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# THE NONLOCAL SOLVABILITY CONDITIONS FOR A SYSTEM WITH CONSTANT TERMS AND COEFFICIENTS OF THE VARIABLE $t$

We consider the Cauchy problem for a system of quasilinear differential equations with constant terms and coefficients of the variable $t$. We investigate the solvability of the Cauchy problem for a system of quasilinear differential equations with constant terms and coefficients of the variable $t$ using the additional argument method. A theorem on the existence and uniqueness of the local solution of the Cauchy problem for a system of quasilinear differential equations with constant terms and coefficients of the variable $t$ is formulated. We obtain sufficient conditions for the existence and uniqueness of a nonlocal solution of the Cauchy problem in original coordinates for a system of quasilinear differential equations with constant terms and coefficients of the variable $t$. A theorem on the existence and uniqueness of the nonlocal solution of the Cauchy problem for a system of quasilinear differential equations with constant terms and coefficients of the variable $t$ is formulated. A theorem on the existence and uniqueness of the nonlocal solution of the Cauchy problem for a system of quasilinear differential equations with constant terms and coefficients of the variable $t$ is proved. The proof of the nonlocal solvability of the Cauchy problem for a system of quasilinear differential equations with constant terms and coefficients of the variable $t$ relies on global estimates.

**Key words**: Cauchy problem, quasilinear system, functions, global estimates.

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## $t$ айнымалы еркін мүшелері мен коэффициенттері бар жүйе үшін локалді емес шешімділік шарттары

$t$ айнымалы еркін мүшелері мен коэффициенттері бар квазисызықты дифференциалдық теңдеулер жүйесі үшін Коши есебін қарастырамыз. Қосымша аргумент әдісі арқылы $t$ айнымалы еркін мүшелері мен коэффициенттері бар квазисызықты дифференциалдық теңдеулер жүйесі үшін Коши есебін шешімділікке зерттейміз. $t$ айнымалы еркін мүшелері мен коэффициенттері бар квазисызықты дифференциалдық теңдеулер жүйесі үшін Коши есебінің локалді шешімі бар және жалғыздығы туралы теорема тұжырымдалған. $t$ айнымалы еркін мүшелері мен коэффициенттері бар квазисызықты дифференциалдық теңдеулер жүйесі үшін бастапқы координаталардағы Коши есебінің локальды емес шешімінің бар болуы мен жалғыздығының жеткілікті шарттарын аламыз. Еркін мүшелері және $t$ айнымалы коэффициенттері бар квазисызықты дифференциалдық теңдеулер жүйесі үшін Коши есебінің локалді емес шешімі бар және жалғыздығы туралы теорема тұжырымдалған. $t$ айнымалы еркін мүшелері мен коэффициенттері бар квазисызықты дифференциалдық теңдеулер жүйесі үшін Коши есебінің локальды емес шешімінің бар болуы және жалғыздығы туралы теорема дәлелденді. $t$ айнымалы еркін мүшелері мен коэффициенттері бар квазисызықты дифференциалдық теңдеулер жүйесі үшін Коши есебінің локальды емес шешімділігінің дәлелі глобалді априорлық бағалауларға негізделген.

**Түйін сөздер**: Коши есебі, квазисызықты жүйе, функциялар, глобалді бағалаулар.

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**Условия нелокальной разрешимости для системы со свободными членами и коэффициентами переменного $t$**

Мы рассматриваем задачу Коши для системы квазилинейных дифференциальных уравнений со свободными членами и коэффициентами переменного $t$. Мы исследуем разрешимость задачи Коши для системы квазилинейных дифференциальных уравнений со свободными членами и коэффициентами переменного $t$ с помощью метода дополнительного аргумента. Сформулирована теорема о существовании и единственности локального решения задачи Коши для системы квазилинейных дифференциальных уравнений со свободными членами и коэффициентами переменного $t$. Мы получаем достаточные условия существования и единственности нелокального решения задачи Коши в исходных координатах для системы квазилинейных дифференциальных уравнений со свободными членами и коэффициентами переменного $t$. Сформулирована теорема о существовании и единственности нелокального решения задачи Коши для системы квазилинейных дифференциальных уравнений со свободными членами и коэффициентами переменного $t$. Доказана теорема о существовании и единственности нелокального решения задачи Коши для системы квазилинейных дифференциальных уравнений со свободными членами и коэффициентами переменного $t$. Доказательство нелокальной разрешимости задачи Коши для системы квазилинейных дифференциальных уравнений со свободными членами и коэффициентами переменного $t$ основано на глобальных оценках.
**Ключевые слова**: задача Коши, квазилинейная система, функции, глобальные оценки.

# 1 Introduction

We consider the system:

$$\begin{cases} \partial_t u\,(t,x) + (a(t)u(t,x) + b(t)v(t,x) + a_1(t))\partial_x u\,(t,x) = f_1(t,x), \\ \partial_t v(t,x) + (c(t)u(t,x) + g(t)v(t,x) + a_2(t))\partial_x v(t,x) = f_2(t,x), \end{cases} \tag{1}$$

where $u(t,x),\ v(t,x)$ are unknown functions, $f_1,\ f_2,\ \ a(t),\ b(t),\ c(t),\ g(t),\ \ a_1(t),\ a_2(t)$ are given functions,

$$a(t) > 0,\ b(t) > 0,\ c(t) > 0,\ g(t) > 0, \quad t \in [0,T],$$

subject to the initial conditions:

$$u(0,x) = \varphi_1(x),\ v(0,x) = \varphi_2(x), \tag{2}$$

where $\varphi_1(x), \varphi_2(x)$ are given functions.

The problem (1), (2) is considered on

$$\Omega_T = \{(t,x)\,|0 \le t \le T, x \in (-\infty,+\infty), T > 0\}.$$

The system (1) appear in various problems in natural sciences, for instance, in describing the spreading of finite intensity perturbation under non-stationary one-dimensional ow of ideal gas [1, 2].

In the present work, we determine sufficient conditions for the existence and uniqueness of a nonlocal solution of the Cauchy problem (1), (2), where $f_1,\ f_2,\ \ a(t),\ b(t),\ c(t),\ g(t),$ $a_1(t),\ a_2(t)$ are given functions,

$$a(t) > 0,\ b(t) > 0,\ c(t) > 0,\ g(t) > 0, \quad t \in [0,T].$$

We investigate the solvability of the Cauchy problem (1), (2) using the additional argument method. The method of an additional argument allows us to obtain sufficient conditions for the existence and uniqueness of a nonlocal solution of the Cauchy problem (1), (2) in original coordinates.

## 2 Material and Methods

We use the additional argument method. For the problem (1), (2) we write the extended characteristic system [3–9]:

$$\frac{d\eta_1(s,t,x)}{ds} = a(s)w_1(s,t,x) + b(s)w_3(s,t,x) + a_1(s), \tag{3}$$

$$\frac{d\eta_2(s,t,x)}{ds} = c(s)w_4(s,t,x) + g(s)w_2(s,t,x) + a_2(s), \tag{4}$$

$$\frac{dw_1(s,t,x)}{ds} = f_1(s,\eta_1), \tag{5}$$

$$\frac{dw_2(s,t,x)}{ds} = f_2(s,\eta_2), \tag{6}$$

$$w_3(s,t,x) = w_2(s,s,\eta_1),\ \ w_4(s,t,x) = w_1(s,s,\eta_2), \tag{7}$$

$$w_1(0,t,x) = \varphi_1(\eta_1(0,t,x)),\ \ w_2(0,t,x) = \varphi_2(\eta_2(0,t,x)),\ \ \eta_i(t,t,x) = x,\ \ i = 1,2. \tag{8}$$

Unknown functions $\eta_i,\ w_j,\ i = 1,2,\ j = \overline{1,4}$, depend not only on $t$ and $x$, but also on additional argument $s$. Integrating equations (3)–(6) with respect to the argument $s$ and taking into considerations conditions (7), (8), we obtain an equivalent system of integral equations:

$$\eta_1(s,t,x) = x - \int_s^t (a(\nu)w_1 + b(\nu)w_3 + a_1(\nu))d\nu, \tag{9}$$

$$\eta_2(s,t,x) = x - \int_s^t (c(\nu)w_4 + g(\nu)w_2 + a_2(\nu))d\nu, \tag{10}$$

$$w_1(s,t,x) = \varphi_1(\eta_1(0,t,x)) + \int_0^s f_1(\nu,\eta_1)d\nu, \tag{11}$$

$$w_2(s,t,x) = \varphi_2(\eta_2(0,t,x)) + \int_0^s f_2(\nu,\eta_2)d\nu, \tag{12}$$

$$w_3(s,t,x) = w_2(s,s,\eta_1),\ \ w_4(s,t,x) = w_1(s,s,\eta_2). \tag{13}$$

Substituting (9), (10) into (11)–(13), we get

$$\begin{aligned} w_1(s,t,x) &= \varphi_1(x - \int_0^t (a(\nu)w_1 + b(\nu)w_3 + a_1(\nu))d\nu) + \\ &+ \int_0^s f_1(\nu, x - \int_\nu^t (a(\tau)w_1 + b(\tau)w_3 + a_1(\tau))d\tau)d\nu, \end{aligned} \tag{14}$$

$$\begin{aligned} w_2(s,t,x) &= \varphi_2(x - \int_0^t (c(\nu)w_4 + g(\nu)w_2 + a_2(\nu))d\nu) + \\ &+ \int_0^s f_2(\nu, x - \int_\nu^t (c(\tau)w_4 + g(\tau)w_2 + a_2(\tau))d\tau)d\nu, \end{aligned} \tag{15}$$

$$w_3(s,t,x) = w_2(s,s,x - \int_s^t (a(\nu)w_1 + b(\nu)w_3 + a_1(\nu))d\nu), \tag{16}$$

$$w_4(s,t,x) = w_1(s,s,x - \int_s^t (c(\nu)w_4 + g(\nu)w_2 + a_2(\nu))d\nu). \tag{17}$$

**Lemma 1** *Let $w_1(s,t,x)$ and $w_2(s,t,x)$ satisfy the system of integral equations (14)–(17). Assume that $w_1(s,t,x)$, $w_2(s,t,x)$ together with their first order derivatives are continuously differentiable and bounded. Then the pair of functions*

$$u(t,x) = w_1(t,t,x),\ \ v(t,x) = w_2(t,t,x)$$

*is a solution to the problem (1), (2) on $\Omega_{T_0}$, where $T_0$ is a constant.*

The Lemma 1 can be proven in the same way as in [9].

The proof of the nonlocal solvability of the Cauchy problem (1), (2) relies on global estimates.

# 3 Existence of a local solution

We denote $\Gamma_T = \{ (s,t,x) | \, 0 \le s \le t \le T,\ x \in (-\infty, +\infty),\ T > 0\}$,

$$C_\varphi = \max\{\sup_R \left|\varphi_i^{(l)}\right| |i = 1,2,\ l = \overline{0,2}\},$$

$$l = \max\{\sup_{[0,T]} a(t),\ \sup_{[0,T]} b(t),\ \sup_{[0,T]} c(t),\ \sup_{[0,T]} g(t)\},$$

$$C_f = \max\{\sup_{\Omega_T} |f_1(t,x)|\,, \sup_{\Omega_T} |f_2(t,x)|\,, \sup_{\Omega_T} |\partial_x f_1(t,x)|\,, \sup_{\Omega_T} |\partial_x f_2(t,x)|\},$$

$$\|G\| = \sup_{\Gamma_T} |G(s,t,x)|\,,\ \|f\| = \sup_{\Omega_T} |f(t,x)|,$$

$\bar{C}^{\alpha_1,\alpha_2,...\alpha_n}(\Omega_*)$ is the space of functions continuous and bounded, together with its derivatives up to order $\alpha_m$ w.r.t. $m$th argument, $m = \overline{1,n}$ on unbounded subset $\Omega_* \subset R^n, n = 1,2....,$

$C([0,T])$ is the space of continuous functions on $[0,T]$.

**Theorem 1** *Suppose that*

$$\varphi_1, \varphi_2 \in \bar{C}^2(R),\ f_1, f_2 \in \bar{C}^{2,2}(\Omega_T),\ a, b, c, g, a_1, a_2 \in C([0,T]),$$

$$T \leqslant \min(\frac{C_\varphi}{4C_f}, \frac{3}{40C_\varphi l}),$$

$$a(t) > 0,\ b(t) > 0,\ c(t) > 0,\ g(t) > 0 \ \textit{on}\ [0,T],$$

$$\varphi_1'(x) \geq 0,\ \varphi_2'(x) \geq 0 \ \textit{on}\ R,$$

$$\partial_x f_1(t,x) \geq 0,\ \partial_x f_2(t,x) \geq 0 \ \textit{on}\ \Omega_T.$$

*Then for each*

$$T \leq \min(\frac{C_\varphi}{4C_f}, \frac{3}{40C_\varphi l}),$$

*the Cauchy problem (1), (2) has a unique solution*

$$u(t,x), v(t,x) \in \bar{C}^{1,2}(\Omega_T)$$

*which can be found from the system of integral equations (14)–(17).*

The Theorem 1 can be proven in the same way as in [4–8].

# 4 Existence of a nonlocal solution

**Theorem 2** *Suppose that*

$$\varphi_1, \varphi_2 \in \bar{C}^2(R),\ f_1, f_2 \in \bar{C}^{2,2}(\Omega_T),\ a, b, c, g, a_1, a_2 \in C([0,T]),$$

$$a(t) > 0,\ b(t) > 0,\ c(t) > 0,\ g(t) > 0 \ \textit{on}\ [0,T],$$

$$\varphi_1'(x) \geq 0,\ \varphi_2'(x) \geq 0 \ \textit{on}\ R,$$

$$\partial_x f_1(t,x) \geq 0,\ \partial_x f_2(t,x) \geq 0 \ \textit{on}\ \Omega_T.$$

*Then for any $T > 0$ the Cauchy problem (1), (2) has a unique solution*

$$u(t,x), v(t,x) \in \bar{C}^{1,2}(\Omega_T)$$

*which can be found from the system of integral equations (14)–(17).*

**Proof.** Differentiating (1) with respect to $x$ and denoting

$$p(t,x)=\partial_x u(t,x),\quad r(t,x)=\partial_x v(t,x),$$

we obtain the system of equations:

$$\begin{cases} \partial_t p+(a(t)u+b(t)v+a_1(t))\partial_x p=-a(t)p^2-b(t)pr+\partial_x f_1, \\ \partial_t r+(c(t)u+g(t)v+a_2(t))\partial_x r=-g(t)r^2-c(t)pr+\partial_x f_2, \\ p(0,x)=\varphi_1'(x),\quad r(0,x)=\varphi_2'(x). \end{cases} \tag{18}$$

We add two equations to the system of equations (9)–(13):

$$\begin{cases} \frac{d\gamma_1(s,t,x)}{ds}=-a(s)\gamma_1^2(s,t,x)-b(s)\gamma_1(s,t,x)\gamma_2(s,s,\eta_1)+\partial_x f_1(s,\eta_1), \\ \frac{d\gamma_2(s,t,x)}{ds}=-g(s)\gamma_2^2(s,t,x)-c(s)\gamma_1(s,s,\eta_2)\gamma_2(s,t,x)+\partial_x f_2(s,\eta_2), \end{cases} \tag{19}$$

subject to the conditions:

$$\gamma_1(0,t,x)=\varphi_1'(\eta_1),\quad \gamma_2(0,t,x)=\varphi_2'(\eta_2). \tag{20}$$

We rewrite (19), (20) as follows:

$$\begin{cases} \gamma_1\,(s,t,x)=\varphi_1'(\eta_1)+\int\limits_0^s[-a(\nu)\gamma_1^2-b(\nu)\gamma_1\gamma_2(\nu,\nu,\eta_1)+\partial_x f_1]d\nu, \\ \gamma_2(s,t,x)=\varphi_2'(\eta_2)+\int\limits_0^s[-g(\nu)\gamma_2^2-c(\nu)\gamma_2\gamma_1(\nu,\nu,\eta_2)+\partial_x f_2]d\nu. \end{cases} \tag{21}$$

As in [4–8], we can prove the existence of a continuously differentiable solution to the problem (21). Therefore,

$$\gamma_1(t,t,x)=p(t,x)=\frac{\partial u}{\partial x},\gamma_2(t,t,x)=r(t,x)=\frac{\partial v}{\partial x}.$$

As in [4,5], we can prove that for all $t$ and $x$ on $\Omega_T$

$$\|u\|\leqslant C_\varphi+TC_f,\ \|v\|\leqslant C_\varphi+TC_f. \tag{22}$$

From (19),(20), we obtain

$$\begin{cases} \gamma_1\,(s,t,x)=\varphi_1'(\eta_1)\exp(-\int\limits_0^s(a(\nu)\gamma_1+b(\nu)\gamma_2)\,d\nu)+ \\ +\int\limits_0^s\partial_x f_1\exp(-\int\limits_\tau^s(a(\nu)\gamma_1+b(\nu)\gamma_2)\,d\nu)d\tau, \\ \gamma_2(s,t,x)=\varphi_2'(\eta_2)\exp(-\int\limits_0^s(c(\nu)\gamma_1+g(\nu)\gamma_2)d\nu)+ \\ +\int\limits_0^s\partial_x f_2\exp(-\int\limits_\tau^s(c(\nu)\gamma_1+g(\nu)\gamma_2)\,d\nu)d\tau. \end{cases} \tag{23}$$

Since

$$a(t)>0,\ b(t)>0,\ c(t)>0,\ g(t)>0 \text{ on } [0,T],$$

$$\varphi_1'(x)\geq 0,\ \varphi_2'(x)\geq 0 \text{ on } R,$$

$$\partial_x f_1(t,x) \geq 0,\ \partial_x f_2(t,x) \geq 0 \text{ on } \Omega_T,$$

it follows from (23) that $\gamma_1 \geq 0,\ \gamma_2 \geq 0$ on $\Gamma_T$. Therefore,

$$\|\gamma_i\| \leqslant C_\varphi + TC_f,\ i = 1, 2.$$

Since $\gamma_1(t,t,x) = \partial_x u,\ \gamma_2(t,t,x) = \partial_x v$, then for all $t$ and $x$ on $\Omega_T$ we obtain the estimates

$$\|\partial_x u\| \leqslant C_\varphi + TC_f,\ \|\partial_x v\| \leqslant C_\varphi + TC_f. \tag{24}$$

As in [4–8], for all $t$ and $x$ we obtain the estimates

$$|\partial^2_{x^2} u| \leq E_1 ch\left(T\sqrt{C_1C_2}\right) + \frac{E_2C_1 + C_3}{\sqrt{C_1C_2}} sh\left(T\sqrt{C_1C_2}\right) + C_1C_4T^2, \tag{25}$$

$$|\partial^2_{x^2} v| \leq E_2 ch\left(T\sqrt{C_1C_2}\right) + \frac{E_1C_2 + C_4}{\sqrt{C_1C_2}} sh\left(T\sqrt{C_1C_2}\right) + C_2C_3T^2, \tag{26}$$

where $E_1,\ E_2,\ C_1,\ C_2,\ C_3,\ C_4$ are constants.

Owing to the global estimates (22), (24)–(26), we can extend the solution to any given segment $[0,T]$. We take $u(T_0,x),\ v(T_0,x)$ for the initial values, using Theorem 1, we extend the solution to the segment $[T_0,\ T_1]$. Then for the initial values we take $u(T_1,x),\ v(T_1,x)$, using Theorem 1, we extend the solution to the segment $[T_1,\ T_2]$. As a result, we can extend the solution to any given segment $[0,T]$ in finitely many steps.

The uniqueness of a solution to the Cauchy problem (1), (2) is proved with the help of estimates similar to those used in the proof of the convergence of successive approximations.

**Example 1** *We consider the system:*

$$\begin{cases} \partial_t u\,(t,x) + ((10t+1)u(t,x) + (9t^6+2)v(t,x) + 150t)\partial_x u\,(t,x) = t + 19arctg2x, \\ \partial_t v(t,x) + ((2t^2+5)u(t,x) + (5t^3+7)v(t,x) - 71t)\partial_x v(t,x) = -\frac{t+17}{e^{5x}+11}, \end{cases} \tag{27}$$

*where $u(t,x),\ v(t,x)$ are unknown functions, subject to the initial conditions:*

$$u(0,x) = \varphi_1(x) = -\frac{1}{e^{19x}+3},\ v(0,x) = \varphi_2(x) = 12 + arctg6x. \tag{28}$$

*The problem (27), (28) is considered on $\Omega_T = \{(t,x)\,|0 \leq t \leq T, x \in (-\infty,+\infty), T > 0\}$.*

*We have*

$$a(t) = 10t+1,\ b(t) = 9t^6+2,\ c(t) = 2t^2+5,\ g(t) = 5t^3+7,$$

$$a_1(t) = 150t,\ a_2(t) = -71t,\ f_1(t,x) = t + 19arctg2x,\ f_2(t,x) = -\frac{t+17}{e^{5x}+11},$$

$$\varphi_1'(x) = \frac{19e^{19x}}{(e^{19x}+3)^2},\ \varphi_2'(x) = \frac{6}{1+36x^2},$$

$$\partial_x f_1(t,x) = \frac{38}{1+4x^2},\ \partial_x f_2(t,x) = \frac{5e^{5x}(t+17)}{(e^{5x}+11)^2}.$$

*Since*

$$\varphi_1, \varphi_2 \in \bar{C}^2(R),\ f_1, f_2 \in \bar{C}^{2,2}(\Omega_T),\ a, b, c, g, a_1, a_2 \in C([0,T]),$$

$$a(t) = 10t + 1 > 0,\ b(t) = 9t^6 + 2 > 0,$$

$$c(t) = 2t^2 + 5 > 0,\ g(t) = 5t^3 + 7 > 0 \ on\ [0,T],$$

$$\varphi_1'(x) = \frac{19e^{19x}}{(e^{19x}+3)^2} > 0,\ \varphi_2'(x) = \frac{6}{1+36x^2} > 0 \ on\ R,$$

$$\partial_x f_1(t,x) = \frac{38}{1+4x^2} > 0,\ \partial_x f_2(t,x) = \frac{5e^{5x}(t+17)}{(e^{5x}+11)^2} > 0 \ on\ \Omega_T,$$

*then by Theorem 2, the Cauchy problem (27), (28) has a unique solution*

$$u(t,x), v(t,x) \in \bar{C}^{1,2}(\Omega_T).$$

*We consider the system (27) subject to the initial conditions:*

$$u(0,x) = \varphi_1(x) = -\frac{1}{e^{23x}+13},\ v(0,x) = \varphi_2(x) = 24 + arctgx. \tag{29}$$

*The problem (27), (29) is considered on* $\Omega_T = \{(t,x)\,|0 \le t \le T, x \in (-\infty,+\infty), T > 0\}$.

*Since*

$$\varphi_1, \varphi_2 \in \bar{C}^2(R),\ f_1, f_2 \in \bar{C}^{2,2}(\Omega_T),\ a, b, c, g, a_1, a_2 \in C([0,T]),$$

$$a(t) = 10t + 1 > 0,\ b(t) = 9t^6 + 2 > 0,$$

$$c(t) = 2t^2 + 5 > 0,\ g(t) = 5t^3 + 7 > 0 \ on\ [0,T],$$

$$\varphi_1'(x) = \frac{23e^{23x}}{(e^{23x}+13)^2} > 0,\ \varphi_2'(x) = \frac{1}{1+x^2} > 0 \ on\ R,$$

$$\partial_x f_1(t,x) = \frac{38}{1+4x^2} > 0,\ \partial_x f_2(t,x) = \frac{5e^{5x}(t+17)}{(e^{5x}+11)^2} > 0 \ on\ \Omega_T,$$

*then by Theorem 2, the Cauchy problem (27), (29) has a unique solution*

$$u(t,x), v(t,x) \in \bar{C}^{1,2}(\Omega_T).$$

# 5 Conclusion

We have obtained sufficient conditions for the existence and uniqueness of a nonlocal solution of the Cauchy problem (1), (2), where $f_1$, $f_2$, $a(t)$, $b(t)$, $c(t)$, $g(t)$, $a_1(t)$, $a_2(t)$ are given functions, $a(t) > 0$, $b(t) > 0$, $c(t) > 0$, $g(t) > 0$, $t \in [0,T]$.

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