total thermal energy alongside with the finite element method. In this case, the procedure for minimizing the total thermal energy involves quadrilateral bilinear finite elements. Partial thermal insulation, the heat flux supplied to the local surface, and the process of heat exchange through the local surface area and ambient temperature are taken into account. Nodal temperature values are determined.
Key words: mathematical model, channel-shaped body (beam), heat flow, cross-section, functional, heat exchange, thermal insulation, temperature distribution field, form functions.

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Күрделі пішінді дененің түйін нүктелеріндегі температураның таралуын сандық пішіндеу

Өздеріңіз білетіндей, іштен жанатын қозғалтқыштардың, газ турбиналық электр станцияларының, атом электр станцияларының, бу генераторларының және өңдеу өнеркәсібінің көптеген бөліктері әртүрлі формада болады және олар әртүрлі жылу әсерлерін сезінеді. Сонымен бірге бұл бөліктерде термиялық кеңею процесі жүреді де нәтижесінде оларда кейбір жағдайларда шекті мәннен асатын мәнмен термиялық кернеу-деформациялық күй пайда болады.

Сондықтан оның бетінің бөліктерінде жылу ағыны және жылу алмасу болған кезде күрделі конфигурацияның ішінара жылу оқшауланған бөліктерінің көлеміндегі тұрақты күйдегі температураның таралу өрісін білу өзекті мәселе болып табылады. Сонымен қатар тұрақты күйдегі жылу өткізгіштік мәселесін шешуде барлық біртекті емес шекаралық шарттарды есепке алу өте қиын. Сондықтан шекті элементтер әдісімен үйлесімде жалпы жылу энергиясының сақталу заңына бағытталған жаңа сандық әдіс ұсынылады. Бұл жағдайда жалпы жылу энергиясын азайту процедурасы төртбұрышты екі сызықты ақырлы элементтерді пайдалана отырып қолданылады. Ішінара жылу оқшаулау, жергілікті жерге берілетін жылу ағыны мен жылу алмасу процесі және қоршаған орта температурасы ескеріледі. Түйінді нүктелерінің температура мәндері анықталады.
Түйін сөздер: математикалық модель (пішін), арна тәрізді дене (арқалық), жылу ағыны, қима, функционал, жылу алмасу, жылу оқшаулау, температураның таралу өрісі, пішін функциялары.

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Численное моделирование поля распределения температуры в конструкционном элементе сложной формы

Как известно, многие детали двигателей внутреннего сгорания, газотурбинных электростанций, парогенераторов атомных электростанций и предприятий обрабатывающей промышленности испытывают тепловые воздействия различной формы. При этом на этих деталях происходит процесс теплового расширения и, как следствие, на них возникает термическое напряженно-деформированное состояние величиной, которая в ряде случаев может превышать предельное значение. В данной статье мы показываем, что знание стационарного поля распределения температуры в объеме частично теплоизолированных деталей сложной конфигурации при наличии теплового потока и теплообмена на участках ее поверхности является актуальной задачей. В то же время учесть все неоднородные граничные условия при решении задачи стационарной теплопроводности очень сложно. Поэтому предлагается новый численный метод, ориентированный на закон сохранения полной тепловой энергии в сочетании с методом конечных элементов. При этом используется процедура минимизации полной тепловой энергии с использованием билинейных конечных элементов четырехугольной формы. Учитываются частичная тепловая изоляция, тепловой поток, подведенный к локальной поверхности, и процесс теплообмена через площадь локальной поверхности и температуру окружающей среды. Определены узловые значения температуры.

Ключевые слова: математическое модель, швеллеро подобное тело (балка), тепловой поток, поперечное сечение, функционал, теплообмен, теплоизоляция, поле распределения температуры, функции формы.

1 Introduction

In the thermomechanical process, the main characteristic that has a significant impact on the strength of the load-bearing structural elements is an intensive temperature increase. Temperature is one of the most important characteristics of the growth process and affects the morphology and crystal structure of heat-resistant alloys. Depending on the parameters of the structure body, the distribution of the temperature field in its different parts is uneven. It should be noted that the simultaneous influence on the distribution of temperature over the volume of the body and such external factors as various forms of local thermal insulation, the property of heat transfer, and the temperature of the heat source. Consequently, during the thermomechanical process, in some parts of the structural elements, the temperature will be acceptable, and in some – critical, which leads to rapid wear of structural elements and to the loss of their physical qualities. In this regard, the exact calculation of the distribution of the temperature field at each nodal point of multidimensional bodies of complex shape is relevant [1- 4].

This article discusses a technique for constructing a mathematical model and the accompanying computational algorithm that allow solving problems of studying the patterns of distributing the temperature field in a complicated-shape structural element where there is a heat flux, heat transfer and partial thermal insulation on their local surfaces.

At present, in our country and abroad, there are many works devoted to the problem of the influence of a thermomechanical process on a change in the structure and composition of the material of any technical installation or design. This article takes into account the simultaneous influence of the heat flow on the body, partial thermal insulation and local heat transfer. A computational algorithm is presented for solving a problem obtained by discretizing bodies of complex shape made of heat-resistant alloys using quadratic finite elements [2, 5].

The purpose of this article is to show the regularity of the distribution of the temperature field using a numerical study of heat transfer in the presence of heat flow, thermal insulation and heat transfer. The objectives of the study are to determine the temperature value at each nodal point of a multidimensional body to develop a computational algorithm based on minimizing the total thermal energy functional.

2 Research methodology

To illustrate the proposed numerical method and the corresponding computational algorithm, consider the following problem. Given a "channel-like"body of unlimited length $-\infty \leq z \leq \infty$ (Figure 1). The outer side and inner surface of which are thermally insulated along the entire length. Through the areas of the upper surface $y = h, 0 \leq x \leq (r+2l)$, $y = h, 0 \leq x \leq (r+2l)$, $-\infty \leq z \leq \infty$ heat exchange with its environment takes place. In this case, the heat transfer coefficient is h_{oc}, and the ambient temperature is T_{oc}. On the surface $y = 0, [(0 \leq x \leq l$ and $(r+l) \leq x \leq (r+2l)]$,$-\infty \leq z \leq \infty$ a heat flux of q - constant intensity is supplied. It is required to determine the steady temperature distribution field in the volume of the structural element under consideration. To do this, first, the initial cross section, which is shown in Figure 1 is discretized by quadrangular finite elements.

Within each finite element, we represent the temperature distribution field as [1, 2, 6]

$$T(x,y) = \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 y = \phi_1(x,y) \cdot T_1 + \phi_2(x,y) \cdot T_2 + \phi_3(x,y) \cdot T_3 + \phi_4(x,y) \cdot T_4 \quad (1)$$

where $\phi_i(x,y)$ are the shape function for a quadrangular finite element with four nodes [1]:

$$\begin{aligned} \phi_1(x,y) &= \tfrac{(b-x)(a-y)}{4ab}; \qquad \phi_2(x,y) = \tfrac{(b+x)(a+y)}{4ab}; \\ \phi_3(x,y) &= \tfrac{(b+x)(a+y)}{4ab}; \qquad \phi_4(x,y) = \tfrac{(b-x)(a+y)}{4ab} \end{aligned} \quad (2)$$

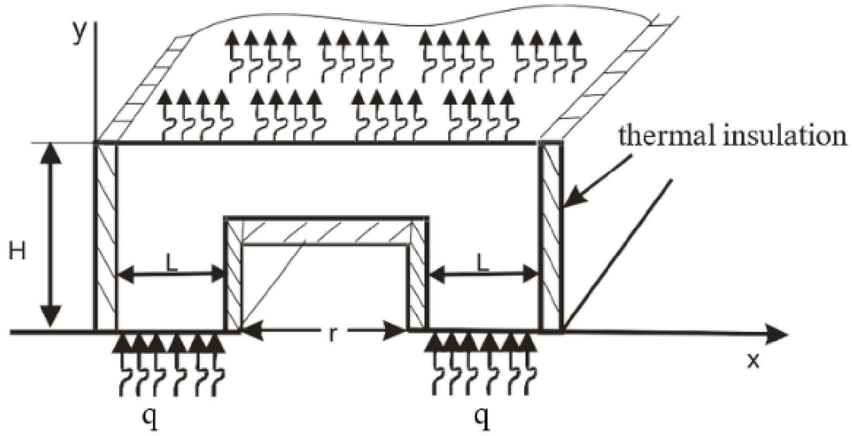

Figure 1: The design scheme of the problem under consideration in the cross section of a structural element

where the size of the finite element along the direction of the coordinate axes x and y is ($2b$, $2a$) (Figure 2)

$$J = \int_V \frac{1}{2}\left[K_{xx}\left(\frac{\partial T}{\partial x}\right)^2 + K_{yy}\left(\frac{\partial T}{\partial y}\right)^2\right] dv + \int_{S(x=0)} qT dS + \int_{S(x=A)} \frac{h}{2}\left(T - T_{oc}\right)^2 dS \quad (3)$$

where V is the volume of the timber in question; *S(x=0)* - the surface area of the beam (x=0), where the heat flow ; $S(x$=A) - the surface area (x=A) of the beam through which heat is exchanged with the environment h; K_{xx}; K_{yy}; $\left(\frac{W}{cm \cdot {}^\circ C}\right)$ - the coefficient of thermal conductivity of the timber under consideration, respectively, in the directions of the coordinate axes ox and oy.

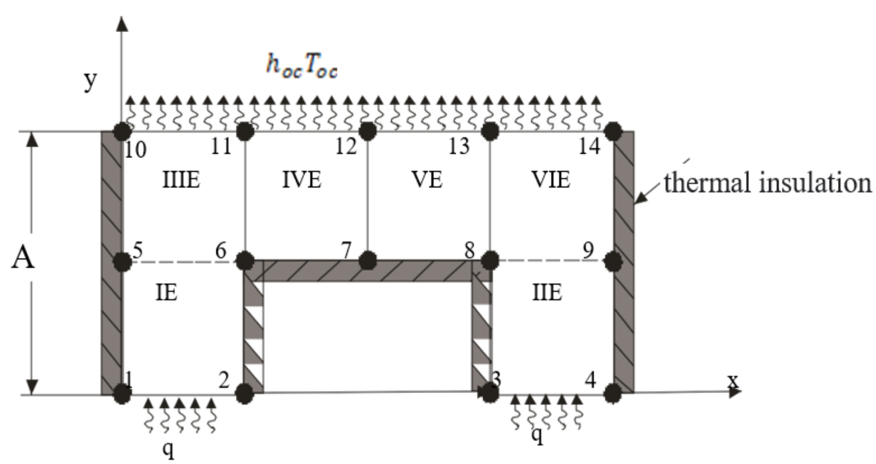

Figure 2: Discretization of the computational domain in the context of a structural element.

3 Results and Discussion

The cross-sectional area of the timber in question (which has the shape of a rectangular quadrangle) is discretized using coordinate lines into quadrangular finite elements. The number of discrete finite elements will be mxn (respectively, on the axes ox and oy). For each element, there is constructed a local coordinate system oxy, so that the origin coincides with the geometric center of the element, as shown in Figure 3. The numbering of the element nodes is shown in this Figure. The coordinates of the element nodes in the local coordinate system will be as follows $1\,(-a;-b)\,;2\,(a;-b)\,;3\,(a;b)\,;4\,(-a;b)$:

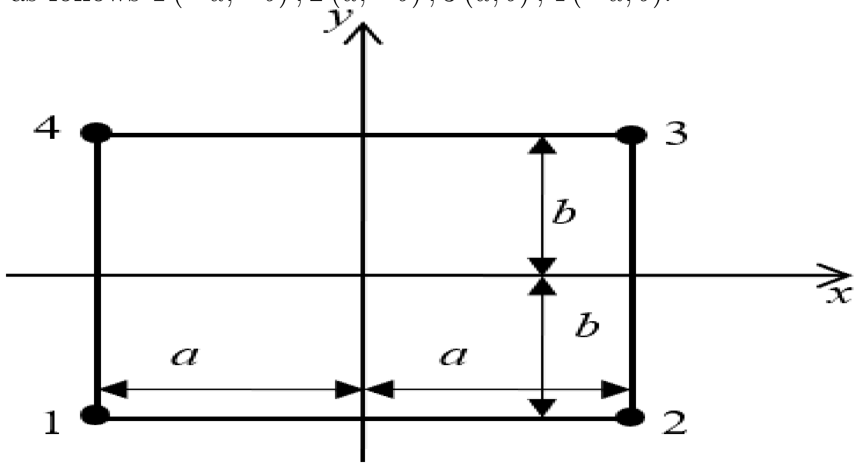

Figure 3: The diagram of constructing a local coordinate system.

These form functions will possess the following properties:

1) in the first node, i.e. when $x = -a;\ y = -b\phi_1\,(-a;-b) = 1;\ \phi_2\,(-a;-b) = \varphi_3\,(-a;-b) = \phi_4\,(-a;-b) = 0;$

2) in the second node, i.e. when $x = a;\ y = -b\ \ \phi_1\,(a;-b) = 0;\quad \phi_2\,(a;-b) = 1;\quad \varphi_3\,(a;-b) = \phi_4\,(a;-b) = 0;$

3) in the third node, i.e. when $x = a; y = b\ \ \phi_1\,(a;b) = 0;\ \ \phi_2\,(a;b) = 0;\ \ \varphi_3\,(a;b) = 1;\ \ \phi_4\,(a;b) = 0;$

4) in the fourh node, i.e. when $x = -a;\quad y = b\ \ \phi_1\,(-a;b) = \phi_2\,(-a;b) = \varphi_3\,(-a;b) = 0;\quad \phi_4\,(-a;b) = 1;$

5) 1) in the first node, i.e. when $x = -a;\ y = -b$
$\varphi_1\,(-a;-b) = 1;\ \ \varphi_2\,(-a;-b) = \varphi_3\,(-a;-b) = \varphi_4\,(-a;-b) = 0;$

2) in the second node, i.e. when $x = a;\ \ y = -b$
$\varphi_1\,(a;-b) = 0;\quad \varphi_2\,(a;-b) = 1;\quad \varphi_3\,(a;-b) = \varphi_4\,(a;-b) = 0;$

3) in the third node, i.e. when $x = a; y = b$
$\varphi_1\,(a;b) = 0;\ \ \varphi_2\,(a;b) = 0;\ \ \varphi_3\,(a;b) = 1;\ \ \varphi_4\,(a;b) = 0;$

4) in the fourh node, i.e. when $x = -a;\quad y = b$
$\varphi_1\,(-a;b) = \varphi_2\,(-a;b) = \varphi_3\,(-a;b) = 0;\quad \varphi_4\,(-a;b) = 1;$

6) $\sum_{i=1}^{4} \frac{\partial \phi_i}{\partial x} = 0$- at any point in a discrete finite element.

In addition, from (1), (2) the values of temperature gradients at any point of a discrete element are easily determined:

$$\frac{\partial T}{\partial x}=\sum_{i=1}^{4}\frac{\partial \phi_i}{\partial x}T_i;\ \frac{\partial T}{\partial y}=\sum_{i=1}^{4}\frac{\partial \phi_i}{\partial y}T_i$$

The expression is also defined:

$$\begin{aligned}&\left(\tfrac{\partial T}{\partial x}\right)^2=\left(\textstyle\sum_{i=1}^{4}\tfrac{\partial \phi_i}{\partial x}T_i\right)^2=\left[-\tfrac{b-y}{4ab}T_1+\tfrac{b-y}{4ab}T_2+\tfrac{b+y}{4ab}T_3-\tfrac{b-y}{4ab}T_4\right]^2=\\&=\tfrac{b^2-2by+y^2}{16a^2b^2}\cdot T_1^2-2\cdot\tfrac{b^2-2by+y^2}{16a^2b^2}\cdot T_1T_2-2\cdot\tfrac{b^2-y^2}{16a^2b^2}\cdot T_1T_3+2\cdot\tfrac{b^2-y^2}{16a^2b^2}\cdot T_1T_4+\\&+\tfrac{b^2-2by+y^2}{16a^2b^2}\cdot T_2^2+2\cdot\tfrac{b^2-y^2}{16a^2b^2}\cdot T_2T_3-2\cdot\tfrac{b^2-y^2}{16a^2b^2}\cdot T_2T_4+\\&+\tfrac{b^2+2by+y^2}{16a^2b^2}\cdot T_3^2-2\cdot\tfrac{b^2+2by+y^2}{16a^2b^2}\cdot T_3T_4+\tfrac{b^2+2by+y^2}{16a^2b^2}\cdot T_4^2;\end{aligned} \tag{4}$$

$$\begin{aligned}&\left(\tfrac{\partial T}{\partial y}\right)^2=\left(\textstyle\sum_{i=1}^{4}\tfrac{\partial \phi_i}{\partial y}T_i\right)^2=\left[-\tfrac{a-x}{4ab}T_1-\tfrac{a+x}{4ab}T_2+\tfrac{a+x}{4ab}T_3+\tfrac{a-x}{4ab}T_4\right]^2=\\&=\tfrac{a^2-2ax+x^2}{16a^2b^2}\cdot T_1^2-2\cdot\tfrac{a^2-x^2}{16a^2b^2}\cdot T_1T_2-2\cdot\tfrac{a^2-x^2}{16a^2b^2}\cdot T_1T_3+2\cdot\tfrac{a^2-2ax+x^2}{16a^2b^2}\cdot T_1T_4+\\&+\tfrac{a^2+2ax+x^2}{16a^2b^2}\cdot T_2^2-2\cdot\tfrac{a^2+2ax+x^2}{16a^2b^2}\cdot T_2T_3-2\cdot\tfrac{a^2-x^2}{16a^2b^2}\cdot T_2T_4+\\&+\tfrac{a^2+2ax+x^2}{16a^2b^2}\cdot T_3^2+2\cdot\tfrac{a^2-x^2}{16a^2b^2}\cdot T_3T_4+\tfrac{a^2-2ax+x^2}{16a^2b^2}\cdot T_4^2;\end{aligned} \tag{5}$$

For clarity of the proposed computational algorithm, we consider the cross section of the timber under consideration as one discrete quadrangular element, as shown in Figure 4.

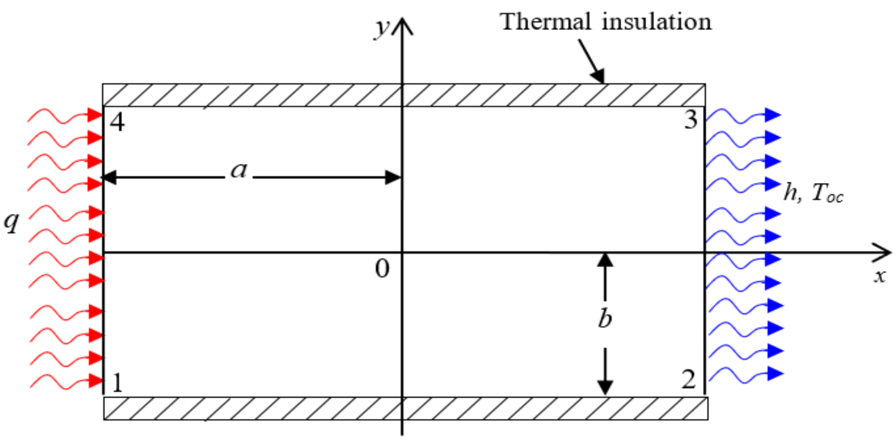

Figure 4: The calculation diagram of the problem.

Now for one discrete element, we calculate the integral over the volume. Here we use the following formula:

$$\int_V f(x;y)\,dV=L\int_{-a}^{a}\int_{-b}^{b}f(x;y)\,dxdy \tag{6}$$

Using (6) we calculate the integral:

$$J_{11}=\int_V\frac{1}{2}\left[K_{xx}\left(\frac{\partial T}{\partial x}\right)^2\right]dV \tag{7}$$

In calculating this integral, we use the expression (4). As a result, we have:

$$1) \int_{-a}^{a}\int_{-b}^{b} \frac{b^2-2by+y^2}{16a^2b^2}T_1^2dxdy = \frac{2aT_1^2}{16a^2b^2}\int_{-b}^{b}\left(b^2-2by+y^2\right)dy =$$

$$= \frac{2aT_1^2}{16a^2b^2}\left[2b^3-0+\frac{2b^3}{3}\right] = \frac{T_1^2}{ab^2}\cdot\frac{8b^3}{3} = \frac{b}{3a}T_1^2; \quad (8)$$

$$2) \int_{-a}^{a}\int_{-b}^{b}\left(-2\cdot\frac{b^2-2by+y^2}{16a^2b^2}\cdot T_1T_2\right)dxdy = -\frac{T_1T_2}{8a^2b^2}\cdot 2a\int_{-b}^{b}\left(b^2-2by+y^2\right)dy =$$

$$= -\frac{T_1T_2}{4ab^2}\left[2b^3+\frac{2b^3}{3}\right] = -\frac{2b}{3a}T_1T_2; \quad (9)$$

$$3) \int_{-a}^{a}\int_{-b}^{b}\left(-2\cdot\frac{b^2-y^2}{16a^2b^2}\cdot T_1T_3\right)dxdy = -\frac{T_1T_3}{8a^2b^2}\cdot 2a\left[2b^3-\frac{2b^3}{3}\right] = -\frac{b}{3a}T_1T_3; \quad (10)$$

$$4) \int_{-a}^{a}\int_{-b}^{b}\left(-2\cdot\frac{b^2-y^2}{16a^2b^2}\cdot T_1T_4\right)dxdy = \frac{b}{3a}T_1T_4; \quad (11)$$

$$5) \int_{-a}^{a}\int_{-b}^{b}\left(\frac{b^2-2by+y^2}{16a^2b^2}\cdot T_2^2\right)dxdy = \frac{2aT_2^2}{16a^2b^2}\left[2b^3+\frac{2b^3}{3}\right] = \frac{b}{3a}T_2^2; \quad (12)$$

$$6) \int_{-a}^{a}\int_{-b}^{b}\left(2\cdot\frac{b^2-y^2}{16a^2b^2}\cdot T_2T_3\right)dxdy = \frac{T_2T_3}{8a^2b^2}\cdot 2a\cdot\frac{4b^3}{3} = \frac{b}{3a}T_2T_3; \quad (13)$$

$$7) \int_{-a}^{a}\int_{-b}^{b}\left(-2\cdot\frac{b^2-y^2}{16a^2b^2}\cdot T_2T_4\right)dxdy = -\frac{b}{3a}T_2T_4; \quad (14)$$

$$8) \int_{-a}^{a}\int_{-b}^{b}\left(\frac{b^2+2by+y^2}{16a^2b^2}\cdot T_3^2\right)dxdy = \frac{T_3^2}{16a^2b^2}\cdot 2a\cdot\frac{8b^3}{3} = \frac{b}{3a}T_3^2; \quad (15)$$

$$9) \int_{-a}^{a}\int_{-b}^{b}\left(-2\cdot\frac{b^2+2by+y^2}{16a^2b^2}\cdot T_3T_4\right)dxdy = -\frac{2b}{3a}T_3T_4; \quad (16)$$

$$10) \int_{-a}^{a}\int_{-b}^{b}\left(\frac{b^2+2by+y^2}{16a^2b^2}\cdot T_4^2\right)dxdy=\frac{b}{3a}T_4^2. \quad (17)$$

Substituting (8) - (17) into (7), we find the integrated form J_{11}:

$$\begin{aligned}
J_{11} &= \int_V \tfrac{1}{2}\left[K_{xx}\left(\tfrac{\partial T}{\partial x}\right)^2\right]dV = \tfrac{LK_{xx}}{2}\int_{-a}^{a}\int_{-b}^{b}\left(\tfrac{\partial T}{\partial x}\right)^2 dxdy = \\
&= \tfrac{LK_{xx}}{2}\int_{-a}^{a}\int_{-b}^{b}\left[\tfrac{b^2-2by+y^2}{16a^2b^2}\cdot T_1^2 - 2\cdot\tfrac{b^2-2by+y^2}{16a^2b^2}\cdot T_1T_2 - 2\cdot\tfrac{b^2-y^2}{16a^2b^2}\cdot T_1T_3+\right.\\
&+2\cdot\tfrac{b^2-y^2}{16a^2b^2}\cdot T_1T_4+\tfrac{b^2-2by+y^2}{16a^2b^2}\cdot T_2^2+2\cdot\tfrac{b^2-y^2}{16a^2b^2}\cdot T_2T_3-2\cdot\tfrac{b^2-y^2}{16a^2b^2}\cdot T_2T_4+\\
&\left.+\tfrac{b^2+2by+y^2}{16a^2b^2}\cdot T_3^2-2\cdot\tfrac{b^2+2by+y^2}{16a^2b^2}\cdot T_3T_4+\tfrac{b^2+2by+y^2}{16a^2b^2}\cdot T_4^2\ \right]dxdy = \\
&= \tfrac{bLK_{xx}}{6a}\left[T_1^2-2T_1T_2-T_1T_3+T_1T_4+T_2^2+T_2T_3-T_2T_4+T_3^2-2T_3T_4+T_4^2\right].
\end{aligned} \quad (18)$$

Examining the last expression, we find that the sum of the coefficients in front of the nodal temperature values will be zero. Indeed, from (18) we find that (1-2-1 + 1 + 1 + 1-1 + 1-2 + 1) = 0.

Next, similarly, we find the integrated form expression

$$J_{22}=\int_V \frac{1}{2}\left[K_{yy}\left(\frac{\partial T}{\partial y}\right)^2\right]dV. \quad (19)$$

Using (5) we find that

$$1) \int_{-a}^{a}\int_{-b}^{b}\frac{a^2-2ax+x^2}{16a^2b^2}T_1^2dxdy=\frac{T_1^2}{16a^2b^2}\cdot 2b\cdot\frac{8a^3}{3}=\frac{a}{3b}T_1^2; \quad (20)$$

$$2) \int_{-a}^{a}\int_{-b}^{b}\left(2\cdot\frac{a^2-x^2}{16a^2b^2}\cdot T_1T_2\right)dxdy=\frac{2T_1T_2}{16a^2b^2}\cdot 2b\cdot\frac{4a^3}{3}=\frac{a}{3b}\cdot T_1T_2; \quad (21)$$

$$3) \int_{-a}^{a}\int_{-b}^{b}\left(-2\cdot\frac{a^2-x^2}{16a^2b^2}\cdot T_1T_3\right)dxdy=-\frac{2aT_1T_3}{6b}=-\frac{a}{3b}\cdot T_1T_3; \quad (22)$$

$$4) \int_{-a}^{a}\int_{-b}^{b}\left(-2\cdot\frac{a^2-2ax+x^2}{16a^2b^2}T_1T_4\right)dxdy=-\frac{2a}{3b}T_1T_4; \quad (23)$$

$$5) \int_{-a}^{a}\int_{-b}^{b}\left(\frac{a^2+2ax+x^2}{16a^2b^2}T_2^2\right)dxdy=\frac{a}{3b}T_2^2; \quad (24)$$

$$6) \int_{-a}^{a}\int_{-b}^{b}\left(-2\cdot\frac{a^2+2ax+x^2}{16a^2b^2}T_2T_3\right)dxdy=-\frac{2a}{3b}T_2T_3; \quad (25)$$

$$7) \int_{-a}^{a}\int_{-b}^{b}\left(-2\cdot\frac{a^2-x^2}{16a^2b^2}\cdot T_2T_4\right)dxdy=-\frac{a}{3b}\cdot T_2T_4; \tag{26}$$

$$8) \int_{-a}^{a}\int_{-b}^{b}\left(\frac{a^2+2ax+x^2}{16a^2b^2}T_3^2\right)dxdy=\frac{a}{3b}T_3^2; \tag{27}$$

$$9) \int_{-a}^{a}\int_{-b}^{b}\left(2\cdot\frac{a^2-x^2}{16a^2b^2}\cdot T_3T_4\right)dxdy=\frac{a}{3b}\cdot T_3T_4; \tag{28}$$

$$10) \int_{-a}^{a}\int_{-b}^{b}\left(\frac{a^2-2ax+x^2}{16a^2b^2}T_4^2\right)dxdy=\frac{a}{3b}T_4^2. \tag{29}$$

Substituting (20) - (29) into (19) we define the integrated form J_{22}:

$$J_{22}=\int_V \tfrac{1}{2}\left[K_{yy}\left(\tfrac{\partial T}{\partial y}\right)^2\right]dV=\tfrac{LK_{yy}}{2}\int_{-a}^{a}\int_{-b}^{b}\left(\tfrac{\partial T}{\partial y}\right)^2 dxdy=$$
$$=\tfrac{aLK_{yy}}{6a}\left[T_1^2+T_1T_2-T_1T_3-2T_1T_4+T_2^2-2T_2T_3-T_2T_4+T_3^2+T_3T_4+T_4^2\right].$$

In this expression, the sum of the coefficients in front of the nodal temperature values will be zero [2, 7, 8]. Then there is found the expression for J_1:

$$J_1=J_{11}+J_{22}=\tfrac{bLK_{xx}}{6a}\left[T_1^2-2T_1T_2-T_1T_3+T_1T_4+T_2^2+T_2T_3-T_2T_4+T_3^2-2T_3T_4+T_4^2\right]+$$
$$+\tfrac{aLK_{yy}}{6a}\left[T_1^2+T_1T_2-T_1T_3-2T_1T_4+T_2^2-2T_2T_3-T_2T_4+T_3^2+T_3T_4+T_4^2\right].$$

Now from (3) we find:

$$J_2=\int_{S(x=-a)}qTdS=Lq\int_{-b}^{b}\left[\phi_1(x;y)T_1+\phi_2(x;y)T_2+\phi_3(x;y)T_3+\phi_4(x;y)T_4\right]|_{x=-a}\,dy=$$
$$=\tfrac{Lq}{2b}\int_{-b}^{b}\left[(b-y)T_1+(b+y)T_4\right]dy=\tfrac{Lq}{2b}\left[2b^2T_1+2b^2T_4\right]=Lqb\left[T_1+T_4\right].$$

From (3) we calculate:

$$\begin{aligned}J_3&=\int_{S(x=a)}\tfrac{h}{2}(T-T_{oc})^2dS=\tfrac{hL}{2}\int_{-b}^{b}\left[\textstyle\sum_{i=1}^{4}\phi_i(x;y)T_i-T_{oc}\right]^2_{x=a}dy=\\&=\tfrac{hL}{2}\int_{-b}^{b}\left[\phi_1(x;y)T_1+\phi_2(x;y)T_2+\phi_3(x;y)T_3+\phi_4(x;y)T_4-T_{oc}\right]^2_{x=a}dy=\\&=\tfrac{hL}{2}\int_{-b}^{b}\Big[\tfrac{b^2+2by+y^2}{4b^2}T_2^2+2\tfrac{b^2-y^2}{4b^2}T_2T_3-2\tfrac{b-y}{2b}T_2T_{oc}+\\&+\tfrac{b^2+2by+y^2}{4b^2}T_3^2-2\tfrac{b+y}{2b}T_3T_{oc}+T_{oc}^2\Big]dy.\end{aligned} \tag{30}$$

Now in (30) we calculate each integral separately:

$$1)\ \int_{-b}^{b}\left[\frac{b^2-2by+y^2}{4b^2}T_2^2\right]dy=\frac{1}{4b^2}\left(2b^3+\frac{2b^3}{3}\right)T_2^2;$$

$$2)\ \int_{-b}^{b}\left[2\cdot\frac{b^2-y^2}{4b^2}T_2T_3\right]dy=\frac{1}{2b^2}\left(2b^3-\frac{2b^3}{3}\right)T_2T_3;$$

$$3)\ \int_{-b}^{b}\left[2\cdot\frac{b-y}{4b^2}T_2T_{oc}\right]dy=\frac{1}{b}\left(2b^2-0\right)T_2T_{oc};$$

$$4)\ \int_{-b}^{b}\left[\frac{b^2+2by+y^2}{4b^2}T_3^2\right]dy=\frac{1}{4b^2}\left(2b^3+\frac{2b^3}{3}\right)T_3^2;$$

$$5)\ \int_{-b}^{b}\left[2\cdot\frac{b+y}{2b}T_3T_{oc}\right]dy=2bT_3T_{oc};$$

$$6)\ \int_{-b}^{b}T_{oc}^2dy=2bT_{oc}^2. \tag{31}$$

Substituting (31) into (30) we find the integrated form J_3:

$$J_3=\int_{S(x=a)}\frac{h}{2}\left(T-T_{oc}\right)^2dS=\frac{hL}{2}\left[\frac{1}{4b^2}\left(2b^3+\frac{2b^3}{3}\right)T_2^2+\frac{1}{2b^2}\left(2b^3-\frac{2b^3}{3}\right)T_2T_3-\right.$$
$$\left.\frac{1}{b}\left(2b^2-0\right)T_2T_{oc}+\frac{1}{4b^2}\left(2b^3+\frac{2b^3}{3}\right)T_3^2-2bT_3T_{oc}+2bT_{oc}^2\right]=$$
$$=\frac{hL}{2}\left[\frac{2b}{3}T_2^2+\frac{2b}{3}T_2T_3-2bT_2T_{oc}+\frac{2b}{3}T_3^2-2bT_3T_{oc}\right]=$$
$$=\frac{hLb}{3}\left[T_2^2+T_2T_3-3T_2T_{oc}+T_3^2-3T_3T_{oc}+3T_{oc}^2\right].$$

It should also be noted here that in the expression in the bracket the sum of the coefficients will be equal to zero.

Given the expressions J_1; J_2 and J_3, from (3) there is determined the final integrated form of the J functional that characterizes the total thermal energy of the timber under study, taking into account the simultaneous presence of a heat flux, thermal insulation and heat transfer:

$$J=J_1+J_2+J_3=\frac{bLK_{xx}}{6a}\left[T_1^2-2T_1T_2-T_1T_3+T_1T_4+T_2^2+T_2T_3-T_2T_4+T_3^2-2T_3T_4+T_4^2\right]+$$
$$+\frac{aLK_{yy}}{6a}\left[T_1^2+T_1T_2-T_1T_3-2T_1T_4+T_2^2-2T_2T_3-T_2T_4+T_3^2+T_3T_4+T_4^2\right]+bLq\left[T_1+T_4\right]+$$
$$+\frac{bLh}{3}\left[T_2^2+T_2T_3-3T_2T_{oc}+T_3^2-3T_3T_{oc}+3T_{oc}^2\right].$$

Further, minimizing the functional J with respect to the nodal values of the temperatureT_1, T_2, T_3 and T_4 we find the resolving system of linear algebraic equations

$$1)\frac{\partial J}{\partial T_1}=0\Rightarrow\frac{bLK_{xx}}{6a}\left[2T_1-2T_2-T_3+T_4\right]+\frac{aLK_{yy}}{6a}\left[2T_1+T_2-T_3-2T_4\right]+bLq=0;$$

$$2)\frac{\partial J}{\partial T_2}=0\Rightarrow\frac{bLK_{xx}}{6a}\left[-2T_1+2T_2+T_3-T_4\right]+\frac{aLK_{yy}}{6a}\left[T_1+2T_2-2T_3-T_4\right]++\frac{bLq}{3}\left[2T_2+T_3-3T_{oc}\right]=0;$$

$$3)\frac{\partial J}{\partial T_3}=0\Rightarrow$$

$$\frac{bLK_{xx}}{6a}\left[-T_1+T_2+2T_3-2T_4\right]+\frac{aLK_{yy}}{6a}\left[-T_1-2T_2+2T_3+T_4\right]++\frac{bLq}{3}\left[T_2+2T_3-3T_{oc}\right]=0;$$

$$4)\frac{\partial J}{\partial T_4}=0\Rightarrow\frac{bLK_{xx}}{6a}\left[T_1-T_2-2T_3+2T_4\right]+\frac{aLK_{yy}}{6a}\left[-2T_1-T_2+T_3+2T_4\right]+bLq=0.$$

For convenience, we discretize with 6 elements. The global numbering of elements and nodes is shown in (Figure 2). Now, for all the finite elements, there is an expression for the J functional that characterizes its total thermal energy, taking into account the existing boundary conditions [2, 6-14].

The integrated form of this functional for all discrete elements is as follows:

$$\begin{aligned}J=&\left(\tfrac{aLK_{xx}}{6b}\right)_{IE}\left[T_1^2-2T_1T_2-T_1T_6+T_1T_5+T_2^2+T_2T_6-T_2T_5+T_6^2-2T_6T_5+T_5^2\right]+\\&+\left(\tfrac{bLK_{yy}}{6a}\right)_{IE}\left[T_1^2+T_1T_2-T_1T_6-2T_1T_5+T_2^2-2T_2T_6-T_2T_5+T_6^2+T_6T_5+T_5^2\right]+\\&+(aLq)_{IE}\left[T_1+T_2\right]+\left(\tfrac{aLK_{xx}}{6b}\right)_{IIE}\left[T_3^2-T_3T_4-T_3T_9+T_3T_8+T_4^2+T_4T_9-T_2T_8+\right.\\&\left.+T_9^2-2T_9T_8+T_8\right]+\left(\tfrac{bLK_{yy}}{6a}\right)_{IIE}\left[T_3^2+T_3T_4-T_3T_9-2T_3T_8+T_4^2-2T_4T_9-T_4T_8+\right.\end{aligned}$$

$$\begin{aligned}&\left.+T_9^2+T_9T_8+T_8^2\right]+(aLq)_{IIE}\left[T_3+T_4\right]+\left(\tfrac{aLK_{xx}}{6b}\right)_{IIIE}\left[T_5^2-2T_5T_6-T_5T_{11}+T_5T_{10}+\right.\\&\left.+T_6^2+T_6T_{11}-T_6T_{10}+T_{11}^2-2T_{11}T_{10}+T_{10}^2\right]+\left(\tfrac{bLK_{yy}}{6a}\right)_{IIIE}\left[T_5^2+T_5T_6-T_5T_{11}-2T_5T_{10}+\right.\\&\left.+T_6^2-2T_6T_{11}-T_6T_{10}+T_{11}^2+T_{11}T_{10}+T_{10}^2\right]+\left(\tfrac{bLh}{3}\right)_{IIIE}\left[T_{11}^2+T_{11}T_{10}-3T_{11}T_e+T_{10}^2-\right.\\&\left.-3T_{10}T_e+3T_e^2\right]+\left(\tfrac{aLK_{xx}}{6b}\right)_{IVE}\left[T_6^2-2T_6T_7-T_6T_{12}+T_6T_{11}+T_7^2-T_7T_{12}-T_7T_{11}+T_{12}^2-\right.\\&\left.-2T_{12}T_{11}+T_{11}^2\right]+\left(\tfrac{bLK_{yy}}{6a}\right)_{IVE}\left[T_6^2+T_6T_7-T_6T_{12}-2T_6T_{11}+T_7^2-2T_7T_{12}-T_7T_{11}+T_{12}^2+\right.\\&\left.+T_{12}T_{11}+T_{11}^2\right]+\left(\tfrac{bLh}{3}\right)_{IVE}\left[T_{12}^2+T_{12}T_{11}-3T_{12}T_e+T_{11}^2-3T_{11}T_e+3T_e^2\right]+\end{aligned}$$

$$\begin{aligned}&+\left(\tfrac{aLK_{xx}}{6b}\right)_{VE}\left[T_7^2-2T_7T_8-T_7T_{13}+T_7T_{12}+T_8^2+T_8T_{13}-T_8T_{12}+T_{13}^2-2T_{13}T_{12}+T_{12}^2\right]+\\&+\left(\tfrac{bLK_{yy}}{6a}\right)_{VE}\left[T_7^2+T_7T_8-T_7T_{13}-2T_7T_{12}+T_8^2-2T_8T_{13}-T_8T_{12}+T_{13}^2+T_{13}T_{12}+T_{12}^2\right]+\\&+\left(\tfrac{bLh}{3}\right)_{VE}\left[T_{13}^2+T_{13}T_{12}-3T_{13}T_e+T_{12}^2-3T_{12}T_e+3T_e^2\right]+\\&+\left(\tfrac{aLK_{xx}}{6b}\right)_{VIE}\left[T_8^2-2T_8T_9-T_8T_{14}+T_8T_{13}+T_9^2+T_9T_{14}-T_9T_{13}+T_{14}^2-2T_{14}T_{13}+T_{13}^2\right]+\\&+\left(\tfrac{bLK_{yy}}{6a}\right)_{VIE}\left[T_8^2+T_8T_9-T_8T_{14}-2T_8T_{13}+T_9^2-2T_9T_{14}-T_9T_{13}+T_{14}^2+T_{14}T_{13}+T_{13}^2\right]+\\&+\left(\tfrac{aLh}{3}\right)_{VIE}\left[T_{14}^2+T_{14}T_{13}-3T_{14}T_e+T_{13}^2-3T_{13}T_e+3T_e^2\right]\end{aligned}$$

Further, minimizing the last functional over nodal values, we obtain the following system of linear algebraic equations with respect to T_i:

$$\frac{\partial J}{\partial T_i} = 0, (i = 1 \div 14).$$

Solving the last system by the Gaussian method, we determine the nodal values of temperatures, and according to them, according to (1), the temperature value at any point of each finite element. Inparticular, withthefollowinginitial [1, 2]:

$K_{xx} = K_{yy} = 72 \left[\frac{W}{cm \cdot {}^{\circ}C}\right]$; $a{=}b{=}1cm$; $q = -100 \left[\frac{W}{cm^2}\right]$; $h_e = 6 \left[\frac{W}{cm^2 \cdot {}^{\circ}C}\right]$;

$T_e = 40^{\circ}C$; $r{=}2\ cm$; $l{=}1cm$.

Wefindthat

$$T_1 = T_4 = 52.895^{\circ}C; T_2 = T_3 = 53.017^{\circ}C; T_5 = T_9 = 50.482^{\circ}C; T_6 = T_8 = 49.874^{\circ}C;$$

$$T_7 = 48.658^{\circ}C; T_{10} = T_{14} = 48.573^{\circ}C; T_{11} = T_{13} = 48.304^{\circ}C; T_{12} = 48.152^{\circ}C.$$

It can be seen from the obtained results that due to the symmetrical formulation of the problem under consideration, the process of the steady distribution of the temperature field in the section of the beam will also be symmetrical.

4 Conclusions

The proposed mathematical model, based on the law of conservation and change of thermal energy, allows us to solve a class of multidimensional problems of steady thermal conductivity for structural elements of any configuration, where there is a heat flux and a temperature, partial thermal insulation, and heat transfer.

Because of the symmetry of the nodal points of the problem under consideration, in this work the results of the numerical solution are symmetrical, i.e., there are the same temperature values.

The exact calculation of distributing the temperature field at each nodal point is determined by formula (1). Based on the energy principle combined with the finite element method, the steady-state temperature distribution field in the volume of a partially thermally insulated beam in the presence of a heat flux and heat exchange is studied numerically. A numerical solution is given for specific initial data. A numerical study of the convergence and accuracy of the obtained numerical solutions is carried out.

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