IRSTI 30.19.21 DOI: https://doi.org/10.26577/JMMCS2024-v123-i3-11

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MODELLING OF A DRILL STRING MOTION UNDER SEISMIC IMPACT

The paper studies the nonlinear dynamics of boreal waves in a seismically active medium. The effect of seismic waves on the vibrations of borehole rings for shallow drilling is examined. It is assumed that the Deformity of borehole beds is finite. A nonlinear linear mathematical model of borehole bed morphology was constructed on the basis of the Ostrovrode-Georgsky-Hamilton principle and the laws of ground motion. Borehole deflection is determined by cross-cutting displacements, longitudinal displacements and the angle of closure of their cross-cutting section. The numerical analysis of the model was carried out using the simulative calculations package Wolfram Mathematica. As a result of the research, the authors found both quantitative and quantitative changes in the boreal column dynamics. A comparative analysis with linear models was carried out. There are quantitative and quantitative changes in the dynamics of borehole columns, manifested in the form of beats caused by seismic waves. They can be caused by the seismic activity of the medium and by the nonlinearity of the borehole curve deformity.
Key words: drill string, dynamics, nonlinearity, vibrations, seismic load.

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Сейсмикалық әсер кезіндегі бұрғылау бағанының қозғалысын модельдеу

Бұл жұмыста сейсмоактивті ортадағы бұрғылау бағандарының сызықтық емес динамикасы зерттеледі. Сейсмикалық толқындардың терең емес бұрғылау бағандарының тербелістеріне әсері қарастырылады. Бұрғылау бағандарының деформациясы ақырғы болып саналады. Остроградский-Гамильтон вариациялық принцип пен жердің қозғалыс заңдылығы негізінде, бұрғылау бағанының қозғалысын баяндайтын сызықсыз теңдеулер жүйесі құрылды. Бағандардың деформациясы көлденең және бойлық орын ауыстыруымен, сондай-ақ олардың қималарының бұралу бұрышымен анықталады. Wolfram Mathematica программалық пакетінің қолданылуымен модельдің сандық талдауы жүзеге асырылды. Зерттеу нәтижесінде авторлар бұрғылау бағандарының динамикасындағы сапалық және сандық өзгерістерді анықтады. Сызықтық модельдермен салыстырмалы талдау жүргізілді.
Бұрғылау бағандарының динамикасында сапалық және сандық өзгерістер сейсмикалық толқындардың әсерінен болатын соққылар түрінде көрінеді. Олар ортаның сейсмикалық белсенділігіне және бұрғылау бағандарының серпімді деформациясының сызықты еместігіне байланысты болуы мүмкін.
Түйін сөздер: бұрғылау бағанасы, динамика, сызықтық емес, тербеліс, сейсмикалық жүктеме.

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Моделирование движения буровой колонны при сейсмическом воздействии

В статье исследуется нелинейная динамика бурильных колонн в сейсмически активной среде. Рассматривается влияние сейсмических волн на колебания бурильных колонн для неглубокого бурения. Предполагается, что деформации бурильных колонн конечны. На основе вариационного принципа Остроградского-Гамильтона и законов движения грунта построена нелинейная математическая модель пространственной деформации бурильных колонн. Деформация колонн определяется поперечными перемещениями, продольными смещениями и углом закручивания их поперечного сечения. Численный анализ модели выполнен с использованием пакета символьных вычислений Wolfram Mathematica. В результате исследования авторы выявили качественные и количественные изменения в динамике бурильной колонны. Проведен сравнительный анализ с линейными моделями.
Наблюдаются качественные и количественные изменения в динамике бурильных колонн, проявляющиеся в виде биений, обусловленных воздействием сейсмических волн. Они могут быть вызваны сейсмической активностью среды и нелинейностью упругой деформации бурильных колонн.
Ключевые слова: буровая колонна, динамика, нелинейность, колебания, сейсмическая нагрузка.

1 Introduction

The drilling of oil and gas production wells is a complex and capital-intensive technological undertaking influenced by various complicated factors. These factors encompass the contact interaction with the walls of the well and bottomhole, influence of own weight, stick-slip phenomena, structure heterogeneity of the drill string, solid and fluid structure interactions, stochastic processes, etc. The drill string vibrations significantly increase the risk of breakdowns, rapid wear of equipment and increased downtime; the combination of these factors accounts for 2–10% of the cost of wells [1].

Currently, the study of the drill string dynamics is of great interest to many authors. Among them, the following areas and works can be distinguished. These are studies of laboratory scaled experimental setups. In [2] provides a detailed review and comparison of existing downscaled laboratory setups for studying drill string dynamics, describes a methodology that goes beyond simple geometric downscaling to a more comprehensive approach that takes into account critical parameters such as stiffness, elasticity, torque and etc., and an improved experimental setup is proposed. There could be noticed the studies of stick slip phenomenon occuring in drilling. Thus, in [3] the model of vertical drill string stick slip vibrations is presented, the model is evaluated through switching the model between the stick and slip phases, three mitigation and active controlling of stick slip vibration strategies are developed and proposed. These are problems of multiparameter analysis of drill strings and sampling of operating modes. Kaplunov, J. et al. [4] developed a multiparameter approach to analyze drilling dynamics for bending vibrations of a rotating elastic beam prestressed simultaneously by an axial compressive force and torque.

An important problem is the consideration of stochastic phenomena during the drilling process. The authors of papers [5–7] established the impact of stochasticity on the drill strings dynamics: in paper [5] for torsional vibration, in paper [6] for the coupled lateral–torsional vibrations and in [7] for the axial-torsional coupled vibration. Proposed stochastic models focus on uncertainties in rock strength during drilling, bit-rock interaction, borehole wall diameter, and BHA unbalance.

The problems of environmental influence on BHA dynamics and its control are being actively studied. In [8], the nonlinear dynamics of a vertical rotating drill string in a supersonic gas flow using the lumped-parameter method was studied, where geometric nonlinearity, axial load and gas flow were considered as complicating factors. The model was expanded taking into account the heterogeneous structure of the drill string, namely the inclusion of stabilizers in [9]. The authors of papers [10–12] modeled the phenomenon of self-oscillations of the drill string as a model with distributed parameters for the cases of longitudinal vibrations and torsional vibrations. They are presented in linear [10, 11] and nonlinear [12] formulations. In [13], the results of studies of longitudinal oscillations of a horizontal drill string are presented. The authors of the work considered the contact interaction of the drill string with the walls of the well. The influence of the inertial force of the bit on the rock is taken into account. The numerical solution is obtained by the lumped-parameter method.The authors of the paper [14] based on the method of summation of modes carried out a simulation of the horizontal movement of the drill string, followed by analysis of mode convergence, evaluation of system stability under parameter changes, and identification of instability. These results obtained were used for developing an effective multivariable control in [15].

In addition to all the above-mentioned and widely discussed complex factors in the literature should be noted the importance of considering the seismicity of the region.

Approximately half of all densely populated regions of the Earth are prone to seismic activity; 10-15% of the entire surface can be classified as highly active seismic zones [16]. According to [17], 19% of the total territory of Kazakhstan is considered seismically active. These regions are home to 40% of the country's population and host over 30% of its industrial and manufacturing assets. Among the industrial facilities located in seismic active zones are several oil and gas extraction sites [17]. In addition, earthquakes can not only be of natural origin, but can also be caused by human economic activity, called induced seismicity [18, 19]. The analysis of the work showed that the dynamics of drill strings in seismic conditions has been poorly studied. Most of the papers consider static underground structures and communications under the influence of seismic waves, such as the works of the authors [20, 21] and others.

Thus, modeling the dynamics of a drilling equipment under seismic conditions is relevant and has theoretical and practical interest.

This article aims to create a mathematical model of this process, find a numerical solution and analyze the influence of seismicity on the motion of the drill string.

2 Mathematical model

Modelling of nonlinear dynamics of drilling columns under seismic conditions is carried out using fundamental laws and methods of nonlinear mechanics of deformable media, seismic resistance theory and laws of soil motion.

Here, a nonlinear model of spatial vibrations of a rotating drill string under the action of seismic waves is developed. The drill string is modeled as a rod element, hinged at the ends and rotating around the oz-axis with an angular velocity ω. The equations of motion of the string are derived based on the Ostrogradsky-Hamilton variational principle:

$$\int_{t_1}^{t_2} \delta \left(T_{kin} - U_p + \Pi\right) dt = 0, \tag{1}$$

where T_{kin} - the kinetic energy, U_p - the potential energy of elastic deformation, and Π - the potential energy of external forces acting on the string. They are defined as:

$$T_{kin} = \frac{1}{2}\rho \int_0^l \int_{A(z)} \left[\left(\frac{\partial U}{\partial t} \right)^2 + \left(\frac{\partial V}{\partial t} \right)^2 + \left(\frac{\partial W}{\partial t} \right)^2 \right] dA(z)dz + \frac{1}{2}\rho \int_0^l \int_{A(z)} \left(\vec{R} \cdot \vec{R} \right) dA(z)dz, \quad (2)$$

$$U_p = \frac{1}{2}\rho \int_0^l \int_{A(z)} \sigma_{ij}\varepsilon_{ij} dA(z)dz, \quad (3)$$

$$\Pi = \int_V \left[P_1 U + P_2 V + P_3 W \right] dV + \int_S \left[q_1 U + q_2 V + q_3 W \right] dS + \int_{S_1} \left[\varphi_1 U + \varphi_2 V + \varphi_3 W \right] dS_1, \quad (4)$$

here U, V, W are the components of the column displacement; $\vec{R}$ is the radius vector of the rotating column; P_i are the volume forces; q_i are the surface forces; φ_i are the end forces; σ_{ij} are the components of the stress tensor; ε_{ij} are the components of the strain tensor. Here, as in the monograph by A.P. Filippov [22], elastic displacements in the case of rod deformation are specified as follows:

$$\begin{aligned} U(x, y, z, t) &= u_1(z, t) + u_2(z, t) + \theta(z, t)y, \\ V(x, y, z, t) &= v_1(z, t) + v_2(z, t) - \theta(z, t)x, \\ W(x, y, z, t) &= w(z, t) - \alpha_y x - \alpha_x y + \frac{\partial \theta(z, t)}{\partial z}\phi(x', y'), \end{aligned} \quad (5)$$

where $u_1(z, t)$, $v_1(z, t)$ are the displacements of the cross-section bending center along the x, y axes due to bending; $u_2(z, t)$, $v_2(z, t)$ are the result of shear; $\theta(z, t)$ are the result of rotation of the cross-section relative to the bending center; $w(z, t)$ are the longitudinal displacements along the oz-axis, which coincides with the drill string axis; ϕ is the Saint-Venant function; $\alpha_x(z, t)$, $\alpha_y(z, t)$ are the rotation angles of the sections around the $ox-$ and $oy-$ axes due to bending.

In constructing the mathematical model of the drill string, the components of displacement due to shear and section deplaning are neglected in formula (5). As a result, we obtain:

$$\begin{aligned} U(x, y, z, t) &= u_1(z, t) + \theta(z, t)y, \\ V(x, y, z, t) &= v_1(z, t) - \theta(z, t)x, \\ W(x, y, z, t) &= w(z, t) - \left(\frac{\partial u_1(z, t)}{\partial z}x + \frac{\partial v_1(z, t)}{\partial z}y \right). \end{aligned} \quad (6)$$

The potential of 3D-elastic deformation (3) is constructed within the framework on the fundamental theory of nonlinear elasticity by V.V. Novozhilov. Here, a second system of simplifications according to Novozhilov V.V. is allowed, where the relative elongations, shifts, and rotation angles are relatively small compared to unity. The components of deformation of the second and higher orders of smallness are neglected, except for the second-order smallness rotation angles. In this case, the components of the deformation tensor are nonlinear and are defined as follows:

$$\begin{aligned} &\varepsilon_{xx} \approx e_{xx} + \frac{1}{2}\left(\omega_y^2 + \omega_z^2\right), && \varepsilon_{xy} \approx e_{xy} - \omega_x\omega_y, \\ &\varepsilon_{yy} \approx e_{yy} + \frac{1}{2}\left(\omega_x^2 + \omega_z^2\right), && \varepsilon_{yz} \approx e_{yz} - \omega_y\omega_z, \\ &\varepsilon_{zz} \approx e_{zz} + \frac{1}{2}\left(\omega_x^2 + \omega_y^2\right), && \varepsilon_{zx} \approx e_{zx} - \omega_z\omega_x. \end{aligned} \tag{7}$$

where e_{ii} are relative elongations along the i-th coordinate axes, ω_i are rotation angles around the i-th axes.

While drilling, the drill string rotates around its axis. Therefore, when modelling the string dynamics, in addition to deformation, it is also necessary to take into account its rotational motions.

To determine the kinetic energy of a rotating elastic drill string, global (fixed $OXYZ$) and local (rotating with the string $Oxyz$) coordinate systems are introduced. The axes Oz, OZ coincide with each other and are directed along the string. Thus, the position of points on the string in the global coordinate system relative to the local one is specified as follows:

$$\begin{aligned} &X = (x+U)\cos\omega t + (y+V)\sin\omega t, \\ &Y = -(x+U)\sin\omega t + (y+V)\cos\omega t, \\ &Z = z + W, \end{aligned} \tag{8}$$

where ω is the angular velocity of the drill string rotation.

The work of external volume forces is widely covered in the works of such famous scientists as Timoshenko S.P., Dzhanalidze G.Yu., Kabulov V.K. This issue was investigated by Rashidov T.R. in relation to the case of seismic impacts and a modification of seismic waves as external forces was carried out in [20, 21].

Taking into account relations (2)-(8) with regard to the soil motion law under seismic impact [20, 21], variation (1) was performed using the Ostrogradsky-Hamilton method. The variation was carried out according to the following variables: lateral displacements u_1, v_1; longitudinal displacement w and twist angle θ.

The system of nonlinear differential equations describing the motion of a drill string under seismic loads is obtained as

$$
\begin{gathered}
\rho A\frac{\partial^2 u_1}{\partial t^2}-\rho I_y\frac{\partial^4 u_1}{\partial z^2\partial t^2}-\rho I_{xy}\frac{\partial^4 v_1}{\partial z^2\partial t^2}+EI_y\frac{\partial^4 u_1}{\partial z^4}-EI_{xy}\frac{\partial^4 v_1}{\partial z^4}-\\
-\frac{EA}{(1-\nu)}\frac{\partial}{\partial z}\left[\left(\frac{\partial u_1}{\partial z}\right)^3+\frac{\partial w}{\partial z}\frac{\partial u_1}{\partial z}\right]+\frac{EI_x(8-6\nu)}{8(1-\nu)}\frac{\partial}{\partial z}\left[\frac{\partial u_1}{\partial z}\left(\frac{\partial \theta}{\partial z}\right)^2\right]-\\
-\frac{EI_y}{2(1-\nu)}\frac{\partial}{\partial z}\left[\frac{\partial^2\theta}{\partial z^2}\left(\frac{\partial u_1}{\partial z}+\frac{\partial v_1}{\partial z}\right)\right]-\frac{E(5-6\nu)}{2(1-\nu)}\left(A\frac{\partial}{\partial z}\left[\frac{\partial u_1}{\partial z}\left(\left(\frac{\partial v_1}{\partial z}\right)^2+\theta^2\right)\right]+\right.\\
\left.+\frac{1}{2}I_{xy}\frac{\partial}{\partial z}\left[\frac{\partial v_1}{\partial z}\left(\frac{\partial \theta}{\partial z}\right)^2\right]\right)-\rho A\omega^2 u_1+2\rho A\omega\frac{\partial v_1}{\partial t}+N(P_1)+N(q_1)+\frac{\partial M_y(P_3)}{\partial z}+\frac{\partial M_y(q_3)}{\partial z}=0,
\end{gathered}
$$

$$
\begin{gathered}
\rho A\frac{\partial^2 v_1}{\partial t^2}-\rho I_x\frac{\partial^4 v_1}{\partial z^2\partial t^2}-\rho I_{xy}\frac{\partial^4 u_1}{\partial z^2\partial t^2}+EI_x\frac{\partial^4 v_1}{\partial z^4}-EI_{xy}\frac{\partial^4 u_1}{\partial z^4}-\\
-\frac{EA}{(1-\nu)}\frac{\partial}{\partial z}\left[\left(\frac{\partial v_1}{\partial z}\right)^3+\frac{\partial w}{\partial z}\frac{\partial v_1}{\partial z}\right]+\frac{EI_y(8-6\nu)}{8(1-\nu)}\frac{\partial}{\partial z}\left(\frac{\partial v_1}{\partial z}\left(\frac{\partial \theta}{\partial z}\right)^2\right)-\\
-\frac{EI_x}{2(1-\nu)}\frac{\partial}{\partial z}\left[\frac{\partial^2\theta}{\partial z^2}\left(\frac{\partial u_1}{\partial z}+\frac{\partial v_1}{\partial z}\right)\right]-\frac{E(5-6\nu)}{2(1-\nu)}\left(A\frac{\partial}{\partial z}\left[\frac{\partial v_1}{\partial z}\left(\left(\frac{\partial u_1}{\partial z}\right)^2+\theta^2\right)\right]+\right.\\
\left.+\frac{1}{2}I_{xy}\frac{\partial}{\partial z}\left(\frac{\partial u_1}{\partial z}\left(\frac{\partial \theta}{\partial z}\right)^2\right)\right)-\rho A\omega^2 v_1-2\rho A\omega\frac{\partial u_1}{\partial t}+N(P_2)+N(q_2)+\frac{\partial M_x(P_3)}{\partial z}+\frac{\partial M_x(q_3)}{\partial z}=0,
\end{gathered}
$$

$$
\begin{gathered}
\rho I_p\frac{\partial^2\theta}{\partial t^2}-\frac{2EI_p}{(1+\nu)}\frac{\partial^2\theta}{\partial z^2}-\frac{3EI_r}{8}\frac{\partial}{\partial z}\left[\left(\frac{\partial\theta}{\partial z}\right)^3\right]+\frac{EA}{(1-\nu)}\theta^3+\frac{2EA\nu}{(1-\nu)}\theta\frac{\partial w}{\partial z}-\\
-\frac{EI_p}{4(1-\nu)}\frac{\partial}{\partial z}\left[\frac{\partial w}{\partial z}\frac{\partial\theta}{\partial z}\right]-\frac{EI_y(11-6\nu)}{8(1-\nu)}\frac{\partial}{\partial z}\left[\frac{\partial\theta}{\partial z}\left(\left(\frac{\partial u_1}{\partial z}\right)^2+\left(\frac{\partial v_1}{\partial z}\right)^2\right)\right]-\\
-\frac{EI_y}{4(1-\nu)}\frac{\partial^2}{\partial z^2}\left[\left(\frac{\partial u_1}{\partial z}+\frac{\partial v_1}{\partial z}\right)^2\right]-\frac{E(5-6\nu)}{4(1-\nu)}\left(\frac{1}{2}I_p\theta\left(\frac{\partial\theta}{\partial z}\right)^2+\frac{1}{2}I_p\theta^2\frac{\partial^2\theta}{\partial z^2}-\right.\\
\left.-2A\theta\left[\left(\frac{\partial u_1}{\partial z}\right)^2+\left(\frac{\partial v_1}{\partial z}\right)^2\right]+2I_{xy}\frac{\partial}{\partial z}\left(\frac{\partial u_1}{\partial z}\frac{\partial v_1}{\partial z}\frac{\partial\theta}{\partial z}\right)\right)-\rho I_p\omega^2\theta+M_z(P_1,P_2)+M_z(q_1,q_2)=0,
\end{gathered}
$$

$$
\begin{gathered}
\rho A\frac{\partial^2 w}{\partial t^2}-EA\frac{\partial^2 w}{\partial z^2}-\frac{EA}{2(1-\nu)}\frac{\partial}{\partial z}\left[\left(\frac{\partial u_1}{\partial z}\right)^2+\left(\frac{\partial v_1}{\partial z}\right)^2\right]-\\
-\frac{2EA\nu}{(1-\nu)}\theta\frac{\partial\theta}{\partial z}-\frac{EI_p}{8(1-\nu)}\frac{\partial}{\partial z}\left[\left(\frac{\partial\theta}{\partial z}\right)^2\right]+N(P_3)+N(q_3)=0,
\end{gathered}
\tag{9}
$$

where E - the elastic modulus; ν - Poisson's ratio; ρ - the density of the material; I - the axial moment of inertia; A - the cross-sectional area of the drill string; $N(P_i)$, $N(q_i)$, $M(q_1,q_2)$, $M(P_1,P_2)$ - the interaction forces between the drill string and the soil, obtained experimentally [20].

Due to the symmetry of the column cross-section along the $x-$ and $y-$ axes, the axial moments of inertia will be equal to each other $I_x = I_y = I$, and the centrifugal moment of inertia will be equal to zero $I_{xy} = 0$. In this model, the seismic load is determined by the law of interaction of the soil with underground objects. It is assumed that the vibrations of the medium determine the motion of underground structures. According to the assumption proposed by G.Yu. Dzhanelidze [23], the forces applied to the surface of the rod and the volume forces can be replaced by distributed forces and moments applied along the axis of the rod, as was done by the authors in [20]:

$$\begin{aligned}
&N(q_1) = -\pi D_h k_x (u_1 - u_0), && N(q_2) = -\pi D_h k_y (v_1 - v_0),\\
&N(q_3) = -\pi D_h k_z (w - w_0), && M_x(q_3) = q\left(\frac{\partial v_1}{\partial z} - \frac{\partial v_0}{\partial z}\right),\\
&M_y(q_3) = q\left(\frac{\partial u_1}{\partial z} - \frac{\partial u_0}{\partial z}\right), && M_z(q_1, q_2) = -\frac{8l}{\beta D_h} R_h^2 k_z (R_h\theta - u_0 + v_0)
\end{aligned} \tag{10}$$

where k_i are the coefficients of elastic interaction with the soil, u_0, v_0, w_0 are the laws of soil motion.

For the case of hinged support of the ends of the drill string, the initial and boundary conditions are specified, respectively:

$$\begin{aligned}
&u_1(z,t)\Big|_{t=0} = v_1(z,t)\Big|_{t=0} = w(z,t)\Big|_{t=0} = \theta(z,t)\Big|_{t=0} = \frac{\partial w(z,t)}{\partial t}\Big|_{t=0} = \frac{\partial \theta(z,t)}{\partial t}\Big|_{t=0} = 0,\\
&\frac{\partial u_1(z,t)}{\partial t}\Big|_{t=0} = C_1, \qquad \frac{\partial v_1(z,t)}{\partial t}\Big|_{t=0} = C_2,
\end{aligned} \tag{11}$$

$$\begin{aligned}
&u_1(z,t)\Big|_{\substack{z=0\\ z=l}} = v_1(z,t)\Big|_{\substack{z=0\\ z=l}} = EI\frac{\partial^2 u_1(z,t)}{\partial z^2}\Big|_{\substack{z=0\\ z=l}} =\\
&EI\frac{\partial^2 v_1(z,t)}{\partial z^2}\Big|_{\substack{z=0\\ z=l}} = EA\frac{w(z,t)}{\partial z}\Big|_{\substack{z=0\\ z=l}} = \frac{2EI_p}{(1+\nu)}\frac{\theta(z,t)}{\partial z}\Big|_{\substack{z=0\\ z=l}} = 0,
\end{aligned} \tag{12}$$

where C_1, C_2 are constants.

3 Numerical results. Analysis

Direct integration of the nonlinear model (9)-(12) is a complex process. Here the authors of the work apply the well-known Bubnov-Galerkin method. It is known as the method of separation of variables and gives good approximations in solving distributed dynamic systems. The authors model the bending shape of the drill string by a spectrum of harmonic forms. The effectiveness of the Bubnov-Galerkin method allows successfully investigating the considered models of drill strings and analyzing their operating modes. According to the method, the displacement components are represented as the sum of the product series of the sought function over time and the basis functions. The latter are selected according to the boundary conditions:

$$\begin{aligned}
&u(z,t) = \sum_{i=1}^{n} \overline{u_i}(t)\sin\left(\frac{i\pi z}{l}\right), \qquad v(z,t) = \sum_{i=1}^{n} \overline{v_i}(t)\sin\left(\frac{i\pi z}{l}\right),\\
&w(z,t) = \sum_{i=1}^{n} \overline{w_i}(t)\cos\left(\frac{i\pi z}{l}\right), \qquad \theta(z,t) = \sum_{i=1}^{n} \overline{\theta_i}(t)\cos\left(\frac{i\pi z}{l}\right).
\end{aligned} \tag{13}$$

As a first approximation, the first basis function of the series (13) is considered. The system of second order nonlinear ordinary differential equations with respect to the new functions $\overline{u_1}(t)$, $\overline{v_1}(t)$, $\overline{w_1}(t)$, $\overline{\theta_1}(t)$ is obtained:

$$\begin{gathered}
a_1u_1''(t)+a_2v_1'(t)+a_3u_1'(t)+a_4u_1(t)\theta_1^2(t)+a_5v_1(t)\theta_1(t)+a_6u_1(t)\theta_1(t)+\\
+a_7u_1(t)w_1(t)+a_8u_1(t)v_1^2(t)+a_9(t)v_1(t)+a_{10}(t)u_1(t)+a_{11}u_1^3(t)=F_a,\\
\\
b_1v_1''(t)+b_2u_1'(t)+b_3v_1'(t)+b_4v_1(t)\theta_1^2(t)+b_5u_1(t)\theta_1(t)+b_6v_1(t)\theta_1(t)+\\
+b_7v_1(t)w_1(t)+b_8v_1(t)u_1^2(t)+b_9(t)u_1(t)+b_{10}(t)v_1(t)+b_{11}v_1^3(t)=F_b,\\
\\
c_1\theta_1''(t)+c_2\theta_1'(t)+c_3\theta_1^3(t)+c_4w_1(t)\theta_1(t)+c_5v_1^2(t)\theta_1(t)+\\
+c_6u_1^2(t)\theta_1(t)+c_7\theta_1(t)+c_8v_1^2(t)+c_9u_1^2(t)+c_{10}u_1(t)v_1(t)=F_c,\\
\\
d_1w_1''(t)+d_2w_1'(t)+d_3(t)w_1(t)+d_4\theta_1^2(t)+d_5u_1^2(t)+d_6v_1^2(t)=F_d,
\end{gathered}\tag{14}$$

a_i, b_i c_i d_i are the coefficients, constants.

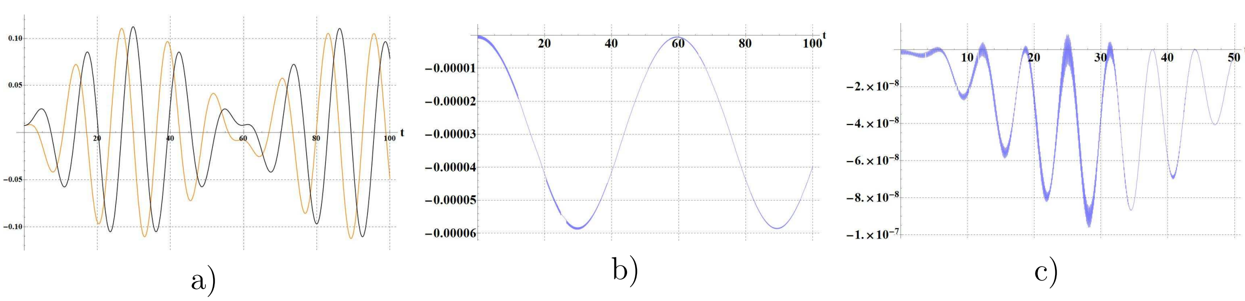
Here and below, for simplicity, we will omit the underline in the sought functions.

Using the Wolfram Mathematica symbolic computing package, the numerical simulation of drill string vibrations under seismic impact was performed.

Calculations were performed for the following parameters of steel drill columns, soil characteristics and seismic waves: Young's modulus of the drill string material (steel) $E = 2.2\times10^5 MPa$; its density $\rho = 7800\ kg/m^3$; $\nu = 0.28$ its Poisson's ratio; $\nu_s = 0.2$ Poisson's ratio of the soil; the outer diameter of the string $D_h = 0.35m$; its inner diameter $d = 0.28m$; the length of the drill string $l = 200m$; the angular velocity of the string rotation $\omega = 30\ rpm$; the string polar moment of inertia $I_p = \pi\left(Dh^4 - d^4\right)/32$; the cross-sectional area $A = \pi\left(Dh^2 - d^2\right)/4$; $\beta_1 = 1 - \dfrac{d^4}{Dh^4}$, $q = \dfrac{5l^3k_3}{8\beta_1 p}$, the coefficients of longitudinal, lateral and vertical interactions of the pipe with the soil: $k_1 = k_2 = 3*10^4 kN/m^3$, $k_3 = 1.5*10^4 kN/m^3$; $u_0 = a_0 e^{-\varepsilon\left(t-\dfrac{x}{C_p}\right)}\sin\omega_0\left(t-\dfrac{x}{C_p}\right)$, where $\varepsilon = 0.3$ is the damping coefficient; the amplitude of the wave $a_0 = 0.08$; the velocity of seismic wave propagation $C_p = 2000m/s$; $\omega_0 = 2\pi/T\ rpm$.

The wave amplitude corresponds to the Chilean earthquake of 1916.

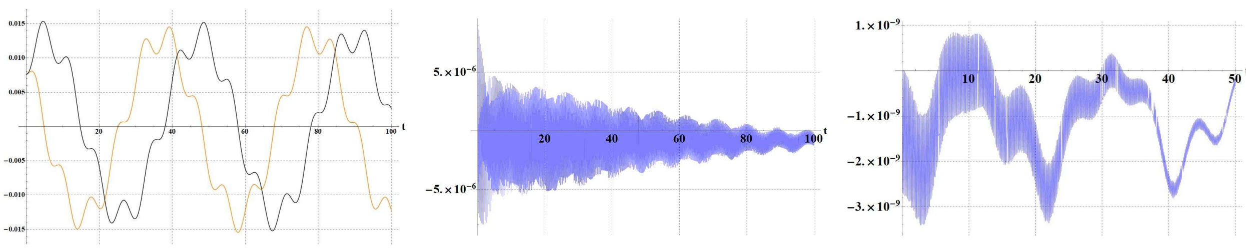
The study results are illustrated in in Figures 1-4. The dynamics of drill strings without seismic impact was studied (Fig. 1).



a) b) c)

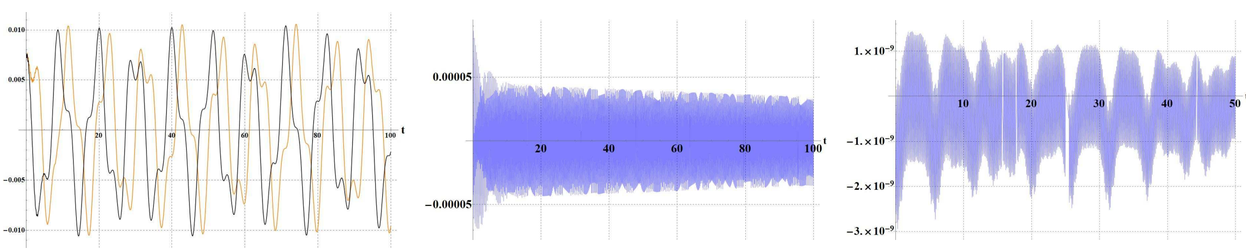
Figure 1: Drill string vibration amplitudes without seismic impact: a) – lateral, b) – longitudinal, c) – torsional

Figure 1 shows that the vibrations are nonlinear. The dominant component in the vibratory process of the column is lateral vibrations. Below, in Figures 2-4, the results of the numerical analysis of drilling column nonlinear vibrations under seismic ground activity are presented for various values of the longitudinal, lateral, and vertical interaction coefficients between the drilling column and the soil k (кН/м^3).



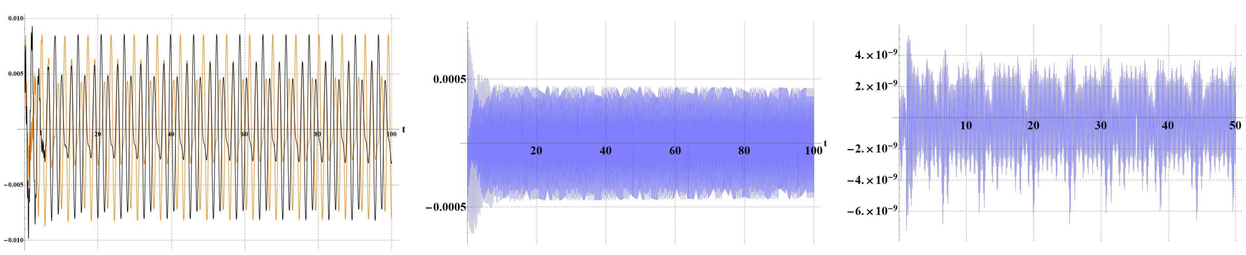
a) b) c)

Figure 2: Amplitudes of drill string vibrations under seismic impact: a) – lateral, b) – longitudinal, c) – torsional vibrations at $k = 30N/m^3$



a) b) c)

Figure 3: Amplitudes of drill string vibrations under seismic impact: a) – lateral, b) – longitudinal, c) – torsional vibrations at $k = 300N/m^3$



a) b) c)

Figure 4: Amplitudes of drill string vibrations under seismic impact: a) – lateral, b) – longitudinal, c) – torsional vibrations at $k = 3kN/m^3$

It was discovered that the amplitude of the column's vibratory process rises as the interaction coefficients between the drilling column and the soil increase. To a greater extent, this is manifested in their longitudinal and torsional vibrations. For smaller values of the coefficients k (Fig. 2 b), the beating mode in longitudinal vibrations tends to a stationary mode. The greater the coefficient k, the more stable the beating mode (Fig. 3-4).

4 Conclusion

The paper focuses on the study of nonlinear drill string dynamics under seismic environmental impact. The authors present the development of a nonlinear mathematical model of moving drill strings spatial deformation as applied to seismic loads. Through the result of numerical modelling, qualitative and quantitative changes in the drill strings dynamics were revealed. The growth of the vibration amplitudes, as well as the changes in the nature of the vibration themselves, manifested as beatings are observed when the seismicity taking into account. This could be justified by a complex process of interaction between the seismically active environment and the rotating moving drill string, the possibility of their resonation and amplification of the disturbance of the general dynamic system. In addition, the very nonlinearity of the mathematical model developed here more fully reflects the complex process under study in comparison with linear models and captures qualitative changes in the drill strings dynamics.

Acknowledgments

This research is funded by the Science Committee of the Ministry of Education and Science of the Republic of Kazakhstan (Grant No. AP 23490543).

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Received: August 08, 2024
Accepted: September 13, 2024