A. Kabulov[1*] ⬥, A. Baizhumanov [2] ⬥, M. Berdimurodov[1] ⬥

[1]Mirzo Ulugbek National University of Uzbekistan, Uzbekistan, Tashkent
[2]O. Zhanibekov South Kazakhstan State Pedagogical University, Kazakhstan, Shymkent
*e-mail: Anvarkabulov1952@gmail.com

ON THE MINIMIZATION OF k-VALUED LOGIC FUNCTIONS IN THE CLASS OF DISJUNCTIVE NORMAL FORMS

In the world, research devoted to adjusting the results of heuristic methods based on forecasting, recognition, classification, and determining the absolute extremum of a multidimensional function is relevant and widely used in such fields as medicine, geology, hydrology, management, and computer technology. In this regard, it is important to construct optimal correctors of heuristic algorithms based on control materials. Therefore, checking the completeness of classes of k-valued logical functions and developing methods and algorithms for minimizing functions in the class of canonical normal forms, estimating the number of monotonic functions of k-valued logic, constructing minimal bases of special classes of correcting functions for correcting incorrect algorithms remains one of the important problems of computational and discrete science. mathematics. Currently, a lot of scientific research is being carried out around the world aimed at expanding the integration of science and industry, in particular the development of the theory of k-valued logical functions for correcting the results of heuristic algorithms. In this case, an important role is played by the construction of formulas in the class of canonical normal forms, the coding of elementary conjunctions and the application of the rules of gluing, absorption and idempotency for them, and checking the completeness of systems of correcting functions. Consequently, the development of effective numerical computational methods and algorithms for constructing correction functions based on k-valued logic to improve the accuracy of the results of heuristic methods is considered a targeted scientific research. The paper considers the representation of k-valued logical functions in the class of disjunctive normal forms. Various classes of monotone functions of k-valued logic are studied. Theorems are proved on the coincidence of abbreviated and shortest disjunctive normal forms of k-valued functions. For a certain class of k-valued monotone functions, we prove an estimate for the number of functions from this class. criteria for the absorption of elementary conjunctions by a first-order neighborhood of disjunctive normal forms of k-valued functions are proved.

Key words: k-valued, minimization, disjunctive normal form, rank, abbreviated d.n.f., monotone function.

А. Кабулов[1*], А. Байжуманов[2], М. Бердимуродов[1]
[1]Мирзо Улугбек атындағы Өзбекстан Ұлттық университеті, Өзбекстан, Ташкент қ.
[2]Ө. Жәнібеков атындағы Оңтүстік Қазақстан мемлекеттік педагогикалық университеті, Қазақстан, Шымкент қ.
*e-mail: Anvarkabulov1952@gmail.com

Дизъюнктивті нормалды формалар класындағы k-мәнді логикалық функцияларды минимизациалау туралы

Әлемде көп өлшемді функцияның абсолютті экстремумын болжау, тану, жіктеу және анықтау негізінде эвристикалық әдістердің нәтижелерін түзетуге арналған зерттеулер өзекті және медицина, геология, гидрология, менеджмент және компьютерлік технологиялар сияқты салаларда кеңінен қолданылады. Осыған байланысты бақылау материалдары негізінде эвристикалық алгоритмдердің оңтайлы корректорларын құру маңызды. Сондықтан k-мәнді логикалық функциялар кластарының толықтығын тексеру және канондық қалыпты формалар класындағы функцияларды минимизациялау әдістері мен алгоритмдерін әзірлеу,

k-мәнді логиканың монотонды функцияларының санын бағалау, арнайы функциялардың минималды негіздерін құру, қате алгоритмдерді түзетуге арналған түзету функцияларының кластары есептеу және дискретті математиканың маңызды мәселелерінің бірі болып қала береді. Қазіргі уақытта бүкіл әлемде ғылым мен өндірістің интеграциясын кеңейтуге бағытталған көптеген ғылыми зерттеулер жүргізілуде, атап айтқанда эвристикалық алгоритмдердің нәтижелерін түзету үшін k- мәнді логикалық функциялар теориясын жасауға бағытталған. Бұл жағдайда канондық қалыпты формалар класындағы формулаларды құру, элементар қосылыстарды кодтау және олар үшін желімдеу, сіңіру және идемпотенттілік ережелерін қолдану, түзету жүйелерінің толықтығын тексеру маңызды рөл атқарады. функциялары. Демек, эвристикалық әдістердің мен алгоритмдерін жасау мақсатты ғылыми зерттеу болып саналады. Жұмыста k-мәнді логикалық функциялардың дисъюнктивтік қалыпты формалар класында ұсынылуы қарастырылады. k-мәнді логиканың монотонды функцияларының әртүрлі кластары зерттеледі. k-мәнді функциялардың қысқартылған және ең қысқа дизъюнктивтік қалыпты түрлерінің сәйкестігі туралы теоремалар дәлелденген. k-мәнді монотонды функциялардың белгілі бір класы үшін біз осы сыныптағы функциялар санының болжамын дәлелдейміз. k-мәнді функциялардың дизъюнктивтік қалыпты формаларының бірінші ретті көршілестігімен элементар қосылыстарды жұту критерийлері дәлелденді.

Түйін сөздер: k-мәнді, минимизация, дизъюнктивтік қалыпты форма, ранг, қысқартылған д.қ.ф., монотонды функция.

А. Кабулов[1*], А. Байжуманов[2], М. Бердимуродов[1]
[1]Национальный университет Узбекистана имени Мирзо Улугбека, Узбекистан, г. Ташкент
[2]Южно-Казахстанский государственный педагогический университет имени У. Жанибекова, Казахстан, г. Шымкент
*e-mail: Anvarkabulov1952@gmail.com

О минимизации k-значных логических функций в классе дизъюнктивных нормальных форм

В мире исследования, посвященные корректировке результатов эвристических методов на основе прогнозирования, распознавания, классификации, определения абсолютного экстремума многомерной функции, актуальны и широко используются в таких областях, как медицина, геология, гидрология, менеджмент, вычислительной технике. В связи с этим важно построение оптимальных корректоров эвристических алгоритмов на основе контрольных материалов. Поэтому проверка полноты классов k-значных логических функций и разработка методов и алгоритмов минимизации функций в классе канонических нормальных форм, оценка количества монотонных функций k-значной логики, построение минимальных базисов специальных классов корректирующих функций для корректировки некорректных алгоритмов остается одной из важных задач вычислительной и дискретной математики. В настоящее время в мире проводится много научных исследований, направленных на расширение интеграции науки и промышленности, в частности развитие теории k-значных логических функций для коррекции результатов эвристических алгоритмов. При этом важную роль играет построение формул в классе канонических нормальных форм, кодирование элементарных конъюнкций и применение для них правил склеивания, поглощения и идемпотентности, проверка полноты систем корректирующих функций. Следовательно, разработка эффективных численных вычислительных методов и алгоритмов построения корректирующих функций на основе k-значной логики для повышения точности результатов эвристических методов считается целевым научным исследованием. В работе рассматривается представление k-значных логических функций в классе дизъюнктивных нормальных форм. Исследуются различные классы монотонных функций k-значной логики. Доказываются теоремы о совпадении сокращенных и кратчайших дизъюнктивных номальных форм k-значных функций. Для определенного класса k-значных монотонных функций доказывается оценка числа функций из этого класса. доказываются критерии поглощения элементарных конъюнкций окрестностью первого порядка дизъюнктивных нормальных форм k-значных функций.

Ключевые слова: k-значная, минимизация, дизъюнктивная нормальная форма, ранг, сокращенная д.н.ф., монотонная функция.

1 Introduction

The methods of k-valued logics are generally necessary for the study of a number of important problems from the most diverse fields: biology, medicine, military affairs, automation, control, planning of experiments etc., everywhere where not only the quantitative relationships between the quantities characterizing the processes under consideration are significant, but also the logical dependences connecting them [1–4]. A multi-valued logical function can be represented as a disjunction (multi-place function "or") $K_1 \vee K_2 \vee ... \vee K_m$, where each term is a conjunction (multi-place function вЂњandвЂњ) of certain variables from the set $\{x_1, \ldots, x_n\}$, taken with or without negation. k-valued function gives a description functioning of the control system, and the formula realizing it, in particular, the disjunctive normal form (d.n.f.), describes the scheme of this system, so that the nodes and elements of the scheme correspond to the terms and letters of the d.n.f. as a rule, k-valued function has many essentially different d.n.f. [5–12]. In mathematical logic they are considered from the qualitative side. With the development of cybernetics, the terms and letters of the d.n.f. began to reflect equipment costs in circuits and this drew attention to the quantitative side. Therefore, one of the problems of k-valued logics dictated by practice is the problem of minimizing multi-valued functions. The results of research in some areas in this area, in particular, minimization in certain systems multi-valued functions are quite widely displayed in the literature [13–15]. Therefore, it should immediately be noted that we will only discuss the minimization of multi-valued functions in the class of d.n.f..

2 Problem statement

Consider the set of multivalued logic functions depending on n variables, etc. the set of functions defined on the set of all vertices of the n-dimensional k-lattice E_n^k and taking values from the set $E_k = \{0, 1, \ldots, k-1\}$. With this interpretation, there is a one-to-one correspondence between multivalued logic functions depending on n arguments and subsets of $N_f \subseteq E_n^k$. The function $f(x_1, \ldots, x_n)$ and the subset $N_f \subseteq E_n^k$ correspond to each other in the case $f(x) = \begin{cases} \gamma, & if\ x \in N_f \\ 0, & if\ x \in E_n^k \backslash N_f \end{cases}$, where

$$\gamma \in E_f \subseteq \{0, 1, \ldots, k-1\} \tag{1}$$

We can assume that the set E_f divides the function f into a number of subfunctions $f_{\gamma_1}(\widetilde{x}), \ldots, f_{\gamma_m}(\widetilde{x})$, and the set N_f into pairwise disjoint subsets $N_{f_{\gamma_1}}, \ldots, N_{f_{\gamma_m}}$, where $m = |E_f|$,

$$f_{\gamma_i}(\widetilde{x}) = \begin{cases} \gamma_i, & if\ f(x) = \gamma_i \\ 0, & if\ f(x) \neq \gamma_i \end{cases}$$

$$N_{f_i} = \left\{ \widetilde{\alpha} : \left(\widetilde{\alpha} \in E_n^k \right) \wedge \left(f(\widetilde{\alpha}) = \gamma_i \right), \left(i = \overline{1, m} \right). \right\}$$

It is easy to see that

$$f(\widetilde{\alpha}) = \max_{\gamma_i} \left\{ f_{\gamma_1}(\widetilde{\alpha}), \ldots, f_{\gamma_m}(\widetilde{\alpha}) \right\} \tag{2}$$

The function $f_\gamma(\widetilde{x})$, which takes only two values (0 *and* γ), will be called quasi-Boolean, and the representation of the function $f(x_1,\ldots,x_n)$ in the form of a (2)-quasi-Boolean representation of the multi-valued logical function $f(x_1,\ldots,x_n)$.

We introduce the function

$$J_M(x) = \begin{cases} k-1, & if \ x \in M \\ 0, & if \ x \notin M \end{cases}, \text{ where}$$

$$M \subseteq E_k = \{0,1,2,\ldots,k-1\} \tag{3}$$

An elementary conjunction (e.c.) is an expression
$\mathfrak{A} = \min[J_{M_1}(x_1),\ldots,J_{M_n}(x_n),\gamma]$, where

$$\varnothing \neq M_j \subseteq E_k, \left(j = \overline{1,n}\right) \tag{4}$$

Further, for brevity, formulas $\max[\mathfrak{A}_1,\ldots,\mathfrak{A}_m]$ will be conventionally denoted as $\mathfrak{A}_1 \vee \ldots \vee \mathfrak{A}_m = \bigvee\limits_{i=1}^{m} \mathfrak{A}_i$: if $\mathfrak{A}_i$ is an analog of e.c., then this formula will be called disjunctive normal form (d.n.f.).

The area of truth of e.c. let's call $\mathfrak{A}$ the region $N_\mathfrak{A}$ in which $\mathfrak{A}$ takes the value γ. It is easy to see that the domain $N_\mathfrak{A} = \prod\limits_{j=1}^{n} M_j$ is a sub lattice (a subset of the set E_n^k) of the lattice E_n^k. With such a geometric consideration, the e.c. the sub lattice $N_\mathfrak{A}$ corresponds to the lattice E_n^k.

Rank e.c. let's $\mathfrak{A}$ call a number $r(\mathfrak{A}) = \sum\limits_{j=1}^{n}(k-|M_j|) = kn - \sum\limits_{j=1}^{n}|M_j|$.

The formula $\mathfrak{M} = \bigvee\limits_{i=1}^{t} \mathfrak{A}_i$ where all $\mathfrak{A}_i, \left(i = \overline{1,t}\right)$ are e.c. will be called the disjunctive normal form.

Note that each set-valued logic function $f(x_1,\ldots,x_n)$ corresponds to a non-empty class of d.n.f. realizing the given function. The set of all intervals corresponding to e.c. a certain d.n.f. from this class determines the covering of N_f by sub lattices of the lattice E_n^k. Hence it follows that the subsets $M \subseteq E_n^k$ can be defined using the d.n.f.

Let $I = \{N_\mathfrak{A}\}$, be some subset of sub lattices from E_n^k.

A sub lattice $N_B \in I$ is said to be maximal with respect to M if there is no sub lattice $N_\mathfrak{A}$ in I such that $N_\mathfrak{A} \neq N_B$ and $N_\mathfrak{A} \supseteq N_B$.

To represent the function $f(x_1,\ldots,x_n)$ in the form of a d.n.f. we considered its quasi-Boolean representation: $f = f_{\gamma_1} \vee \ldots \vee f_{\gamma_m}$ and $\gamma_1 < \gamma_2 < \ldots < \gamma_m$.

Note that for the same function $f(\widetilde{x})$ there can be several equivalent quasi-Boolean representations. Indeed, $f = f_{\gamma_1} \vee \ldots \vee f_{\gamma_m} = f^* = f^*_{\gamma_1} \vee \ldots \vee f^*_{\gamma_m}$ where $N_{f^*_{\gamma_i}} = N_{f_{\gamma_i}} \cup Q_i$, $Q_i \subseteq \bigcup\limits_{j>i} N_{\gamma_j}, \left(i = \overline{1,m}\right)$.

We will consider only one "maximum" representation $f' = f'_{\gamma_1} \vee \ldots \vee f'_{\gamma_m}$ where $N_{f'_{\gamma_i}} = \bigcup\limits_{j=1}^{n} N_{f_{\gamma_j}}, \left(i = \overline{1,m}\right)$.

Select all maximal sub lattices $N_{B_j^i}, \left(i = \overline{1,m}\right)$ contained in $N_{f'_{\gamma_i}}$ that have non-empty intersection with $N_{f_{\gamma_i}}$ and such that the value of B_j^i is equal to γ_i in $N_{B_j^i}, \left(i = \overline{1,m}\right)$. D.n.f. $\mathfrak{M} = \bigvee\limits_{i=1}^{m} \bigvee\limits_{j=1}^{I_i} B_j^i$ is called the reduced disjunctive normal form of the function $f(\widetilde{x})$.

A covering of a set N_f by maximal sub lattices is said to be irreducible if, after the removal of any of its sub lattices, it ceases to be a covering. A d.n.f. realizing a function f is called dead-end if it corresponds to an irreducible cover of the set N_f.

Consider a multivalued logic function $F(x_1, \ldots, x_n)$ defined at $M \subseteq E_n^k : F(\widetilde{x}) = \gamma_j$, if $\widetilde{x} \in M_j$, $(j = \overline{1, m})$, $m < k$, $\gamma_j \in E_k$, $M = \bigcup\limits_{i=0}^{m} M_i$ and $M_i \bigcap M_j = \varnothing$ at $(i \neq j,\ i,\ j = \overline{0, m})$. And $\gamma_1 < \ldots < \gamma_m$, $\gamma_0 = 0$.

Thus, $F(x_1, \ldots, x_n)$ is defined by specifying pairwise disjoint sets $M_0, \ldots, M_m$. The function $F(\widetilde{x})$ is defined, generally speaking, not on the entire set E_n^k. There are different before the definition in the class of functions $F(\widetilde{x})$, multi-valued logic, not equivalent to each other.

Our task is to find the simplest ones, in a certain sense, before definitions.

For $F(\widetilde{x})$, select all maximal intervals $N_{B_j^i}$, $\left(i = \overline{1, m},\ j = \overline{0, I_i}\right)$ contained in $E_n^k \backslash \bigcup\limits_{v=0}^{i-1} M_v$ that have non-empty intersection with M_i such that the value of B_j^i is equal to γ_i.

D.n.f. $\mathfrak{M} = \bigvee\limits_{i=1}^{m} \bigvee\limits_{j=1}^{I_i} B_j^i$ is called the reduced normal form for $F(\widetilde{x})$. It is easy to see that d.n.f. $\eta_{\Sigma T F}$ is uniquely determined by the function F.

Let us now indicate the points at which, when the values of the function F change (transition to F'), the values of $\eta_{\Sigma T F}$ change (transition to $\eta_{\Sigma T F'}$).

3 Monotone functions of k-valued logic

Let's consider some order on the set ε_k. For two sets $\Im = (\alpha_1, \alpha_2, \ldots \alpha_n)$ and $\widetilde{\beta} = (\beta_1, \beta_2, \ldots \beta_n)$ the precedence relation $\Im \leq \widetilde{\beta}$ is satisfied if $\alpha_i \leq \beta_i$ in this order for any $i = \overline{1, n}$.

Definition 1. A k-valued logic function $f(x_1, x_2, \ldots, x_n)$ is said to be monotonic with respect to a given order if for any tuples α and β such that $\left(\Im \leq \widetilde{\beta}\right)$ we have $f(\alpha) \leq f(\beta)$.

If $0 < 1 < 2 < \ldots < k - 1$, then the set of functions that are monotone in this order constitutes the class of monotone functions of k-valued logic.

Theorem 1. Abbreviated d.n.f. monotone function f_γ of k-valued logic in n variables

a) consists of e.c. K^A, and only elementary formulas of the form $J_{[a, k-1]}(x)$, $0 \leq a \leq k-1$ are used;

b) Is the only minimal (shortest) d.n.f. of the function f.

Proof.

a) Let $K = J_{T_1}(x_1) \cdot J_{T_2}(x_2) \cdot \ldots \cdot J_{T_n}(x_n) \cdot \gamma$, where $T_j = [a_j, k-1]$, $j \neq i$, $0 \leq a_j \leq k-1$, $T_i = [b_i] \bigcup [a_i, k-1]$, $b_i \leq a_i$.

Then the conjunction K, and hence the function f_γ, takes the value γ on the set $\widetilde{a} = (a_1, a_2, \ldots, a_{i-1}, a_i, a_{i+1}, \ldots, a_n)$.

b) It follows from the monotonicity condition for the function f_γ that for any set $\widetilde{b}$ such that $\widetilde{b} \geq \widetilde{a}$, $f_\gamma\left(\widetilde{b}\right) = \gamma$ therefore, there exists an e.c. $K' = J_{T'_1}(x_1) \cdot J_{T'_2}(x_2) \cdot \ldots \cdot J_{T'_n}(x_n) \cdot \gamma$, where $T'_j = T_j$, $j \neq i$, $T'_i = [b_i, k-1]$, for which $U_K \subset U_{K'} \subseteq U_{f_\gamma}$, and e.c. K is not maximal for U_{f_γ}.

c) As proved, each maximum e.c. K functions has the form $K = J_{[a_1,k-1]}(x_1) \cdot J_{[a_2,k-1]}(x_2) \cdot \ldots \cdot J_{[a_n,k-1]}(x_n) \cdot \gamma$, $0 \leq a_j \leq k-1$, $j = \overline{1,n}$.

Let us show that the set $\widetilde{a} = (a_1, a_2, \ldots, a_n)$ is core for the function f_γ, etc. in the abbreviated d.n.f. the function f_γ has no e.c., except for K, which takes the value γ on this set.

Indeed, if in the reduced d.n.f. function f_γ was an e.c. K', which takes the values γ on the tuple $\widetilde{a}$, then it would follow from the monotonicity condition for the function f_γ that the e.c. K' takes the value γ on all tuples $\widetilde{b}$ such that $\widetilde{b} \geq \widetilde{a}$, then $U_K \subseteq U_{K'}$, which contradicts the maximum e.c. K.

The theorem is proved.

Corollary. Abbreviated d.n.f. monotone function f of k-valued logic in n variables consists of e.c. K^A and is the only minimal d.n.f. functions f.

Proof. A monotone function f has the following obvious properties: for any comparable collections $\widetilde{a} \in N_{f_\gamma}$ and $\widetilde{b} \in N_{f_\theta}$ we have $\widetilde{b} > \widetilde{a}$ for $\gamma < \theta$. Therefore, e.c. K, is included in the abbreviated d.n.f. functions f_γ ($\gamma \in \{\varepsilon_k/0\}$) do not contain elementary formulas of type $J_T(x)$, where the set T is a disconnected set of points from ε_k.

The second assertion is proved by the method of theorem 1, and the core sets for the function f are the sets $\widetilde{a} = (a_1, a_2, \ldots, a_n)$ in the e.c. $K = J_{[a_1,b_1]}(x_1) \cdot J_{[a_2,b_2]}(x_2) \cdot \ldots \cdot J_{[a_n,b_n]}(x_n) \cdot \gamma$.

Corollary proven. On the set ε_k we introduce a partial order $0 < 1$, $0 < 2$, $\ldots, 0 < k-1$ where i is incomparable with j if $i, j \in \{\varepsilon_k/0\}$.

The set of functions of k-valued logic f in n variables, monotone in a given order, is combined into the class S. Let us estimate the cardinality of the class S.

The set ε_k is associated with a basic graph - a directed graph with K vertices corresponding to the elements of the set ε_k, in which there is an arc (i,j) if and only if $i > j$.

Let us introduce the Z axis on the plane. Associate each point A of the plane with the number Z_A, the projection of A onto the Z axis. In particular, if the basis graph is drawn on the plane, then each vertex corresponds to the number Z_i. An image of a basic graph is called admissible if for any arc (i,j) of the graph $Z_i - Z_j \geq 1$.

Consider a random variable $\xi = Z_A$, where the point A can fall into any vertex of the graph with probability $\frac{1}{k}$ then the expectation is $M\xi = Z_{cp} = \frac{Z_1 + \ldots + Z_k}{k}$ and the variance is $D\xi = \frac{1}{k} \sum_{i=1}^{k} (Z_i - Z_{cp})^2$.

We will consider the image of the graph shifted so that $M\xi = 0$.

In older articles an estimate is obtained for finding the number $\psi(n)$ of monotone functions of n variables from an arbitrary partially ordered set of k elements:

$$\psi(n) = d^{\frac{1}{\sqrt{2\pi D}} \cdot \frac{k^n}{\sqrt{n}}(1+\varepsilon(n))} \tag{5}$$

where $\varepsilon(n) \to 0$ at $n \to \infty$; $D = \inf D\xi$; $d = \max(|H_1|, \ldots, |H_s|)$, $H_0 \leq H_1 \leq \ldots \leq H_{s+1}$, all $H_i \subseteq \varepsilon_k$, $s \geq 1$, $|H_0| = |H_{s+1}| = 1$ and $H_i \neq H_j$, at $i \neq j$ ($H_i \leq H_j$ if $a \leq b$ for any $a \in H_i$, $b \in H_j$), the maximum is taken over all possible chains. This estimate is also valid for the class of functions S.

For the order in ε_k we have:

$$
\begin{cases}
Z_1 - Z_0 \geq 1 \\
\cdots \quad \cdots \quad \cdots \\
Z_{k-1} - Z_0 \geq 1
\end{cases}
\tag{6}
$$

$$
Z_{cp} = \frac{Z_0 + Z_1 + \ldots + Z_{k-1}}{k} = 0, \; D\xi = \frac{1}{k}\sum_{i=0}^{k-1} Z_i{}^2
\tag{7}
$$

It follows that $Z_0 \leq -\frac{k-1}{k}$ and $Z_i \geq \frac{1}{k}$, $i = \overline{1, k-1}$, so $D\xi \geq \frac{k-1}{k^2}$ and $D \geq \frac{k-1}{k^2}$.

On the set ε_k for the introduced order there is only $(k-1)$ chain

$$
\begin{cases}
\{0\} < \{0,1\} < \{1\} \\
\cdots\cdots\cdots \\
\{0\} < \{0, k-1\} < \{k-1\}
\end{cases}
\tag{8}
$$

It is obvious that in the considered case $d = 2$.

Consequently

$$
\left\{ \psi(n) = d^{\frac{1}{\sqrt{2\pi(k-1)}} \cdot \frac{k^{n+1}}{\sqrt{n}} (1+\varepsilon(n))} \right.
\tag{9}
$$

where $\varepsilon(n) \to 0$ at $n \to \infty$.

For functions of class S, e.c. consists of elementary formulas of the form $J_T(x)$, where $T \subseteq \{\varepsilon_k/0\}$.

If for monotone functions f of k-valued logic the abbreviated d.n.f. is the only minimal one, then for functions of the class S this property does not hold, which is demonstrated by the following.

Example. $k = 3$, $n = 3$.

Let the function $f(x_1, x_2, x_3)$ take the values 1 on the sets $(0,1,1), (1,1,1), (1,2,1), (2,1,1), (1,2,2)$ and 0 in other cases.

Abbreviated d.n.f. function f has the form:

$$
D_C(f) = J_1(x_2) \cdot J_1(x_3) \vee J_1(x_1) \cdot J_2(x_2) \cdot J_{[1,2]}(x_3)
\tag{10}
$$

and the minimum:

$$
D_M(f) = J_1(x_2) \cdot J_1(x_3) \vee J_1(x_1) \cdot J_{[1,2]}(x_2) \cdot J_1(x_3)
\tag{11}
$$

The process of transition from the abbreviated d.n.f. functions f of a k-valued logic to a dead-end one can be divided into elementary steps, each of which is a removal from the d.n.f. D obtained in the previous step, one e.c. K. Removed e.c. is such that $U_K \subseteq \bigcup_{j=1}^{m} U_{K_j}$, where K_j are some e.c. from d.s.f. D different from K.

In older articles the criterion for covering an interval by the sum of other intervals for functions f of k-valued logic is described. For functions f of class S, this criterion has a simpler form.

E.c. K_1 and K_2 are called orthogonal if $K_1 \cdot K_2 = 0$. In other words, conjunctions K_1 and K_2 are orthogonal if and only if $U_{K_1} \bigcap U_{K_2} = \varnothing$. Obviously, when studying the absorption, some sets of e.c. $\{K_j\}$, $j = \overline{1,m}$ e.c. K it suffices to consider only those e.c. that are non-orthogonal to K. To check orthogonality, it is easiest to use the following properties: two e.c. are orthogonal if and only if there exists a variable x_j for which $T_j^1 \bigcap T_j^2 = \varnothing$ is satisfied in the elementary formulas $J_{T_j^1}(x_1)$ and $J_{T_j^2}(x_2)$.

D.n.f. D realizing the function f absorbs the e.c. K if $K(\widetilde{x}) \le D(\widetilde{x})$ for any $\widetilde{x} = \varepsilon_k^n$.

So let $K = J_{T_1}(x_1) \cdot J_{T_2}(x_2) \cdot \ldots \cdot J_{T_t}(x_t) \cdot \gamma$.

Obviously, K can be absorbed only by those sets of e.c. $\{K_j\}$, which take values from $\{0, \gamma\}$, so let's consider the absorption process using the example of the quasi-Boolean function f_γ.

For each e.c. $\{K_j\}$, $j = \overline{1,m}$ construct an e.c. K_j, replacing the elementary formulas $J_T(x)$ occurring in K_j with $J_{[1,k-1]}(x)$.

It's obvious that $U_{K_j} \subseteq U_{K_j}$, $U_{K_j} \cap (U_D/U_{K_j}) = \varnothing$.

Let us introduce into consideration the set $\varepsilon_k^{n,t}$- the collection of all sets from ε_k^n, in which the t first coordinates take values from $\{\varepsilon_k/0\}$, and the rest are arbitrary.

Theorem 2. The disjunction $D = \overset{m}{\underset{j=1}{\vee}} K_j$ absorbs the e.c. K if and only if $\overset{m}{\underset{j=1}{\vee}} K_j = \gamma$ for any $\widetilde{x} = \varepsilon_k^{n,t}$, etc. if $\overset{m}{\underset{j=1}{\vee}} K_j = \varepsilon_k^{n,t}$.

Proof. *Need.* Let D absorb the e.c. K. Let us prove that in this case $\overset{m}{\underset{j=1}{\vee}} K_j = \gamma$ for any $\widetilde{x} = \varepsilon_k^{n,t}$. Let us assume that this is not the case, etc. there is a collection $\Im$ such that $\overset{m}{\underset{j=1}{\vee}} K_j(\Im) = 0$. Denote by $x_{i_1}, \ldots, x_{i_p}$ the variables that are not included in any of the e.c. from D. Obviously, the values of the remaining variables do not affect the value of the expression $\overset{m}{\underset{j=1}{\vee}} K_j$.

The value of the function f on the tuples $\Im$ will be denoted by $[f]\Im$. Then one can write

$$\left[\overset{m}{\underset{j=1}{\vee}} K_j\right]\{\Im\} = 0 \tag{12}$$

where $\{\Im\}$ is the set of sets whose $(x_1, x_2, \ldots, x_t)$ coordinates take all possible values from $\{\varepsilon_k/0\}$, and the remaining $(n-t)$ coordinates are such that (12) is satisfied. hence we get that $[K_j]\{\Im\} = 0$ for all $j = \overline{1,m}$, so $[D]\Im = 0$.

Let us determine the values of the remaining variables entering K ($K \ne 0$ on U_D, since K is not orthogonal to D) so that K turns into γ on these sets. The intersection of these two sets determines the values of all variables in such a way that D takes the value 0 on this set, and e.c. K value γ. This contradicts the condition $K(\widetilde{x}) \le D(\widetilde{x})$ for any $\widetilde{x} = \varepsilon_k^n$, hence the assumption that $\overset{m}{\underset{j=1}{\vee}} K_j \ne \gamma$ on $\varepsilon_k^{n,t}$ is false, and the necessity of the condition of the theorem is proved.

Adequacy. Let the condition of the theorem be satisfied etc., $\overset{m}{\underset{j=1}{\bigcup}} U_{K_j} = \varepsilon_k^{n,t}$. Then $U_K \subseteq$

$\overset{m}{\underset{j=1}{\bigcup}} U_{K_j}$, but $U_K \bigcap \left(\overset{m}{\underset{j=1}{\bigcup}} U_{K_j} / \overset{m}{\underset{j=1}{\bigcup}} U_{K_j}\right) = \varnothing$. Consequently. $U_K \subseteq \overset{m}{\underset{j=1}{\bigcup}} U_{K_j}$.

The theorem has been proven.

4 Conclusions

The paper proposes a representation of k-valued functions in the class of disjunctive normal forms. Monotone functions of k-valued logic are investigated. We prove theorems on the coincidence of abbreviated and shortest d.n.f. k-valued functions. For a certain class of k-valued monotone functions, the number of functions from this class is calculated. Criteria for the absorption of elementary conjunctions by a first-order neighborhood are proposed.

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