

2-бөлім

Раздел 2

Section 2

Механика

Механика

Mechanics

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COMPLEX RESISTANCE OF A COMPRESSED-BENT ROD TAKING INTO ACCOUNT ELASTIC COMPLIANCE OF ITS SUPPORT

This work deals with the urgent problem in the mechanics of a deformable solid: studying factors of the stress-strain state of a single-span statically indeterminate beam with complex boundary conditions that is under conditions of complex resistance (axial compression with plane transverse bending). To solve the problem, both analytical methods (the method of forces in the form of “five” support moments, the method of initial parameters) and the numerical finite difference method with a “linear” grid with density $n=8$ were used. The necessary resolving equations and matrices are given to take into account changes in the rigidity parameter of the right hinged support and variations in concentrated and uniformly distributed loads, both along the axis of the beam and across it. In the final form, diagrams of deflections, bending moments and shear forces were constructed for specific values of bending rigidity and the degree of elastic compliance of the right hinge-yielding support. Reliability of the theoretical principles and applied results obtained by the authors is confirmed on the basis of the given alternative calculation methods.

Key words: complex resistance, elastic compliance of supports, stress-strain state, statically indeterminate structure, five-moment equation, fictitious reaction of supports, resolution matrix, finite difference method, boundary conditions, initial method parameters.

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Серпімді-иілгіш тіректі ескергендегі сығыла-иілген өзектің күрделі кедергісі

Бұл жұмыс деформацияланатын қатты дене механикасының өзекті мәселесі - күрделі кедергі жағдайында орналасқан күрделі шекаралық шарттары бар бір аралықты статикалық анықталмаған арқалықтың кернеулі-деформациялық күйінің факторларын зерттеуге арналған (жазық көлденең иілумен осьтік сығылу). Мәселені шешу үшін аналитикалық әдістер де (“бес”тірек моменттері түріндегі күштер әдісі, бастапқы параметрлер әдісі) және жиілігі $n=8$ болатын “сызықтық”торы бар сандық ақырлы айырым әдісі қолданылды. Қажетті шешу теңдеулер мен матрицалар оң жағы топсалы тіректің қатаңдық параметрінің өзгеруін және арқалық өсінің бойымен де өзгертін шоғырланған және біркелкі таралған жүктеме-лердің өзгеруіне мүмкіндік береді. Соңында иілу қатаңдығының нақты мәндері үшін майысу, иілу моменттері мен көлденең күштерінің және оң жақ топсалы-икемді тіректерінің серпімді икемді дәрежесінің эпюралары тұрғызылды. Авторлар алған теориялық қағидалар мен қолданбалы нәтижелердің сенімділігі берілген балама есептеу әдістерінің негізінде расталады.

Түйін сөздер: күрделі кедергі, тіректердің серпімді илгіштігі, кернеулі-деформациялық күй, статикалық анықталмаған конструкция, бес моменттік теңдеу, тіректердің жалған реакциясы, шешу матрицасы, ақырлы айырым әдісі, шекаралық шарттар, бастапқы параметрлер әдісі.

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Сложное сопротивление сжато-изогнутого стержня с учетом упругой податливости ее опоры

Данная работа посвящена актуальной задаче механики деформируемого твердого тела – исследование факторов напряженно-деформируемого состояния (НДС) однопролетной статически неопределимой балки со сложными граничными условиями, находящейся в условиях сложного сопротивления (осевое сжатие с плоским поперечным изгибом). Для решения поставленной задачи применены как аналитические методы (метод сил в виде «пяти» опорных моментов, метод начальных параметров), так и численный метод конечных разностей при «линейной» сетке с плотностью $n = 8$. Приведены необходимые разрешающие уравнения и матрицы, позволяющие учитывать изменение параметра жесткости правой шарнирной опоры и варьирования сосредоточенными и равномерно-распределенными нагрузками, как вдоль оси балки, так и поперек её. В конечном виде построены эпюры прогибов, изгибающих моментов и поперечных сил для конкретных значений изгибной жесткости и степени упругой податливости правой шарнирно-податливой опоры. Достоверность полученных авторами теоретических положений и прикладных результатов подтверждена на основе приведенных альтернативных методов расчета.

Ключевые слова: Сложное сопротивление, упругая податливость опор, напряженно-деформированное состояние, статически неопределимая конструкция, уравнение пяти моментов, фиктивная реакция опор, разрешающая матрица, метод конечных разностей, граничные условия, метод начальных параметров.

Purpose and tasks of the study. The purpose of this work is to study the issues of the stress-strain state of a rod structure in the form of a statically indeterminate beam with an elastic compliant support at the right end by analytical and numerical methods based on the equations of five support moments and using the method of initial parameters. The following tasks are solved:

- reviewing scientific literature at the time of the study;
- using the numerical finite difference method with the formation of resolution matrices with the “density” of the linear grid $n=8$;
- using the five moment equations to obtain the initial moment diagram (taking into account the compliance of the right support);
- checking reliability of the obtained theoretical and applied results based on an alternative method of initial parameters;
- studying the effect of the rigidity of the right support on the parameters of the stress-strain state of the studied single-span beam under the simultaneous action of both axial and transverse loads.

1 Introduction

Structures used in various branches of technology (construction, transport, mechanical engineering, mine and underground construction) are often in the state of complex resistance (a combination of several types of stress: tension-compression, bending, torsion, etc.). The task is complicated in the presence of support points that have a high compliance coefficient, which significantly affects the stress-strain state of the structures under study.

It is known that the level of stress-strain state of structures is significantly affected by the presence of so-called extra connections, the number of which determines the degree of their static indetermination.

The analysis of the stress-strain state of such structures is widely reflected in the scientific works of domestic and foreign scientists. The essence of the problem lies in obtaining final results with complex loading patterns (both in the axial and transverse directions), in the presence, alongside with rigid supports, of supports that have significant compliance in the direction of their settlement. Elastic compliance is characterized by damping (cushioning) of supports, which affects the magnitude of deflections and internal forces in statically indeterminate beams (single-span and multi-span). Among the results obtained in this area of studies, the following can be noted.

In works [1, 2], oblique bending of beams under eccentric compression and torsion is considered, and the calculation of cylindrical helical springs for their axial compression is also given. Work [3] sets out a method of calculating the strength of structural elements subject to variants of complex resistance (eccentric tension-compression, oblique bending); various reference data for carrying out relevant engineering calculations are also provided here.

In work [4], the compliance of compressed rods with elastic support is studied, taking into account their supercritical behavior; in this case, large deflections of rods with hinged supports are studied (the problem of nonlinear elastic resistance). Based on the results of the analytical study, resolving nonlinear integral-differential equations were obtained; the variant of loss of stability in the form of a mechanical “clap” was analyzed.

Study [5] provides an analysis of the stress-strain state of a rod on an elastic foundation with initial deflections and elastic-yielding fastenings at the ends (the following types of deformation are taken into account: bending, linear, shear). In a particular case, for linearly elastic fastenings at the ends of the rod, its supercritical behavior was also studied.

In work [6], the problem of supercritical behavior of an elastically fixed (non-rotating) rod is considered on the basis of the iterative method in determining its displacements.

In works [7-9], the flexibility of supporting steel beams on columns (racks) was studied. In works [10, 11], a method was proposed to take into account compliance in the nodes of metal structures. Study [12] deals with the issues of regulating the level of supports in metal structures by changing the values of their rigidity (or compliance).

The authors of works [13-15] studied the operation of continuous beams of constant and variable cross sections both when they are supported on an elastic continuous space, and when they are supported on elastic-yielding supports; The study was carried out using the analytical method of five supporting points. The Winkler model was used for the elastic foundation of the structure.

Based on the above review of previously published scientific works, this article continues

studying this field of mechanics of a deformable solid for cases of complex resistance in the presence of an elastic-yielding support at one of the ends of a single-span beam.

In this regard, this work sets the purpose of studying structures taking into account the above factors.

2 Theoretical propositions and calculation methods

The object of analysis is a single-span statically indeterminate beam in the compressed-bent state in the presence of a hinged support with the corresponding compliance coefficient “C” (Fig. 1, a).

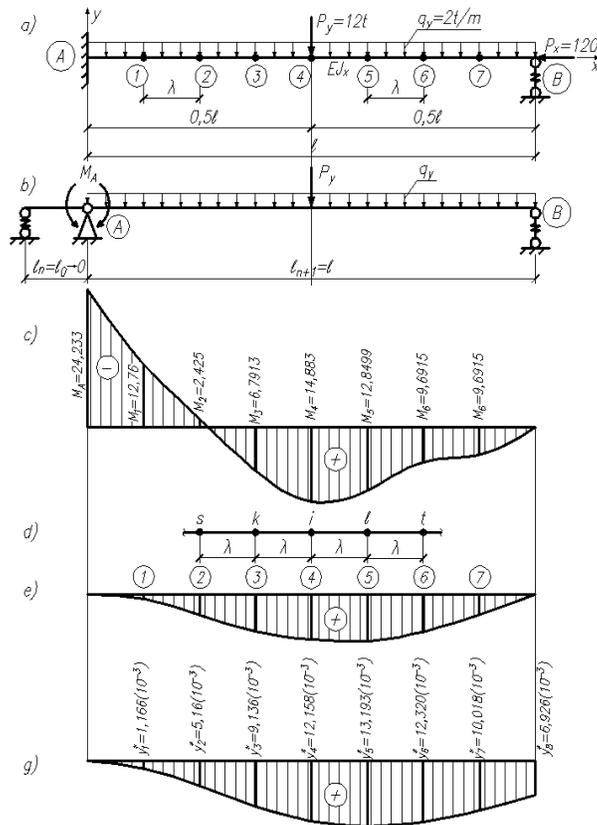


Figure 1: Design diagram of a compressed-bent beam with an elastic-yielding support B: a) design diagram of a beam with an elastic-yielding support B; b) the basic force method system; c) diagram of bending moments (M_0) from shear forces P_y, q_y ; d) a fragment of a regular linear grid; f) diagram of deflections (y_c, m) (without taking into account the settlement of the support B); g) calculated diagram of deflections (M) taking into account B settlement

To solve the purpose, there should be used the numerical finite difference method [16. 17]. The initial differential equation of compressed-bent rods has the form [18]:

$$\frac{d^2y}{dx^2} + \lambda^2 y = -\frac{M_0}{EJ}. \tag{1}$$

where $y = y(x)$ is the required function of the beam deflection

$$\alpha^2 = \frac{N_x}{EJ} \quad (2)$$

is the parameter of the longitudinal axial load P_x effect (Fig. 1,a) ($N_x = -P_x$); M_0 is a bending moment from the initial transverse load P_y, q_y (Fig.1,a); EJ is the beam flexural rigidity.

Before applying the finite difference method to implement equation (1), it is necessary to construct a diagram of bending moments from a given load P_y, q_y , taking into account the elastic compliance of support "B". To do this, there should be used the method of equation of five support moments [18, 19]. The basic system of the force method is given in Figure 1, b (C is the compliance coefficient of the support B).

For node A (Fig. 1, b) ($M_B = 0; M_O = 0$): ($\ell_n = \ell_0 = 0; \ell_{n+1} = \ell$)

$$\delta_{A.A}M_A + \delta_{A.B}M_B + \Delta_{AP} = 0. \text{ or } \delta_{A.A}M_A + \Delta_{AP} = 0. \quad (3)$$

$$\delta_{A.A} = \delta_{n.n} = \frac{\ell}{3EJ} + \frac{C}{\ell^2}; \Delta_{AP} = \Delta_{nP} = \frac{A_A^\Phi}{EJ} + \frac{C}{\ell}R_B. \quad (4)$$

$$A_A^\Phi = \frac{3}{48}P_y\ell^2 + \frac{q_y\ell^3}{24} = \frac{3}{48}12(6)^2 + \frac{2(6)^3}{24} = 27 + 18 = 45(tm^2)$$

$$R_B = \frac{P_y}{2} + \frac{q_y\ell}{2} = 6 + 6 = 12(t).$$

$$\delta_{A.A} = 0.560483 \cdot 10^{-3}. \Delta_{A.P} = \frac{45}{3.8 \cdot 10^3} + \frac{0.87 \cdot 10^{-3}}{6} \cdot 12 = 13.58205 \cdot 10^{-3}.$$

According to (3):

$$M_A = -\frac{\Delta_{A.P}}{\delta_{A.A}} = -\frac{13.58205 \cdot 10^{-3}}{0.560483 \cdot 10^{-3}} = 24.233 (tm). \quad (5)$$

Let's write down equation (1) in finite differences for the i -th node of the regular linear grid (Fig.1, d):

$$\frac{1}{\lambda^2} (y_k - 2y_i + y_\ell) + \alpha_i^2 y_i = -\frac{M_{O.i}}{EJ}.$$

or

$$y_i (-2 + \alpha_i^2 \lambda^2) + (y_k + y_\ell) = - \frac{M_O \lambda^2}{EJ} \quad (6)$$

Let's pre-calculate the load parameters $\alpha_i (i = 1.2.3.7.8)$ ($N_i = \pm P_x = 120 \text{ t}$)

$$\alpha_i \lambda^2 = (0.125\ell)^2 \frac{N_i}{EJ} = 0.015625 \left(\frac{120 \cdot 36}{3.8 \cdot 10^3} \right) = 0.01776316;$$

$$\Delta_{1P} = \frac{M_{0.1}}{EJ} = \frac{12.7673}{3.8 \cdot 10^3} = 3.3598 \cdot 10^{-3} (0.75)^2 = 1.8899 \cdot 10^{-3};$$

$$\Delta_{2P} = 0.3593 \cdot 10^{-3}; \quad \Delta_{3P} = -1.0053 \cdot 10^{-3}; \quad \Delta_{4P} = -2.2036 \cdot 10^{-3};$$

$$\Delta_{5P} = -1.9021 \cdot 10^{-3}; \quad \Delta_{6P} = -1.436 \cdot 10^{-3}; \quad \Delta_{7P} = -0.8006 \cdot 10^{-3}.$$

The system of resolving finite-difference equations for the beam obtained from expression (5) (Fig. 1, a) is given in Table 1.

Table 1: Matrix for calculating a compressed-bent rod (Fig. 1, a)

	y_1	y_2	y_3	y_4	y_5	y_6	y_7	Right part
1	-1.982237	1						$1.8899 \cdot 10^{-3}$
2	1	-1.982237	1					$0.3593 \cdot 10^{-3}$
3		1	-1.982237	1				$-1.0053 \cdot 10^{-3}$
4			1	-1.982237	1			$-2.2031 \cdot 10^{-3}$
5				1	-1.982237	1		$-1.9021 \cdot 10^{-3}$
6					1	-1.982237	1	$-1.436 \cdot 10^{-3}$
7						1	-1.982237	$-0.8005 \cdot 10^{-3}$

In the matrix form system of linear algebraic equations (SLAE) (6) presented in Table 1 will take the form

$$A \cdot \vec{y} = \vec{P}. \quad (7)$$

The SLAE (6) solution gives the vector of deflections at the nodes of the linear grid (Fig.1, a), that is:

$$\vec{y} = A^{-1} \cdot \vec{P} \quad (8)$$

A^{-1} is the reverse matrix.

To assess reliability of the obtained results of deflections \vec{y} (9), let's construct a diagram of deflections for the beam (Fig. 2) (without taking into account elastic compliance of the support B (taking $c = 0$):

$$\delta_{A.A} = \frac{\ell}{3EJ} = 0.526316 \cdot 10^{-3}; \quad \Delta_{A.P} = 11.842 \cdot 10^{-3}; \quad (9)$$

According to (3): $M_A = -\frac{11.842 \cdot 10^{-3}}{0.52631 \cdot 10^{-3}} = -22.5 \text{ tm}$.

Comparing values (5, 10), one establishes that the presence of an elastic-yielding support “B” in the beam (Fig. 2, a) increases the value of the bending moment M_A in relation to the rigid support by (7.7%):

$$\delta = \frac{24.233 - 22.5}{22.500} \cdot 100\% = 7.7\%.$$

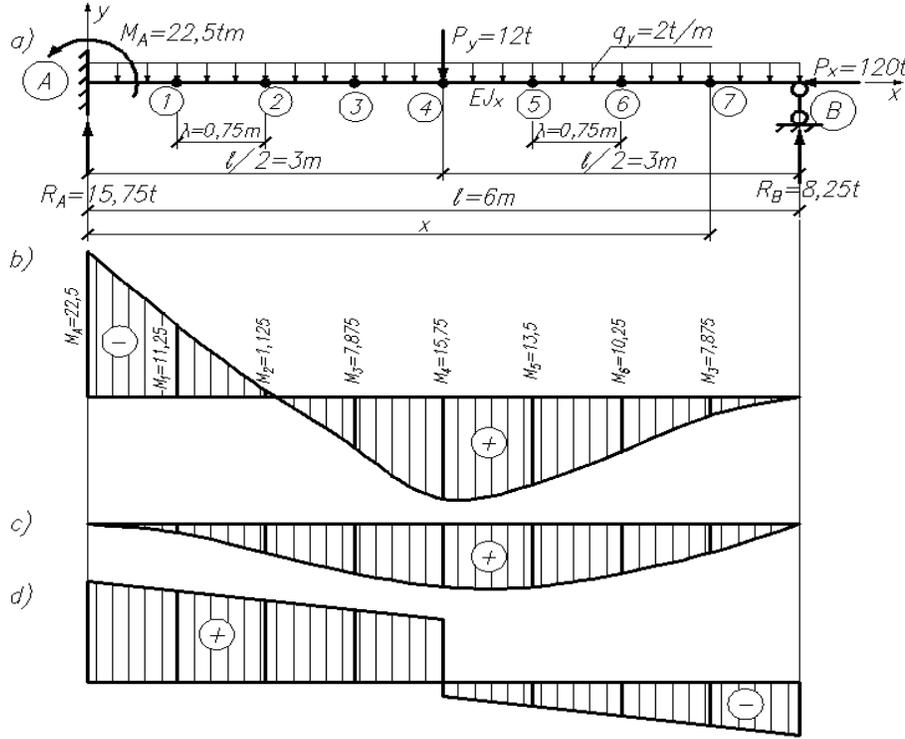


Figure 2: Calculation of a compressed-bent beam with rigid support B: a) design diagram of a beam with rigid supports A, B; b) diagram of bending moments M_x , tm (without taking into account the axial landing forces of the support B); c) diagram of deflections y_i (without taking into account the axial force P_x); d) diagram of transverse forces (without taking into account the axial force P_x)

Figure 2, b shows a diagram of the bending moments of a compressed-bent beam with absolutely rigid supports A and B (without taking into account the effect of the axial force P_x , constructed by the initial parameters method [20]). ($M_0 = M_A = -22.5 \text{ tm}$; $Q_0 = Q_A = R_A = 15.571 \text{ t}$) ($y_0 = y_A = 0$; $\theta_0 = \theta_A = 0.0$):

$$EJy = -22.5 \frac{(x-0)^2}{2} + 15.75 \frac{(x-0)^3}{6} - 12 \frac{(x-3)^3}{6} - 2 \frac{(x-0)^4}{24};$$

or

$$EJy = -11.25 (x)^2 + 2.625(x)^3 - 2(x-3)^3 - 0.0833(x)^4; \quad (10)$$

- a) $(x = 0.75m): y_1 = \frac{1}{3.8 \cdot 10^3} (-5.247069) = -1.38081 \cdot 10^{-3} (m)$;
 b) $(x = 1.5m): y_2 = -4.4408 \cdot 10^{-3} (m)$;
 c) $(x = 2.25m): y_3 = -7.6813 \cdot 10^{-3} (m)$;
 d) $(x = 3.0m): y_4 = -9.7698 \cdot 10^{-3} (m)$;
 e) $(x = 3.75m): y_5 = -9.7626 \cdot 10^{-3} (m)$;
 f) $(x = 4.5m): y_6 = -7.771 \cdot 10^{-3} m$;
 g) $(x = 5.25m): y_7 = -4.2944 \cdot 10^{-3} (m)$.

The diagram of the beam deflections with absolutely rigid supports A and B (without taking into account the effect of the axial force P_x) is shown in Figure 2. c.

Let's calculate the ordinates of the diagram $M_x^{(2)}$ (Fig. 2,b):

$$M_7 = R_B \cdot 0.75 - q \frac{(0.75)^2}{2} = 8.25 \cdot 0.75 - (0.75)^2 = 5.625 (tm);$$

$$M_6 = 10.225 (tm); \quad M_5 = 13.50 (tm); \quad M_4 = 15.75 (tm); \quad (11)$$

$$M_3 = 7.785 (tm); \quad M_2 = -1.125 (tm); \quad M_1 = -11.25 (tm); \quad M_A = M_{on} = -22tm.$$

Let's build the diagram of the beam transverse forces with absolutely rigid supports A and B (without taking into account the effect of the axial force P_x):

$$Q_A = R_A = 15.75t; \quad Q_1 = 14.25; \quad Q_2 = 12.75; \quad Q_3 = 11.25; \quad (12)$$

$$Q_4^{leB} = 15.75 - 2.3 = 9.75; \quad Q_4^{right} = 9.75 - P_y = 9.75 - 12 = -2.25.$$

3 Research results

Based on equation (7) using the data in Table 1, the values of deflections at the nodes of the linear grid were obtained (Fig. 1, a) taking into account the elastic compliance of the hinge support "B":

$$\begin{aligned} y_1^* &= 1.013 \cdot 10^{-3} (m); \quad y_2^* = 3.914 \cdot 10^{-3} (m); \quad y_3^* = 7.167 \cdot 10^{-3} (m); \\ y_4^* &= 9.402 \cdot 10^{-3} (m); \quad y_5^* = 9.417 \cdot 10^{-3} (m); \quad y_6^* = 7.514 \cdot 10^{-3} (m); \\ y_7^* &= 4.161 \cdot 10^{-3} (m). \end{aligned} \quad (13)$$

The diagrams of deflections y_i^* ($i = 1.2.3.4.5.6.7$) are presented in Figure 1,e.

Let's determine the settlement (displacement. deflection) of the elastic-yielding support "B" [$c=0.87m/t$]; the value of the support compliance coefficient]: ($R_B = 7.961 t$ (Fig. 1,b)

$$y_B = R_B = 7.61 \cdot 0.87 \cdot 10^{-3}. \text{ or } y_B = 6.926 \cdot 10^{-3} (m). \quad (14)$$

Taking into account expression (14) based on the method of initial parameters [20], let's construct a diagram of deflections y_i^* (Fig. 1.f), calculating the ordinates of this diagram (the origin is at point "B"). Along the cross section ($m-n$) (Fig. 1.b) there is:

$$EJy = -EJ \cdot 6.926 \cdot 10^{-3} + Q_0 EJx + \frac{7.961}{6} \cdot x^3 + \frac{12}{6} \cdot (x-3)^3 + \frac{2}{24} \cdot x^4 \quad (15)$$

With the initial conditions, $x = 6m$; $y_A = 0.0$; from (15) there are determined the θ_0 values:

$$EJ\theta_0 = -16.38 \quad (16)$$

By substituting (16) into (15) (with $EJ = 3.8 \cdot 10^{-3}tm^2$) there is obtained:

$$y = 10^{-3}(-6.926 - 4.31x + 0.3492x^3 - 0.5623(x-3)^3 - 0.02137 \cdot x^4). \quad (17)$$

According to (17) there are calculated deflections at the design nodes $i = 1.2.3.4.5.6.7$. B:

$$\begin{aligned} y_A &= 0.0; y_1 = 1.6627 \cdot 10^{-3}; y_2 = 5.1607 \cdot 10^{-3}; y_3 = 9.1369 \cdot 10^{-3}; \\ y_4 &= 12.1585 \cdot 10^{-3}; y_5 = 13.1936 \cdot 10^{-3}; y_6 = 12.3206 \cdot 10^{-3}; \\ y_7 &= 10.018 \cdot 10^{-3}; y_B = 6.926 \cdot 10^{-3}. \end{aligned} \quad (18)$$

Based on [18], p. 467-468 and with the use of equations of the initial parameters method, $K = 1.15 \cdot 10^3$ (t/m) is the spring stiffness coefficient in the support "B", the origin of the point (at point "B"). The initial parameters (at point "B") are as follows:

$$M(0) = 0; Q(0) = Ky(0); y(0) \neq 0; y'(0) \neq 0; \quad (19)$$

$$P_y = 12t; P_x = 120t; K = 1.15 \cdot 10^3 \left(\frac{T}{M} \right); EJ = 3.8 \cdot 10^3 (tm^2); \ell = 6m;$$

$R_B = 7.961t$ (support reaction); $\alpha^2 = \frac{P_x}{EJ} = \frac{120t}{3.8 \cdot 10^3 Tm^2}$ is the parameter of the axial force P_x ; $\alpha = 0.1777$.

The boundary conditions (at the point A):

$$y(\ell) = 0; y'(\ell) = 0. \quad (20)$$

The equation of the method of initial parameters ([18], formulas 2.35 ÷ 2.38, p. 461) (taking into account condition (19)):

$$y(z) = y(0) + y'(0) \cdot \frac{\sin \alpha z}{\alpha} - \frac{Ky(0)}{\alpha^3 EJ} (\alpha z - \sin \alpha z) + \frac{R_B}{6EJ} z^3 - \frac{P_y}{6EJ} (z-3)^3 - \frac{q_y}{24EJ} z^4; \quad (21)$$

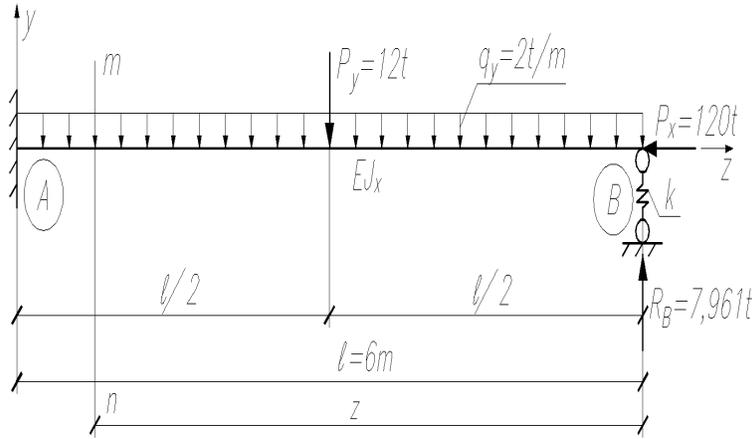


Figure 3: Design diagram of the beam for the method of initial parameters

$$y'(z) = y'(0)\cos\alpha z - \frac{Ky(0)}{\alpha^2 EJ} (1 - \cos\alpha z) + \frac{R_B}{2EJ} z^2 - \frac{P_y}{2EJ} (z - 3)^3 - \frac{q_y}{6EJ} z^3. \quad (22)$$

Based on condition (20), there are written down equations (21, 22)

$$0 = y(0) + y'(0) \cdot \frac{\sin\alpha l}{\alpha} - \frac{Ky(0)}{\alpha^3 EJ} (\alpha l - \sin\alpha l) + \frac{R_B}{6EJ} l^3 - \frac{P_y}{6EJ} (l - 3)^3 - \frac{q_y}{24EJ} l^4. \quad (23)$$

$$0 = y'(0)\cos\alpha l - \frac{Ky(0)}{\alpha^2 EJ} (1 - \cos\alpha l) + \frac{R_B}{2EJ} l^2 - \frac{P_y}{2EJ} (l - 3)^3 - \frac{-q_y}{6EJ} l^3. \quad (24)$$

By solving (23, 24) together, there is obtained:

$$\alpha = \sqrt{31.58 \cdot 10^{-3}} = 0.1777; \sin\alpha l = 0.860; \cos\alpha l = 0.4840;$$

Instead of (23, 24), there is obtained:

$$\begin{cases} -9.1933 \cdot y(0) + 4.936 \cdot y'(0) = 0.0426 \\ -4.9826 \cdot y(0) + 0.4801 \cdot y'(0) = 0.03316. \end{cases} \quad (25)$$

From here:

$$y(0) = -7.097 \cdot 10^{-3} \cdot y'(0) = -4.588 \cdot 10^{-3} \quad (26)$$

By substituting (26) into equations (23, 22), there is obtained

$$y(z) = 10^{-3} \cdot (-7.097 - 25.819 \cdot \sin(0.1777z) + 382.78 [0.1777z - \sin(0.1777z)]) + 0.52632(z - 3)^3 - 0.02193z^4 \quad (27)$$

- the equation of the compressed-bent rod deflections with elastic compliant support B.

$$y'(z) = 10^{-3} \cdot (-4.588\cos\alpha z + 68.017(1 - \cos\alpha z) - 1.5789(z - 3)^2 - 0.08772z^3) \quad (28)$$

- the equation for the angle of rotation.

Based on (27), let's build the diagram of deflections for a compressed-bent beam with elastic compliant support B (Fig. 1.f).

Table 2: ($\alpha = 0.1777$). Parameters of equations (27, 28)

$z.M$	$\sin\alpha z$	$\alpha z - \sin\alpha z$	αz	$\cos\alpha z$
0.75 (7 node)	0.130	0.003275	0.1333	0.991
1.5 (6 node)	0.257	0.0035	0.2666	0.9651
2.25 (5 node)	0.3890	0.010825	0.3998	0.9211
3.0 (4 node)	0.508	0.0251	0.5331	0.8612
3.75 (3 node)	0.6210	0.045	0.6664	0.785
4.5 (2 node)	0.717	0.08265	0.7997	0.6967
5.25 (1 node)	0.802	0.13025	0.9329	0.595

a) ($z = 0.75$) (7 node): $y_7 = 10^{-3}(-9.2068)$;

b) ($z = 1.5$) (6 node): $y_6 = 10^{-3}(-12.503774)$;

c) ($z = 2.25$) (5 node): $y_5 = 10^{-3}(-13.559)$;

d) ($z = 3.0$) (4 node): $y_4 = 10^{-3}(-11.2333)$;

e) ($z = 3.75$) (3 node): $y_3 = 10^{-3}(-10.4627)$;

f) ($z = 4.5$) (2 node): $y_2 = 10^{-3}(-4.7720)$;

g) ($z = 5.25$) (1 node): $y_1 = 10^{-3}(-0.3434)$;

$$K = 1.15 \cdot 10^3 \text{ (m)}; \quad c = \frac{1}{K} = 0.87 \cdot \frac{10^{-3}m}{T}; \quad EJ_x = 3.8 \cdot 10^3 \text{ (tm}^2\text{)}$$

Let's calculate the bending moments according to formulas ([18]. 2. 37. p. 461):

$$M(z) = -0.1777 \cdot 10^3 \cdot 4.588 \cdot 10^{-3} \sin\alpha z - \frac{1.15 \cdot 10^3 \cdot 7.097 \cdot 10^{-3}}{0.1777} \sin\alpha z - 12(z - 3) - z^2 \quad (29)$$

a) ($z = 6m$) (A node): $M_A = 42.0307 \cdot 0.877 - 12 \cdot 3 - 36 = -35.13067 \text{ (tm)}$;

b) ($z = 5.25m$) (1 node): $M_1 = -20.8539 \text{ (tm)}$;

c) ($z = 4.5m$) (2 node): $M_2 = 42.0307 \cdot 0.717 - 12 \cdot 1.5 - 20.25 = -8.114 \text{ (tm)}$;

d) ($z = 3.75m$) (3 node): $M_3 = 42.0307 \cdot 0.621 - 12 \cdot 0.75 - 14.0625 = 3.0386 \text{ (tm)}$;

e) ($z = 3.0m$) (4 node): $M_4 = 42.0307 \cdot 0.508 - 12 \cdot 0 - 9 = 12.3516 \text{ (tm)}$;

f) ($z = 2.25m$) (5 node): $M_5 = 42.0307 \cdot 0.389 - 5.0625 = 11.2874 \text{ (tm)}$;

g) ($z = 1.5m$) (6 node): $M_6 = 42.0307 \cdot 0.257 - 2.25 = 8.852 \text{ (tm)}$;

h) ($z = 0.75m$) (7 node): $M_7 = 42.0307 \cdot 0.13 - 0.5625 = 4.9015 \text{ (tm)}$;

i) $M_B = 0$ (hinge in the support).

$$Q(z) = 7.611(\cos\alpha z) - 12 - 2z - P_x y'(z); (P_x = 120t) \quad (30)$$

$$Q_0 = 1.15 \cdot y(0).$$

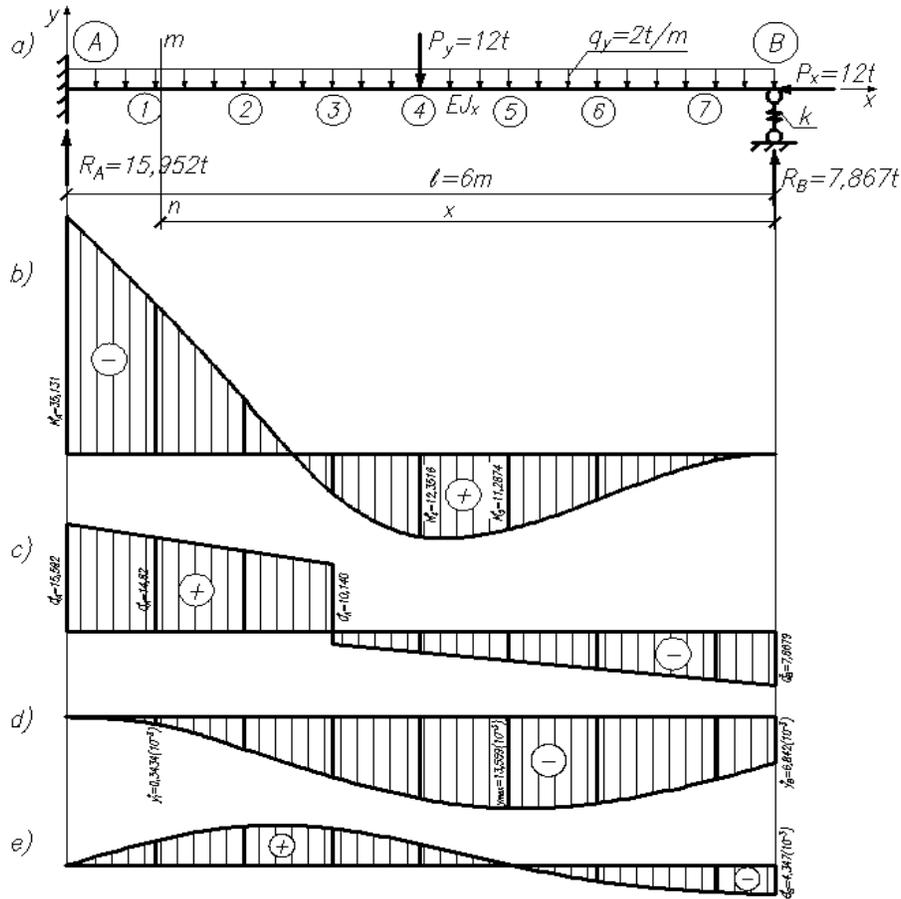


Figure 4: Designing a beam with the use of the initial parameters method: a) design diagram of the beam; b) design diagram of the moments taking into account the compliance of the support B (M_i^* , tm); c) design diagram of transverse forces Q_i^* (taking into account the axial force and compliance of the support B); d) design diagram of deflections (y_i , m) taking into account the settlement of the support B; e) design diagram of the rotation angles (y_i' , rad) taking into account the settlement of the support B

Table 3: Parameters of equations (29, 30)

z	0.0	0.75	1.5	2.25	3.0	3.75	4.5	5.25	6.0
$y'(z) \cdot 10^3$	-4.347	-3.9776	-2.3501	0.1413	3.1211	5.508	5.8871	4.1305	0.0
$P_x y'(z)$	0.522	0.4496	0.266	-0.016	-0.353	-0.623	-0.666	-0.4676	0.0
$Q(z)(t)$	-4.389	-	-	-	-	-	-	-	-
$\cos \alpha z$	1.00	0.991	0.9651	0.9211	0.8612	0.785	0.6967	0.595	0.484

Based on [21], p. 45:

1) Expression for deflections $n = \sqrt{\frac{P_x}{EJ}} = \sqrt{\frac{120}{3.8 \cdot 10^3}} = 0.1777(M^{-1})$ – the axial force parameter, (y_0, θ_0, M_0, Q_0 are the initial parameters of the method).

2) The resolving equation is:

$$y = y_0 + \frac{\sin nx}{n} \theta_0 + \frac{\cos nx - 1}{n^2 EJ} M_0 + \frac{\sin nx - nx}{n^3 EJ} Q_0 + \frac{1}{n^3 EJ} \sum_i^n P_i [n(x - a_i) - \sin n(x - a_i)] + \frac{q}{n^4 EJ} \left(\frac{n^2 x^2}{2} + \cos nx - 1 \right). \quad (31)$$

where

$$\theta = \cos nx \theta_0 - \frac{\sin nx}{n EJ} M_0 + \frac{\cos nx - 1}{n EJ} Q_0 + \frac{1}{n^2 EJ} \sum_i^n P_i [1 - \cos n(x - a_i)] + \frac{q}{n^3 EJ} (nx - \sin nx). \quad (32)$$

Based on Figure 3 and equations (31, 32) there is (the beginning of coordinates at the point B):

a) Initial conditions (at the point B):

$$M_0 = 0; \quad Q_0 = K \cdot y_0 = 1.15 \cdot 10^3 y_0, \quad (33)$$

$K = 1.15 \cdot 10^3 t/m$ is the coefficient of the support B rigidity, $y_0 = y_B$.

b) The boundary conditions (with $x = \ell = 6m$) (at the point A):

$$M(\ell) \neq 0; \quad Q(\ell) \neq 0; \quad y(\ell) = 0. \quad \theta(\ell) = 0. \quad \theta_0 = y'_B. \quad (34)$$

Taking into account (33, 34) and according to (31, 32) there is (along the m - n section (Figure 4), see Table 2)):

$$\begin{aligned} 0 &= y_0 + 5.6275 \sin(1.0662) \theta_0 - P_y \frac{1}{(0.1777)^3 \cdot 3.8 \cdot 10^3} \times \\ &\times \{0.1777(6 - 3) - \sin[(0.1777) \cdot (6 - 3)]\} + \frac{\sin(1.0662) - 0.1777 \cdot 6}{(0.1777)^3 \cdot 3.8 \cdot 10^3} \theta_0 \times \\ &\times (1.15 \cdot 10^3) - \frac{q}{(0.1777)^4 \cdot 3.8 \cdot 10^3} \left[\frac{(0.1777)^2 \cdot 6^2}{2} + \cos(1.0662) - 1 \right] \\ 0 &= \cos(1.0662) \theta_0 - P_y \frac{1}{(0.1777)^2 \cdot 3.8 \cdot 10^3} [1 - \cos[(0.1777)(6 - 3)]] - \\ &- \frac{q}{(0.1777)^3 \cdot 3.8 \cdot 10^3} (0.1777 \cdot 6 - \sin(1.0662)) + \frac{\cos(1.0662) - 1}{(0.1777)^2 \cdot 3.8 \cdot 10^3} \times \end{aligned}$$

$$\times (1.15 \cdot 10^3) y_0.$$

or

$$\begin{cases} -9.323y_0 + 4.923\theta_0 = 0.041495. \\ -4.94526y_0 + 0.484\theta_0 = 0.031988. \end{cases} \Rightarrow \begin{cases} y_0 = -6.842 \cdot 10^{-3} (m) \\ \theta_0 = -4.347 \cdot 10^{-3} (rad). \end{cases}$$

Let's calculate the ordinates of the beam transverse force diagram Q_i^* (taking into account the axial force P_x and elastic compliance of the support B) (Fig. 4, c), ([21], formulas 24, 25, p.46) ($P_x = 120T$):

$$Q_x = \frac{dM}{dx} = n^2 EJ \cos nx \theta_0 + \cos nx \theta_0 + P_y (\cos n(x - a_i) + \frac{q_y}{n} \sin nx) + P_x \theta(z);$$

$$\begin{aligned} Q_x &= (0.1777)^2 \cdot 3.8 \cdot 10^3 \cos(1.0662) (-4.347 \cdot 10^{-3}) + \\ &+ \cos(1.0662) (-1.15 \cdot 10^3 \cdot 6.842 \cdot 10^{-3}) + 12 \cdot \cos(0.5331) + \\ &+ 11.255 \sin(1.0662) + 120 \cdot \theta(z); \end{aligned}$$

a) $x = 6m$; $Q_A = -0.025246 - 3.808 + 10.334 + 9.693 + 0 = 15.953t$;

б) $x = 0$; $Q_B = -7.8679t$;

Checking the calculation (checking the equilibrium of forces on the Y axis): $\sum F_{ky} = 0$;

$$Q_A + Q_B - P_y - q_y \cdot \ell = 0; 15.952 + 7.8679 + 12 + 2.6 = 0;$$

$$23.8200 \approx 24.$$

The error is: $\delta = 0.750\% < [\delta = 5\%]$ (the calculation was made correctly).

The Q_i^* ordinates are calculated in Table 4 (taking into account the data of Tables 2, 3; the beginning of coordinates is the node B, Fig. 4, a).

Table 4: Ordinates of the Q_i^* diagram

No.	x, m	$0.1777 \times (nx)$	$(x-3) \times (n_1 x)$	$(x-3) \cdot 0.1777 (nx)$	$\sin nx$	$\cos nx$	$\cos(n_1 x)$	$Q_i^* \cdot T$
A	6	1.0662	3.0	0.5331	0.86	0.484	0.8612	15.952
1	5.25	0.933	2.25	0.400	0.802	0.595	0.9211	14.6201
2	4.5	0.800	1.5	0.266	0.717	0.6967	0.9651	13.1398
3	3.75	0.666	0.75	0.133	0.621	0.785	0.9911	11.6735
4	3.0	0.5331	0.0	0.0	0.508	0.8612	1.0	- 1.8608
5	2.25	0.400	-	-	0.389	0.9211	-	- 3.3657
6	1.5	0.266	-	-	0.257	0.9651	-	- 4.9386
7	0.75	0.1333	-	-	0.13	0.991	-	- 6.4023
B	0.0	0.0	-	-	0.0	1.000	-	- 7.8679

$$Q_x = -0.521613\cos(0.1777x) - 7.8683\cos(0.1777x) + \\ + 12\cos[(0.1777(x-3))] + 11.255\sin(0.1777x) + 120 \cdot \theta(x).$$

$$Q_x = -8.3899\cos(0.1777x) + 12\cos[(0.1777(x-3))] + \\ + 11.255\sin(0.177x) + 120 \cdot \theta(x).$$

Based on the values in Table 4, there was built the Q_i^* diagram (Fig. 4, c).

4 Conclusions

1) In this work, a study of the stress-strain state (SSS) of a single-span statically indeterminate compressed-bent beam was carried out that is under conditions of complex resistance, taking into account the elastic compliance of the hinge support B.

2) The initial diagram of bending moments that was constructed by the force method based on the equation of five support moments, taking into account the compliance coefficient of the support B with the value $c = 0.87 \cdot 10^{-3} \left(\frac{m}{t}\right)$ (Fig. 1, c).

3) The diagram of beam deflections (Fig. 1,f). taking into account along with the transverse forces P_y , q_y and the action of the axial force P_x is constructed taking into account elastic compliance of support B with the use of the numerical finite difference method with a regular linear grid of density $n=8$.

4) In Figure 2, c, d, the calculated diagrams of bending moments and transverse forces are given, taking into account longitudinal-transverse bending and elastic-compliance of supports B.

5) In Figure 2 c, d, the calculated diagrams of bending moments and transverse forces are given. taking into account longitudinal-transverse bending and elastic-compliance of supports "B".

6) Based on the studies carried out, the following was established:

a) taking into account the action of the axial force P_x alongside with the transverse forces P_y, q_y , as well as taking into account the effect of the elastic compliance of support B ($c = 0.87 \cdot 10^{-3} \left(\frac{m}{t}\right)$ is the compliance coefficient) compared to the absolute rigidity of this support, significantly changes the nature and ordinate values of the bending moment diagram (compare Fig. 1,c and Fig. 4,b);

b) the diagrams of transverse forces do not significantly change the nature and magnitude of the ordinates of the diagram of transverse forces (compare Fig. 2,d and Fig. 4,d); the deflection diagrams (compare Fig. 1,e and Fig. 4,d) differ significantly. In this case, the magnitude of the deflections at point 5 is by (1,39) times greater when taking into account the settlement of the support B;

c) the diagram of rotation angles, taking into account the axial force P_x and the elastic settlement of the support B, has the largest value at node 2 (see Fig. 4,e); in this case, the angle of rotation at point 5 approaches zero; here the deflection reaches its maximum.

Figure 2 c,d shows the design diagrams of bending moments and transverse forces taking into account the longitudinal-transverse bending and elastic compliance of supports B.

7) The theoretical principles and applied results presented in this study can serve as the basis for calculations and design of core elements of various buildings and their structures that are used in the field of construction, mechanical engineering, transport, mine and underground construction.

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