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MATHEMATICAL MODELLING OF THE PROCESS OF NATURAL GAS TRANSPORTATION VIA PIPE NETWORKS USING CROSSING-BRANCH METHOD

This research is among the most relevant research on the problems of natural gas transportation via pipe networks. The increased demand for natural gas in Kazakhstan is associated with a greater level of environmental friendliness; as a result, many power-generating stations use natural gas as their main source of energy. Modernization of existing thermal power plants is necessary to improve the environmental situation in the country. There are three main groups of gas pipeline systems considered in the literature: collection, transmission, and distribution systems. In this article, we present detailed research on the transmission process and develop useful approaches. Over the past few years, a huge amount of research has been conducted on many problems of decision-making in the gas industry and, in particular, on optimizing the pipeline network. In this paper, we consider dynamical models, highlighting aspects of modelling and the most relevant solutions to date. This research can serve as a useful tool for understanding the evolution of many real-world applications and the most recent advances in solution methodologies emerging in this complex field of research. The results of this research can be used to develop technologies for automating calculations, planning, and optimizing natural gas transportation.
Key words: natural gas transportation, mathematical modelling, nonlinear model.

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Қиылысу-тармақ әдісін қолдана отырып, табиғи газды құбыр желілері арқылы тасымалдау процесін математикалық модельдеу

Қарастырылып отырған зерттеу жұмысы табиғи газды құбыр желілері арқылы тасымалдау мәселелері бойынша ең өзекті зерттеулердің бірі болып табылады. Қазақстандағы табиғи газға деген сұраныстың артуы қоршаған ортаға зиянсыздық деңгейінің жоғарылауымен байланысты; нәтижесінде көптеген электр станциялары негізгі энергия көзі ретінде табиғи газды пайдаланады. Елдегі экологиялық жағдайды жақсарту үшін жұмыс істеп тұрған жылу электр станцияларын жаңғырту қажет. Әдебиеттерде қарастырылған газ құбырлары жүйелерінің үш негізгі тобы бар: жинау, тасымалдау және тарату жүйелері. Бұл мақалада біз тасымалдау процесі туралы егжей-тегжейлі зерттеулерді ұсынамыз және пайдалы тәсілдерді әзірлейміз. Соңғы бірнеше жылда газ саласындағы шешімдерді қабылдаудың көптеген мәселелері бойынша, атап айтқанда, құбыр желісін оңтайландыру бойынша үлкен көлемдегі зерттеулер жүргізілді. Бұл жұмыста біз динамикалық модельдерді қарастырамыз, модельдеудің аспектілерін және бүгінгі таңдағы ең өзекті шешімдерді атап өтеміз. Бұл зерттеу көптеген нақты әлем қолданбаларының эволюциясын және осы күрделі зерттеу саласында пайда болған шешім әдістемелеріндегі ең соңғы жетістіктерді түсіну үшін пайдалы құрал бола алады. Бұл зерттеулердің нәтижелерін есептеулерді автоматтандыру, жоспарлау және табиғи газды тасымалдауды оңтайландыру технологияларын әзірлеу үшін пайдалануға болады.

Түйін сөздер: табиғи газды тасымалдау, математикалық модельдеу, сызықты емес модель.

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Математическое моделирование процесса транспортировки природного газа по трубопроводным сетям методом пересечения-ветви

Данное исследование относится к числу наиболее актуальных исследований по проблемам транспортировки природного газа по трубопроводным сетям. Повышенный спрос на природный газ в Казахстане связан с повышением уровня экологичности; в результате многие электростанции используют природный газ в качестве основного источника энергии. Модернизация существующих теплоэлектростанций необходима для улучшения экологической ситуации в стране. В литературе рассматриваются три основные группы газопроводных систем: системы сбора, транспортировки и распределения. В этой статье мы представляем подробное исследование процесса передачи и разрабатываем полезные подходы. За последние несколько лет было проведено огромное количество исследований по многим проблемам принятия решений в газовой отрасли и, в частности, по оптимизации трубопроводной сети. В данной статье мы рассматриваем динамические модели, освещая аспекты моделирования и наиболее актуальные на сегодняшний день решения. Это исследование может послужить полезным инструментом для понимания эволюции многих реальных приложений и последних достижений в методологиях решения, возникающих в этой сложной области исследований. Результаты исследования могут быть использованы при разработке технологий автоматизации расчетов, планирования и оптимизации транспортировки природного газа.

Ключевые слова: транспортировка природного газа, математическое моделирование, нелинейная модель.

1 Introduction

This research is intended to build mathematical and computer simulations of non-stationary modes with the optimization of the gas transportation process by choosing the most effective control strategy and control actions on the technological equipment of the main gas pipelines of the gas transmission system.

Over the past couple of centuries, fossil fuels have been the primary source of energy and essential to global economic growth. Originally, coal was the main source of energy, but oil later replaced it and became an important factor in maintaining civilization.

In the modern world, the rise in prices for non-renewable energy sources is breaking records in the entire history of their production. It is advisable to connect this with the deteriorating environmental situation in the world. A gradual transition to clean types of energy, such as solar, wind, etc., is planned by most developed countries. But immediate transformation of energy systems is impossible and takes time. Therefore, the most promising direction is less polluting energy sources such as natural gas.

Complex gas-dynamic processes occurring in a pipeline in transient conditions require a more comprehensive solution to problems such as management, design and operation of gas transmission systems. It should also be noted that gas transport is carried out in large diameter pipes and under high pressure.

The characteristics of the gas in the pipeline change in real time, which significantly complicates the modeling process and does not allow one to estimate the gas parameters well enough to make a decision. Gas dynamics is described by a system of three partial differential equations based on the laws of conservation of energy, mass and momentum. But for practical purposes, the obtained numerical solutions of this system do not give satisfactory results.

An acute discussion about physical phenomena on graphs continues, particularly regarding the context of gas dynamics [1], [2] including the publications of the Pipeline Interest Group [3] or textbooks such as [4], [5].

In particular, accurate but computationally cheap prediction of gas dynamics in pipeline networks is a major industrial problem and has been studied for several years [6], [7]. In gas dynamics, the predominant physical phenomenon is pressure loss due to the effects of hydraulic friction on the pipe walls. Currently, there are several models that describe this effect with varying degrees of accuracy. Most of these models are given on isothermal Euler equations, i.e., a system of nonlinear partial differential equations for each pipe. The main difficulty lies in nonlinear dynamics, i.e., the nonlinearity of the system of differential equations, which limits the possibility of efficient and accurate modelling of pipelines and pipeline networks.

We consider the problem of modelling and simulating gas dynamics in pipes. The key point in modeling is the non-stationary nature of the process, which significantly affects the construction of optimal control, reducing the quality of the resulting solutions. Stationary modes are well studied and are often used in modeling natural gas transport, but they reduce the plausibility and reality of such processes, which leads to low confidence in the results obtained for management and decision-making.

Obtaining new capacity while maintaining emissions regulations is one of the difficult problems facing countries. The most optimal solution to switch to gas leads to the construction of gas-fired thermal power plants and gas power plants. The natural gas power generation vector is a reliable hedge against the variability of renewable energy sources such as wind and solar [8], [9]. Such conditions for energy supply are a challenge for gas transportation systems, since different volumes of consumption must be included in the modes of the gas transportation process in order to provide the necessary capacities in a timely manner. New approaches should be considered to replace those that existed in conditions of stable gas supplies, when gas from the field was supplied to consumers in accordance with established volumes [10], [11].

Such contracts ensured a nearly constant supply of gas [12]. Modeling and optimization of the transportation process was limited to the consideration of stationary processes [13], [14]. But now, consideration of the stationary case for gas-dynamic systems is becoming unsatisfactory. Several approaches to such research are encompassed in [15], [16].

The article is structured as follows:

- Section 2 represents the description and statement of methods.
- Section 3 provides a description of the main computational procedures and their relationship. The results of the calculations obtained using the dynamical model are shown in graphs.
- Section 4 discusses the meaning, importance, and relevance of the research results.
- Conclusions on the main outcomes of this study are finally presented in Section 5.

2 Methods

The following is a mathematical model that most fully describes the process of gas movement in a pipe, i.e., Newton's equation of motion

$$\frac{\partial P}{\partial x} + g\rho \frac{dh}{dx} + \frac{\lambda}{2DS^2}\frac{|M|M}{\rho} + \rho \frac{DW}{dt} = 0, \tag{1}$$

where P is pressure, g is acceleration due to gravity, ρ is density, $\frac{dh}{dx}$ is the slope of the pipeline, λ is the coefficient of hydraulic friction, D is the inner diameter, S is the cross-sectional area, M is mass flow rate,

$$\frac{DW}{dt} = \frac{\partial w}{\partial t} + w\frac{\partial w}{\partial x},$$

and x is the length of the pipe.

The continuity equation is as follows:

$$S\frac{\partial \rho}{\partial t} + \frac{\partial M}{\partial x} = 0, \tag{2}$$

and the energy equation is written in the form of temperature $\theta = \theta(x)$ under the assumption of zero temperature gradient over time. Also, the equation of state is

$$P = zR\theta\rho, \tag{3}$$

where R is the gas constant and z is the compressibility factor.

The coefficient of hydraulic friction can be calculated using Chen's formula [17], which is often used in practice:

$$\frac{1}{\sqrt{\lambda}} = -2\log\left(\frac{\varepsilon/D}{3.7065} - \frac{5.0452}{N_{Re}}\log\left(\frac{1}{2.8257}\left(\frac{\varepsilon}{D}\right)^{1.1098} + \frac{5.8506}{0.8981 N_{Re}}\right)\right).$$

Here, N_{Re} is the Reynolds number, $N_{Re} = \rho u D/\mu$, μ is the dynamic viscosity of the gas, u is velocity, and ε is the roughness of the pipe.

Typically, isothermal models are used to model gas transport, ignoring temperature changes along the length of the pipeline and over time.

There are many formulas for calculating the compressibility coefficient; for example, see the source for an overview of existing methods [18]. Based on many studies, the standards for calculating the gas compressibility factor based on the parameters and composition of natural gas SGERG-88 [19], GERG-2004 [20], and the latest version of the standard GERG-2008 [21] were constructed; these take into account most of the dependent factors.

In accordance with [22], a simplified formula is used in practice; the compressibility coefficient of natural gas at a pressure of up to $15MPa$ and temperature in range of $250 - 400K$, z is calculated by the formula

$$z = 1 + A_1 P_{red} + A_2 P_{red}, \tag{4}$$

where

$$A_1 = -0.39 + 2.03T_{red} - 3.16T_{red}^2 + 1.0T_{red}^3,$$

$$A_2 = 0.0423 - 0.1812T_{red} + 0.2124T_{red}^2,$$

$$P_{red} = PP_{cr},$$

$$T_{red} = TT_{cr}.$$

The critical pressure and temperature of the gas mixture can also be determined from a known density under standard conditions ($T = 293.15K$, $P = 101325Pa$):

$$P_{cr} = 0.1737 \cdot (26.813 - \rho_{st}),\ T_{cr} = 155.24 \cdot (0.564 + \rho_{st}),$$

where ρ_{st} is the gas density under standard conditions (set according to the gas passport).

The basis for all simplifications under the assumption of an isothermal process, and, therefore, also the classical dynamics of a gas in a pipe, is described by the Euler equations:

$$\rho_t + q_x = 0,$$

$$q_t + \left(\frac{q^2}{\rho} + a^2\rho\right)_x = -\lambda\frac{q|q|}{2D\rho} - g\rho h_x.$$

Here, q is the gas flow, $q = \rho u$, and $a^2 = zRT$. The first equation is the law of conservation of mass, and the second equation describes momentum. The system represents a hyperbolic equilibrium law involving the effects of friction and gravity (described by gh_x). The computational complexity for modelling a pipeline network using these equations is high. The first approximation of this system can be obtained by performing an approximate estimate of the term $\partial\left(q^2/\rho\right)$ and excluding it from the equation. The resulting system is written as follows:

$$q_t\left(a^2\rho\left(1 + \frac{q^2}{\rho^2 a^2}\right)\right)_x = -\lambda\frac{q|q|}{2D\rho} - g\rho h_x.$$

This system, known as the Weymouth equations, reduces to solving a linear system of hyperbolic partial differential equations for each pipe.

In the research implementation, we consider non-isothermal (1) - (3) models using the compressibility factor (4).

Adopting the space discretization according to Figure **??**, the equation of motion (1) is represented as

$$\rho_{i+1} - \rho_i + \frac{\lambda_i\left|\hat{w}_i\right| L_i}{2D_i S_i \tilde{w}_{zi}^2} M_i + g\frac{\Delta h_i}{\tilde{w}_{\mathrm{zi}}^2}\hat{\rho}_i + \frac{\tilde{\rho}_i + b^*\bar{\rho}_{i+1}\bar{\rho}_i}{\tilde{\theta}_i}\Delta\theta_i + \frac{L_i}{S_i\tilde{w}_{zi}^2}\frac{dM_i}{dt} = 0, (i = 1, 2, \ldots, n), \quad (5)$$

where

$$\hat{w}_i = \frac{\bar{M}_i}{S_i\bar{\rho}_i},$$

$$\tilde{w}_{\mathrm{zi}}^2 = R\tilde{\theta}_i\bar{Z}_i\bar{Z}_{i+1},$$

$$\hat{\rho}_i = \frac{\bar{p}_i + \bar{p}_{i+1}}{2R\bar{\theta}_i\sqrt{\bar{Z}_i\bar{Z}_{i+1}}},$$
$$\tilde{\theta}_i = 0.5\left(\bar{\theta}_i + \bar{\theta}_{i+1}\right), \tilde{\rho}_i = 0.5\left(\bar{\rho}_i + \bar{\rho}_{i+1}\right),$$
$$\bar{p}_i = R\bar{\theta}_i\bar{Z}_i\bar{\rho}_i,$$
$$\bar{Z}_i = \frac{1}{1+\bar{\rho}_i\left(b^* - a^*\bar{\theta}_i\right)}, \Delta\theta_i = \theta_{i+1} - \theta_i,$$
$$\Delta h_i = h_{i+1} - h_i, \frac{\hat{\rho}_i}{\tilde{w}_{\text{zi}}^2} = \frac{2\bar{\rho}_i\bar{\rho}_{i+1}}{\left(\bar{p}_i + \bar{p}_{i+1}\right)\sqrt{\bar{Z}_i\bar{Z}_{i+1}}}.$$

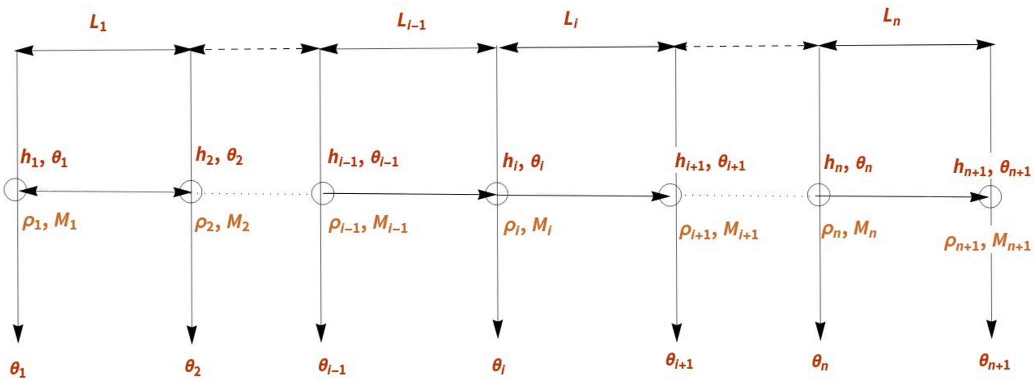

Figure 1: Discretization scheme by space

By following the paper [23], we write a discretized system of (2) - (3):

$$\begin{aligned}
&\frac{d\rho_1}{dt} + \frac{2}{L_1S_1}M_1 = -\frac{2}{L_1S_1}Q_1,\\
&\frac{d\rho_i}{dt} + \frac{2}{L_{i-1}S_{i-1}+L_iS_i}\left(M_i - M_{i-1}\right) = -\frac{2}{L_{i-1}S_{i-1}+L_iS_i}Q_{i,}(i=2,3,...,n)\\
&\frac{d\rho_{n+1}}{dt} + \frac{2}{L_nS_n}\left(-M_n\right) = -\frac{2}{L_nS_n}Q_{n+1}.
\end{aligned} \tag{6}$$

The integration is based on a modified implicit integration method [24] with a nonsymmetric difference:

$$\frac{Y_{n+1}-Y_n}{\Delta t} = A\left((1-v)Y_{n+1} + vY_n\right) + \frac{1}{2}B\left(U_{n+1}+U_n\right), \tag{7}$$

where U_n and Y_n are the values at time t, U_{n+1}, Y_{n+1} are the values at time $t+\Delta t$ which is the integration time-step, and i is the coefficient of nonsymmetric difference.

We write the system of (5) - (6) for $i=2$ in the following form:

$$\alpha_2\frac{d}{dt}M_2 + \alpha_1M_2 + \rho_3 - \rho_2 = r_2, \tag{8}$$

$$\beta \frac{d}{dt}\rho_2 + M_2 - M_1 + Q_2 = 0, \tag{9}$$

$$\alpha_1 = \frac{\lambda_2 \left|\hat{w}_2\right| L_2}{2D_2 S_2 \tilde{w}_{z2}^2}, \alpha_2 = \frac{L_2}{S_2 \tilde{w}_{z2}^2}, r_2 = -g\frac{\Delta h_2}{\tilde{w}_{z2}^2}\hat{\rho}_2 - \frac{\tilde{\rho}_2 + b^* \bar{\rho}_3 \bar{\rho}_2}{\tilde{\theta}_2}\Delta\theta_2, \beta = L_1 S_1 + L_2 S_2. \tag{10}$$

Here, values with a bar above represent values at the previous time step. Then, equations (8) - (10) can be written using the implicit scheme (7):

$$a_2 M_2 + \rho_3 - \rho_2 = f_2, \tag{11}$$

$$d_2 \rho_2 + M_2 - M_1 = e_2, \tag{12}$$

$$\begin{aligned} a_2 &= \frac{\alpha_2}{(1-v)\Delta t} + \alpha_1, \\ f_2 &= \frac{r_2 + \bar{r}_2}{2(1-v)} + \left(\frac{\alpha_2}{(1-v)\Delta t} + \frac{v}{1-v}\alpha_1\right)\bar{M}_2 + \frac{v}{1-v}\left(\bar{\rho}_2 - \bar{\rho}_3\right), \\ d_2 &= \frac{\beta}{(1-v)\Delta t}, \\ e_2 &= -\frac{Q_2 + \bar{Q}_2}{2(1-v)} + \frac{\beta}{(1-v)\Delta t}\bar{\rho}_2 + \frac{v}{1-v}\left(\bar{M}_1 - \bar{M}_2\right). \end{aligned} \tag{13}$$

As a result, we obtain a system of linear equations with a tridiagonal matrix for equations (11) - (13):

$$\begin{pmatrix} a_1 & 1 & & & & & & \\ -1 & d_2 & 1 & & & & & \\ & -1 & a_2 & 1 & & & & \\ & & \cdot & \cdot & \cdot & & & \\ & & & \cdot & \cdot & \cdot & & \\ & & & & \cdot & \cdot & \cdot & \\ & & & & & -1 & d_n & 1 \\ & & & & & & -1 & a_n \end{pmatrix} \cdot \begin{pmatrix} M_1 \\ \rho_2 \\ M_2 \\ \cdot \\ \cdot \\ \cdot \\ \rho_n \\ M_n \end{pmatrix} = \begin{pmatrix} f_1 \\ e_2 \\ f_2 \\ \cdot \\ \cdot \\ \cdot \\ e_n \\ f_n \end{pmatrix} + \begin{pmatrix} \rho_1 \\ 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ -\rho_{n+1} \end{pmatrix}. \tag{14}$$

Equation (14) gives formulas for M_1 and M_n:

$$M_1 = A + B\rho_1 + \mathrm{C}\rho_{n+1}, \tag{15}$$

$$-M_n = U + V\rho_1 + \mathrm{W}\rho_{n+1}. \tag{16}$$

First, let us build formulas for the coefficients of (15) using the left tridiagonal matrix algorithm [25]:

$$\begin{cases} M_1 = \eta_1, \\ \rho_{i+1} = \xi_i M_i + \eta_i, \\ M_{i-1} = \xi_i \rho_i + \eta_i, \ (i = 1,\ 2,\ \ldots,\ n-1), \end{cases},$$

where the coefficients are written as follows:

$$\xi_{n-2} = \frac{a_{2n-2}}{c_{2n-1}},\ \eta_{n-2} = \frac{f_{2n-1}}{c_{2n-1}},\ \xi_i = \frac{a_i}{c_i - b_i \xi_{i+1}}\,(i = 2n-3,\ 2n-4, \ldots,\ 1).$$

Simplifying η_i $(i = 1,\ 2, \ldots,\ n-1)$, we obtain formulas for A, B, and C:

$$B = \frac{1}{c_1 - b_1 \cdot \xi_1}, \tag{17}$$

$$C = -\frac{b_{2n-2}}{c_{2n-1}} \cdot \frac{b_{2n-3}}{c_{2n-2} - b_{2n-2} \cdot \xi_{2n-2}} \cdot \frac{b_{2n-4}}{c_{2n-3} - b_{2n-3} \cdot \xi_{2n-3}} \cdot \ldots \cdot \frac{b_1}{c_2 - b_2 \cdot \xi_2} \cdot \frac{1}{c_1 - b_1 \cdot \xi_1}, \tag{18}$$

$$A = \left(\ldots \left(\left(\left(b_{2n-2} \cdot \frac{f_{2n-3}}{c_{2n-1}} + f_{2n-2} \right) \cdot \frac{b_{2n-3}}{c_{2n-2} - b_{2n-2} \cdot \xi_{2n-2}} + f_{2n-3} \right) \times \right.\right. \tag{19}$$

$$\left.\left. \times \frac{b_{2n-4}}{c_{2n-3} - b_{2n-3} \cdot \xi_{2n-3}} + f_{2n-4} \right) \cdot \ldots \cdot \frac{b_1}{c_2 - b_2 \cdot \xi_2} + f_1 \right) \cdot \frac{1}{c_1 - b_1 \cdot \xi_1}$$

Second, let us build formulas for the coefficients of (16) using the right tridiagonal matrix algorithm [25]:

$$\begin{cases} M_n = \beta_n, \\ \rho_i = \alpha_i M_{i+1} + \beta_i, \\ M_i = \alpha_i \rho_{i+1} + \beta_i,\ (i = 1,\ 2, \ldots,\ n-1) \end{cases},$$

where

$$\alpha_1 = \frac{b_1}{c_1},\ \beta_1 = \frac{f_1}{c_1},\ \alpha_{i+1} = \frac{b_i}{c_i - a_i \alpha_i}\,(i = 1,\ 2, \ldots,\ 2n-2).$$

Simplifying β_i, we obtain formulas for U, V, and W:

$$W = \frac{1}{c_{2n-1} - a_{2n-2} \cdot \alpha_{2n-2}} \tag{20}$$

$$V = \frac{a_1}{c_1} \cdot \frac{a_2}{c_2 - a_1 \cdot \alpha_1} \cdot \frac{a_3}{c_3 - a_2 \cdot \alpha_2} \cdot \ldots \cdot \frac{a_{2n-2}}{c_{2n-2} - a_{2n-3} \cdot \alpha_{2n-3}} \cdot \frac{1}{c_{2n-1} - a_{2n-2} \cdot \alpha_{2n-2}} \tag{21}$$

$$U = -\left(\left(\left(\left(\left(a_1 \cdot \frac{f_1}{c_1} + f_2 \right) \cdot \frac{a_2}{c_2 - a_1 \cdot \alpha_1} + f_3 \right) \cdot \frac{a_3}{c_3 - a_2 \cdot \alpha_2} + f_4 \right) \times \ldots \times \right.\right.\right. \tag{22}$$

$$\left. \times \frac{a_{2n-2}}{c_{2n-2} - a_{2n-3} \cdot \alpha_{2n-3}} + f_{2n-1} \right) \cdot \frac{1}{c_{2n-1} - a_{2n-2} \cdot \alpha_{2n-2}}.$$

In general, these formulas can be written as follows:

$$M_{\alpha\beta} = A + B\rho_\alpha + C\rho_\beta, \tag{23}$$

$$-M_{\alpha\beta} = U + V\rho_\alpha + W\rho_\beta. \tag{24}$$

Here, $M_{\alpha\beta}$ elements describe crossing points in the transmission network (see Figure 3).

3 Numerical results

We use a model for gas dynamics in pipe networks by the crossing-branch method. We present the derivation of the model as well as numerical results illustrating its validity properties.

The calculation scheme consists of three main steps, as shown in Figure 2.

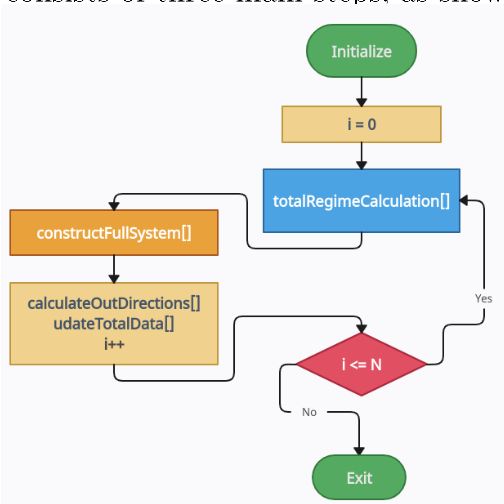

Figure 2: Calculation scheme by space and time

Next we present numerical results on an artificial sample network without compressors. Future work will be dedicated to the model considering compressors along the entire network.

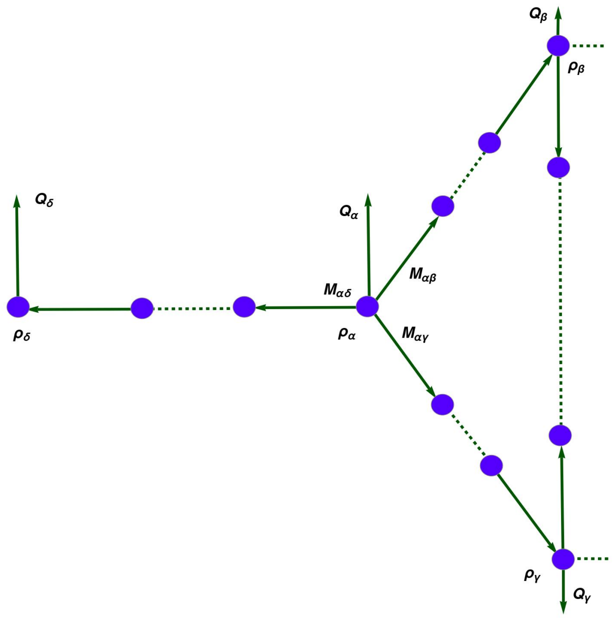

Figure 3: Crossing-branch scheme

To study the model, two calculation scenarios were built:

1. Building a steady state with stable input data.
2. Constructing a dynamic mode with the simulation of the growth of the gas flow at one inlet point of the main pipeline with a constant inlet pressure.

Using the crossing-branch method from [23]– [24] in Figure 3, we obtain numerical results for the artificial gas pipes network, as shown in Figure 4.

The method used in this study can be applied to pipeline networks without involving graphs. It is easy to see in the network figure that the mode of gas flow through pipes can be set as an array of sets of points following the given orientation of the gas flow. Then, we get an array of dimension 17, each element of which consists of a set of network points. For example, for our artificial network, we get the array in Table **??**.

We present an example synthetic network in Figure 4 consisting of a tree with 29 nodes connected by 28 edges with a total length of 370.6 km, 3 gas fields, and 7 terminals or withdrawals, not containing compressors.

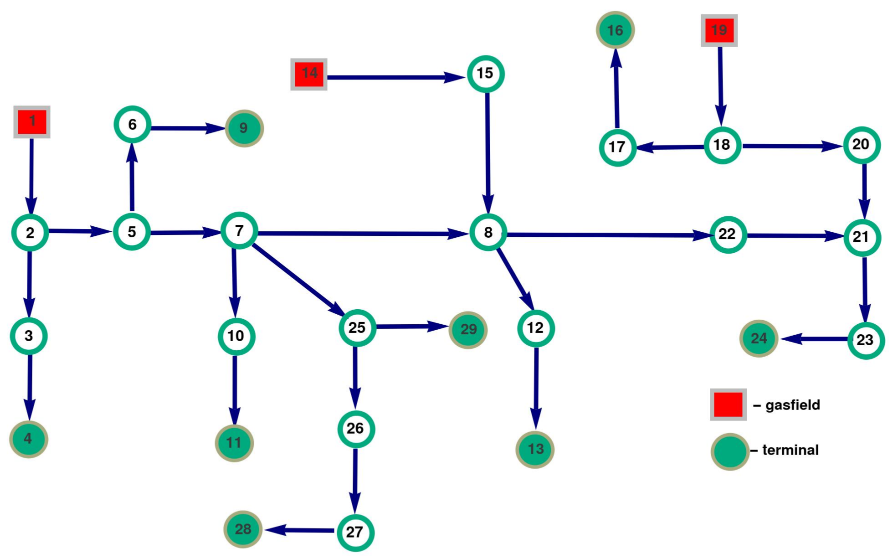

Figure 4: Gas pipelines network

Let us write the equation for the cross vertex α:

$$d_\alpha \rho_\alpha + \sum_{i \in Q_\alpha} M_{\alpha i} + Q_\alpha = e_\alpha, \tag{25}$$

where Q_α is the set of neighboring cross vertices. If we assume that the data on the output points are known, then we get an equation independent of ρ_δ. By combining all equations for cross vertices by applying formulas (17)–(24), we obtain the linear system with respect to vectors of unknown densities at the cross vertices of the graph:

$$S\rho = R. \tag{26}$$

`totalRegimeCalculation[]` constructs the initial regime using input data of the regime. According to the scheme in Figure 2, the procedure `constructFullSystem[]` builds matrix

S from (26) and solves the system. N depends on the simulation time, i.e., from the time interval of the modelling of the transportation process. As the compressors are not connected to the constructed gas pipeline network, the gas supply and the volume of consumption must be equal. Here, the loss of gas during transportation is considered insignificant due to micro-cracks in the pipes.

By ignoring gas parameters of output points of the network in (25), we must evaluate the separate numerical procedures for that like `calculateOutDirections[]`. This calculation is used a linear non-isothermal model [26].

The last block of calculations includes the procedure `updateTotalData[]`, which updates all gas parameters and applies appropriate conversions, such as density to pressure.

Table 1: Table of the regime

Table 1: Table of the regime

Branch #	**Set of points**	**Branch #**	**Set of points**	**Branch #**	**Set of points**
1	$\{1,2\}$	7	$\{7,8\}$	13	$\{21,23,24\}$
2	$\{2,3,4\}$	8	$\{14,15,8\}$	14	$\{7,25\}$
3	$\{2,5\}$	9	$\{8,12,13\}$	15	$\{25,26,27,28\}$
4	$\{5,6,9\}$	10	$\{18,17,16\}$	16	$\{25,29\}$
5	$\{5,7\}$	11	$\{19,18\}$	17	$\{21,22,8\}$
6	$\{7,10,11\}$	12	$\{18,20,21\}$		

Numerical experiments were performed with time step $\tau = 0.01$ sec and $h = 1$ km and the duration of the simulation was 5.5 hours. Accounting for transient withdrawals, and accordingly the assumption of transient injections, we consider a regime with time-dependent dynamical information, where one of the injection point gas parameters, such as mass flow or pressure, is given dynamically.

Figures 5 and 6 show results for the numerical solution. Below are the calculated pressure values in the start point of branches and graphical form for each branch separately.

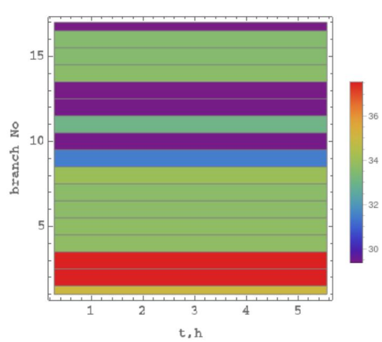

Figure 5: Pressure values in the start point of branches

The results for densities in crossing nodes {2, 5, 7, 25, 8, 18, 21} are shown in Figures 7 and 8.

Calculations were performed using the software system Wolfram Mathematica. The computational procedures shown in scheme Figure 2 were implemented in various Wolfram

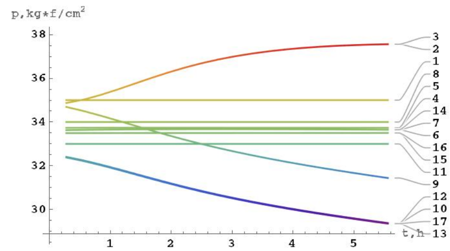

Figure 6: Pressure values in the graphical form for each branch separately

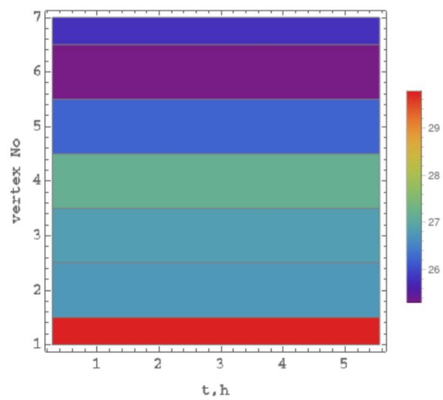

Figure 7: Density values in crossing points

notebooks, which can be instantly launched. The source code has been uploaded to GitHub and is available at the link [27].

4 Discussion

The outcomes of this research have provided insight into the investigation of the natural gas transportation process. Due to the unstable flow of natural gas into the pipeline system and changes in consumption, modeling the transportation process is a complicated problem. However using the considered dynamic model, we can obtain gas flow parameters at each point in the network dynamically. The crossing-branch method, which utilizes supply and consumption data, enables the development of the state of a dynamic system at a specific time.

Gas pipelines are often operated in transient modes due to the time-varying needs of consumers for natural gas and gas supplies. For a certain period of operation of the gas pipeline with specified gas parameters, depending on time, at the input and output points, pipeline dispatchers are faced with the task of optimizing the transition process to minimize fuel consumption at compressor stations in real time. For optimal management and control of the transport network, it is preferable to use dynamic models since they describe the dynamics of transient processes and allow efficient use of fuel gas.

Currently, the authors have conducted calculations to simulate the operation of gas compressor units, which will eventually be included in the complete transport network. The model discussed in this article will allow optimizing fuel consumption in such scenarios,

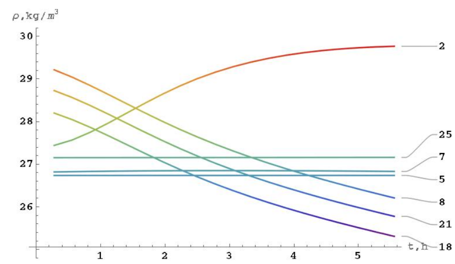

Figure 8: Density values in crossing points in the graphical form

reducing compressor speed, i.e. decreasing the load of these units while maintaining the equipment within the permissible operating range. These results should be considered when planning the implementation of gas transmission networks including equipment such as gas compressor units.

It should be noted that there is one important disadvantage of this model. As can be seen from formulas (18)-(19) and (21)-(22), it demands significant computational resources for large n. However for practical purposes (when n is less than 1000, mainly for real pipeline networks, since the number of vertices does not reach large values) this algorithm performs well and provides satisfactory computational results in an acceptable time.

5 Conclusions

The main problem is that gas movement without high pressure is impossible. If the gas enters with high pressure, then during movement, as a result of friction against the walls of the pipes, the pressure drops and the speed decreases. Therefore, compressor stations are installed in all onshore gas transportation networks. The main customers of this process are consumers and suppliers. The goal of this and future research is to control gas transportation in such a way as to meet the needs of consumers when operating compressors in economical mode, minimizing fuel gas consumption. The main element of the gas network is the pipe, followed by compressors to increase the gas pressure. The pipeline network contains many valves - regulators that can be opened and closed, which provide control and control of the direction and volume of gas. To get an idea of the size of such a gas transportation infrastructure, we consider the pipeline network of Kazakhstan. The total length of Kazakhstan's main gas pipelines is more than 19 thousand kilometers, on which 56 compressor stations operate, and 316 gas pumping units are installed [28]. Recently, many scientific papers have been devoted to theoretical studies of the optimization of gas network transients using mathematical optimization tools. After the gas consumption planning process, the operating modes of various main gas pipelines are built - the direction and volume of flows, the load on the network and the load of compressor stations that provide the gas with the required high pressure are taken into account. Specialist dispatchers monitor compressors and, based on their knowledge and experience, determine the loading of units for stable operation of the transport process.

Further research will be devoted to the construction of mathematical and computer modeling of non-stationary modes in order to optimize the gas transportation process by choosing the most optimal control policy and control influences on the technological equipment of main gas pipelines.

Acknowledgments

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