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MATHEMATICAL MODELING OF FISH RESOURCES ASSESSMENT

In the field of fisheries, one of the key tasks is to determine the total and commercial stocks of fish resources in a water body, as well as to establish the maximum allowable catch of fish to maintain the sustainability of the ecosystem. Mathematical models play an important role in this process, providing effective resource management and predicting fish population dynamics, which ensures the sustainable development of fisheries and the protection of aquatic ecosystems from depletion. The aim of this study is to develop a new mathematical model for assessing changes in fish resource stocks using the cohort method. The work proposes an alternative approach to calculating changes in fish resource stocks based on statistical theory. The model developed in this work represents a new approach to calculating fish resources for practical application, which can be useful for fisheries enterprisers and government bodies involved in fish resource management.
The advantage of this work lies in the development of a mathematical model based solely on known statistical data and data obtained from fisheries laboratories in the Republic of Kazakhstan. Modern statistical methods were applied to solve the mathematical model, which allow for the identification of unknown factors affecting the dynamics of fish populations, thereby increasing the accuracy of forecasts and the efficiency of resource management.

Key words: mathematical model, cohort, distribution function, fish stock, permitted catch, biomass.

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Балық ресурстар қорын бағалауды математикалық модельдеу

Балық шаруашылығы саласында басты міндеттердің бірі су қоймасындағы балық ресурстарының жалпы және тауарлық қорын анықтау, сондай-ақ экожүйенің тұрақтылығын сақтау үшін балық аулаудың рұқсат етілген шекті мөлшерін белгілеу болып табылады. Бұл үдерісте ресурстарды тиімді басқаруды және балық популяциясының динамикасын болжауды қамтамасыз ететін математикалық модельдер маңызды рөл атқарады, бұл балық шаруашылығын тұрақты дамытуға және су экожүйесін сарқылудан қорғауға мүмкіндік береді.
Бұл зерттеудің мақсаты когорттық әдісті қолдана отырып, балық қорларындағы өзгерістерді бағалаудың жаңа математикалық моделін жасау болып табылады. Жұмыс статистикалық теорияға негізделген балық қорларындағы өзгерістерді есептеудің баламалы тәсілін ұсынады. Бұл жұмыста құрастырылған модель балық ресурстарын басқарумен айналысатын балық шаруашылығы кәсіпорындары мен мемлекеттік органдар үшін пайдалы болуы мүмкін практикалық қолдану үшін балық ресурстарын есептеудің жаңа тәсілі ұсынылады.

Бұл жұмыстың артықшылығы - тек белгілі статистикалық мәліметтер мен Қазақстан Республикасының аумағындағы балық шаруашылығы зертханаларынан алынған мәліметтер негізінде ғана математикалық модель құру. Математикалық модельді шешу үшін балық популяциясының динамикасына әсер ететін белгісіз факторларды анықтауға мүмкіндік беретін заманауи статистикалық әдістер қолданылды, бұл болжамдардың дәлдігін және ресурстарды басқару тиімділігін арттырады.
Кілттік сөздер: математикалық модель, когорта, таралу функциясы, балық қоры, рұқсат етілген аулау, биомасса.

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Математическое моделирование оценки запаса рыбных ресурсов

В сфере рыбного хозяйства одной из ключевых задач является определение общего и промыслового запасов рыбных ресурсов в водоеме, а также установление предельно допустимого улова рыбы для поддержания устойчивости экосистемы. Математические модели играют важную роль в этом процессе, обеспечивая эффективное управление ресурсами и прогнозирование динамики популяции рыб, что позволяет обеспечить устойчивое развитие рыбного хозяйства и защиту водных экосистем от истощения.
Целью настоящего исследования является разработка новой математической модели оценки изменения запасов рыбных ресурсов с применением когортного метода. В работе предложен альтернативный подход расчета изменения запаса рыбных ресурсов на основе статистической теории. Построенная в настоящей работе модель представляет собой новый подход расчета рыбных ресурсов для практического применения, который может быть полезен для рыбохозяйственных предприятий и государственных органов, занимающихся управлением рыбными ресурсами.
Преимущество настоящей работы заключается в разработке математической модели исключительно на основе известных статистических данных и данных, полученных от лабораторий рыбных хозяйств на территории Республики Казахстан. Для решения математической модели были применены современные статистические методы, которые позволяют определить неизвестные факторы, влияющие на динамику популяций рыбных ресурсов, что повышает точность прогнозов и эффективность управления ресурсами.
Ключевые слова: математическая модель, когорта, функция распределения, рыбные запасы, разрешенный улов, биомасса.

1 Introduction

Assessment of the stock of natural resources, including fish resources, has always been and is an urgent problem due to economic and environmental aspects. There are a large number of mathematical models for assessing the stock of fish resources [7, 10, 13, 16, 18]. Moreover, each model has its own strengths and weaknesses. The main reason for the large number of models is the lack of all necessary and correct information. The works [1-3, 9, 11] provide descriptions and comparisons of mathematical models for assessing fish stocks using empirical methods. One of the main methods for mathematical modeling of the fish life cycle process is the cohort model (cohort analysis) [14]. In fish population modeling, a cohort is defined as a group of fish of the same age or year of birth. Cohort analysis involves dividing the research object into groups according to various characteristics, the so-called cohorts. Cohort analysis can be divided into two groups, the first is deterministic, the second is stochastic. The stochastic method assumes that some or all of the variables and parameters are subject to errors of one kind or another, and attempts to rationally distribute these errors. Deterministic methods assume that all parameters and variables are measured without errors [5, 8]. The work [6] provides a review of existing cohort models based on fish catch and mortality equations. Cohort models of the state of fish stocks are based on the concept of populations as a collection of specimens of individual generations, the number of each of which decreases under the influence of fishing and natural causes.

There are many modifications of cohort models [15]. For their implementation, it is necessary to have data on commercial catches of the species, their age and size (weight) composition over a long period of years, as well as abundance indices, the stock-replenishment relationship, terminal values of fishing mortality and the assumption of separability of fishing mortality. Depending on the composition of the available data, one or another cohort model is chosen.

There is also a formal theory of fish life developed by F.I. Baranov, in which a mathematical model of fish stocks and fishery optimization was built. This theory is quite widely used in ichthyology. According to the theory of F.I. Baranov, it is possible to regulate fishing by selecting a certain intensity and selectivity of fishing, i.e. it is possible to bring the stock into a state that ensures its stable existence, maximum biological production and economic efficiency of sustainable fishing [4, 17].

According to the research of the famous ichthyologist R.A. Karev. [12] traditional mathematical models have a number of disadvantages such as changes in population analysis can worsen the results, lack of possibility of high-quality results, lack of interactive interaction between the model and the person, inaccessibility of mathematical models for practitioners. And the author in his work proposes a descriptive model of the population dynamics of sturgeon, devoid of the above shortcomings, and a new method for assessing natural reproduction (natural reproduction of fish from caviar to juveniles in nature) in an unclear situation.

One of the key areas of research is the dynamic modeling of fish populations. Work [19] presents a mathematical model that describes the size of the cod population in the northern seas. In their work, the authors take into account factors such as climate change and human impacts, offering a tool for predicting changes in long-term population sustainability.

Modeling is an important issue in understanding and assessing the dynamics of fish populations and the effectiveness of fisheries management. The development of accurate and reliable models is becoming a necessity in the face of a changing environment and increasing pressure from human activities. This paper presents a mathematical model that describes the long-term dynamics of several fish populations in a fishery water body. The model is based on the generally accepted "cohort"concept and takes into account the factors of annual mortality and the impact of fishing, which allows more accurate assessment of changes in the population.

2 Mathematical model

The purpose of this study is to construct a mathematical model to determine the dynamics of changes in the total amount of fish X^k and the production quantity X^k_{Com} of fish, as well as their biomass W^k (total stock) and W^k_{Com} (commercial stock), respectively, in a certain year. Moreover, any mathematical model is constructed depending on the available initial data.

As practice shows, on the territory of the Republic of Kazakhstan the main source of input (initial) information for calculating and constructing a mathematical model of total and commercial fish stocks, as well as their biomass, are control fishing using the set net method, permitted fishing and statistical data, which can be expressed as follows:

1. Amount of i-th species of j-th age fish caught in the k-th year using the set net method denotes A^k_{ij};

2. The mass of the i-th species of fish of the j-th age caught in the k-th year using the set net method is w^k_{ij};

3. Amount of caviar from the i-th species of fish of the j-th age caught in the k-th year using the net set method;

4. Amount of females, males, juvenile species caught of the i-th species of fish of the j-th age in the k-th year using the set net method;

5. Allowed catch of the i-th fish species in the k-th year is B^k_i;

6. Adjustment coefficient between permitted and actual fishing of the i-th fish species is β_i;

7. Survival rate coefficient of caviar (appearance of fry) z^k_{ic} and survival rate coefficient up to one year is z^k_{i0} (from statistical data).

where $i = \overline{1, m}$ - type of fish, $j = \overline{0, n_i - 1}$ - age, $k = \overline{0, n-1}$ - year of observation.

Let's construct a cohort mathematical model to determine the dynamics of changes in the amount of fish in the following form:

$$x^{k+1}_{\mathrm{ij+1}} = x^k_{\mathrm{ij}} z^k_{\mathrm{ij}} - (1 + \beta_i) B^k_{\mathrm{ij}} \tag{1}$$

where, x^k_{ij} is amount of i-th species of fish of j-th age in k-th year;

z^k_{ij} is survival rate coefficient of i-th species of fish of j-th age in k-th year. It is determined by the influence of various factors: environmental conditions, natural mortality, the presence of predators and food availability.

β_i is adjustment coefficient between permitted and actual fishing of the i-th fish species. This coefficient takes into account the difference between the officially permitted fishing volume and the actual catch, which may be associated with various factors, such as recreational fishing, poaching, as well as other illegal (not accounted for) fishing methods;

B^k_{ij} is amount of permitted fishing of i-th species of fish of j-th age in k-th year, determined by the relation $B^k_{ij} = \frac{A^k_{ij}}{A^k_i} B^k_i$, $i = \overline{1, m}$, $j = \overline{0, n_i - 1}$, $k = \overline{0, n-1}$.

Since the method for obtaining input data (set net method) provides information on the amount of fish caught starting from the age of 2, we consider the method for solving equation (1) in two stages.

In the first stage, we build a model for fish age 2 and more years. If we write down any amount of the i-th species of fish of the j-th age in the k-th year $x_{i,j}^k$ to the initial element, then equation (1) will be rewritten in the following form

$$x_{ij}^k =$$

$$\begin{cases} x_{ij-k}^0 \left(\prod_{l=0}^{k-1} z_{ij-k+l}^l \right)^{H_1} - (1+\beta_i) H_1 \left(\sum_{h=1}^{k-1} \frac{A_{ij-k+h}^l}{A_i^k} B_i^l \prod_{m=h}^{k-1} z_{ij-k+m}^m + \frac{A_{ij}^k}{A_i^k} B_i^j \right), j > k \\ x_{i2}^{k-j+2} \left(\prod_{l=2}^{j-1} z_{il}^{k-j+l} \right)^{H_2} - (1+\beta_i) H_2 \left(\sum_{h=3}^{j-1} \frac{A_{ih}^{k-j+h}}{A_i^k} B_i^{k-j+h} \prod_{l=h}^{j-1} z_{il}^{k-j+l} + \frac{A_{ij}^k}{A_i^k} B_i^j \right), j \le k \end{cases} \tag{2}$$

where

$$H_1 = \begin{cases} 1, & k \ge 1 \\ 0, & k = 0 \end{cases}, \quad H_2 = \begin{cases} 1, & j \ge 3 \\ 0, & j = 2 \end{cases}$$

or

$$\begin{cases} x_{i3}^{k+1} = x_{i2}^k z_{i2}^k - (1+\beta_i) B_{i2}^k \\ x_{i4}^{k+1} = x_{i3}^k z_{i3}^k - (1+\beta_i) B_{i3}^k \\ \ldots \\ x_{ij+1}^{k+1} = x_{ij}^k z_{ij}^k - (1+\beta_i) B_{ij}^k \\ \ldots \\ 0 = x_{in_i-1}^k z_{in_i-1}^k - (1+\beta_i) B_{in_i-1}^k \end{cases} \tag{3}$$

Let us introduce the following notation

$$x_{ij}^k = A_{ij}^k \gamma_i, \tag{4}$$

where A_{ij}^k is the amount of the i-th species of fish of the j-th age caught by the set net method in the k-th year;

γ_i is the distribution coefficient of the i-th fish species for a specific fishery water body; $i = \overline{1,m}, j = \overline{2, n_i - 1}, k = \overline{0, n-1}$.

Let us rewrite equation (3) taking into account equation (4) in the following form

$$\begin{cases} A_{i3}^{k+1} \gamma_i = A_{i2}^k \gamma_i z_{i2}^k - \frac{A_{i2}^k}{A_i^k} (1+\beta_i) B_i^k \\ A_{i4}^{k+1} \gamma_i = A_{i3}^k \gamma_i z_{i3}^k - \frac{A_{i3}^k}{A_i^k} (1+\beta_i) B_i^k \\ \ldots \\ A_{ij+1}^{k+1} \gamma_i = A_{ij}^k \gamma_i z_{ij}^k - \frac{A_{ij}^k}{A_i^k} (1+\beta_i) B_i^k \\ \ldots \\ 0 = A_{in_i-1}^k \gamma_i z_{in_i-1}^k - \frac{A_{in_i-1}^k}{A_i^k} (1+\beta_i) B_i^k \end{cases} \tag{5}$$

After simple transformations from (5), we determine the survival rate coefficients as follows

$$\begin{cases} z_{i2}^k = \frac{A_{i3}^{k+1}}{A_{i2}^{k+1}} + \frac{B_i^k}{A_i^k \frac{\gamma_i}{1+\beta_i}} = \frac{B_i^k}{A_i^k} y_i + \frac{A_{i3}^{k+1}}{A_{i2}^k} \\ z_{i3}^k = \frac{A_{i4}^{k+1}}{A_3^{k+1}} + \frac{B_i^k}{A_i^k \frac{\gamma_i}{1+\beta_i}} = \frac{B_i^k}{A_i^k} y_i + \frac{A_{i4}^{k+1}}{A_{i3}^k} \\ \dots \\ z_{ij}^k = \frac{A_{ij+1}^{k+1}}{A_{ij}^{k+1}} + \frac{B_i^k}{A_{ij}^k \frac{\gamma_i}{1+\beta_i}} = \frac{B_i^k}{A_{ij}^k} y_i + \frac{A_{ij+1}^{k+1}}{A_{ij}^k} \\ \dots \\ z_{i,n_i-1}^k = \frac{B_i^k}{A_{i,n_i-1}^k \frac{\gamma_i}{1+\beta_i}} = \frac{B_i^k}{A_{i,n_i-1}^k} y_i \end{cases} \tag{6}$$

where $y_i = \frac{1+\beta_i}{\gamma_i}$

Since our main equation has three degrees of freedom (i, j, k), then in the system of equations (6) we can fix i, j and represent it as follows

$$\begin{cases} z_{ij}^0 = \frac{B_i^0}{A_i^0} y_i + \frac{A_{ij+1}^1}{A_{ij}^0} \\ z_{ij}^1 = \frac{B_i^1}{A_i^1} y_i + \frac{A_{ij+1}^2}{A_{ij}^1} \\ \dots \\ z_{ij}^k = \frac{B_i^k}{A_{ij}^k} y_i + \frac{A_{ij+1}^{k+1}}{A_{ij}^k} \\ \dots \\ z_{ij}^{n-1} = \frac{B_i^{n-1}}{A_{ij}^{n-1}} y_i + \frac{A_{ij+1}^n}{A_{ij}^{n-1}} \end{cases} \tag{7}$$

where $i = \overline{1, m}, j = \overline{2, n_i - 1}$.

The transition from (6) to (7) was made in order to find an adequate estimate of the values of y_i, since y_i can take any value, but should not change sharply from year to year.

Solve the system of equations (7) using the following algorithm:

1. Find the intersection points of all equations of system (7) for fixed i, j;
2. Collect all points of all ages (at $j = \overline{2, n_i - 1}$) for the i-th fish species;
3. Find the mathematical expectation, the dispersion of points from section above 2, and build a normal distribution;
4. The value of the mathematical expectation is taken as the total value of y_i, after which all z_{ij}^k are determined, and therefore are determined, values x_{ij}^k.

Next year and from year to year, additional points are added, therefore, values of distribution and mathematical expectation change, and the value of y_i will be further refined. After specifying all the coefficients, we find the survival rate coefficient z_{ij}^k, and therefore obtain the fish quantity x_{ij+1}^{k+1}.

Based on the found survival rate coefficients z_{ij}^k, additional causal chain analysis can be carried out, both horizontally and vertically. A horizontal change implies a change by year (ecology, droughts, lack of food supply, etc.). Analysis of horizontal changes allows to evaluate the effectiveness of the remote control system and other related aspects.

Now let's move on to the second stage. Let's write an equation to determine the amount of fish up to the 1st year inclusive.

$$x_{i0}^{k+1} = \sum_{j=m_i}^{n_i-1} \mu_{ij}^k x_{ij}^k z_{ic}^k \tag{8}$$

where, $\mu_{i,j}^k$ is coefficient determining by the product of the amount of caviar of one female and the proportion of all females of the i-th fish species of the j-th age in the k-th year, which are obtained from set net methods;

m_i is age of first spawning of the i-th fish species.

Then, from equation (8), knowing the value $x_{i,0}^k$ and found values x_{i2}^k, z_{i2}^k from the first stage, we will write the following equation to find the amount of fish up to 2 years old

$$x_{i2}^{k+3} = \sum_{j=m_i}^{n_i-1} \mu_{i,j}^k x_{ij}^k z_{ic}^k z_{i0}^k z_{i1}^{k+1} z_{i2}^{k+2} \tag{9}$$

From equation (9) we determine z_{i1}^{k+1}:

$$z_{i1}^{k+1} = \frac{x_{i2}^{k+3}}{\sum_{j=m_i}^{n_i-1} \mu_{i,j}^k x_{ij}^k z_{ic}^k z_{i0}^k z_{i2}^{k+2}}$$

The total amount of fish in a fishery water body can be determined as

$$X^k = \frac{1}{m} \sum_{i=1}^{m} \frac{X_i^n}{r_i^k}$$

where, X^k is an amount of total fish stock in the k-th year of observation; m is an amount of fish species in the fishery water body; r_i^k is a distribution of all fish species by amount in the fishery water body in the k-th year $\sum_{j=1}^{m} r_i^k = 1$, information obtained from net set methods.

And the total biomass is determined by the following equation

$$W^k = \sum_{i=1}^{m} W_i^k = \frac{1}{m} \sum_{i=1}^{m} \sum_{j=0}^{n_i-1} \frac{w_{i,j}^k x_{i,j}^k}{g_i^k}$$

where, W^k is a biomass of the total fish stock in the k-th year of observation;

W_i^k is a biomass of the total stock of the i-th fish species in the k-th year of observation;

$w_{i,j}^k$ is a mass of caught fish of the i-th species of j-th age in the k-th year;

g_i^k is a distribution of all fish species by weight in the fishery water body in the k-th year $\sum_{j=1}^{m} g_i^k = 1$, information obtained from net set methods.

Taking into account that the commercial fish stock is determined from the first spawning run, this indicator is individual for each species of fish, knowing the amount of fish by the age, we can determine the commercial fish stock in the fishery water body using the following formula

$$X^k_{\text{Com}} = \frac{1}{m} \sum_{i=1}^{m} \sum_{j=m_i}^{n_i-1} \frac{x^k_{i,j}}{r^k_i},$$

$$W^k_{\text{Com}} = \frac{1}{m} \sum_{i=1}^{m} \sum_{j=m_i}^{n_i-1} \frac{w^k_{i,j} x^k_{i,j}}{g^k_i}$$

where, X^k_{Com} is an amount of commercial fish stock in the k-th year of observation; W^k_{Com} is a biomass of commercial fish stock in the k-th year of observation.

3 Conclusion

In this work, based on the cohort analysis, a new mathematical model has been contructed to determine changes in the amount of total and commercial fish stocks and their biomass in a particular reservoir. To solve the model, systems modeling methods and data analysis were used, which made it possible to take into account complex factors influencing the dynamics of fish populations, such as survival rates and the level of fishing in the fishery water body. Modern methods of statistical analysis were used as a research method. A unique method for finding uncertainty coefficients is proposed. This work is important in the development of tools for the sustainable management of fisheries resources in fishery water body and contributes to the effective use of fisheries in the Republic of Kazakhstan.

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