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S.S. Zhumatov[1], **Zh.M. Kadirbayeva**[1,2*], **Z.S. Kobeyeva**[3]
[1]Institute of Mathematics and Mathematical Modeling, Kazakhstan, Almaty
[2]Kazakh National Women's Teacher Training University, Kazakhstan, Almaty
[3]Shymkent University, Kazakhstan, Shymkent
*e-mail: zhkadirbayeva@gmail.com

CONSTRUCTION OF A SET OF DIFFERENTIAL EQUATIONS SYSTEMS AND STABILITY IN THE VICINITY OF A PROGRAM MANIFOLD

The problem of constructing a entire set of differential equation systems for a given program manifold is addressed. Necessary and sufficient conditions have been compiled to ensure that the program manifold is integral to the systems being developed. These conditions form a rectangular linear algebraic system in relation to the required functions. Using the property of a rectangular matrix, the set of differential equation systems was constructed. Additionally, the problem of designing indirect automatic control systems with rigid feedback is explored. Since the given program is not always executed perfectly due to initial or ongoing disturbances, it is reasonable to require stability of the program manifold with respect to a certain function. This leads to the analysis of the stability of the system of equations in relation to the given program manifold. Through the construction of Lyapunov functions for the system in canonical form, sufficient conditions for the absolute stability of the program manifold are derived. These results can be applied to the design of stable automatic indirect control systems.

Key words: absolute stability, program manifold, indirect automatic control systems, Lyapunov function, Lurie-Letov type system.

С.С. Жұматов[1], Ж.М. Қадырбаева[1,2*], З.С. Көбеева[3]
[1]Математика және математикалық моделдеу институты, Қазақстан, Алматы қ.
[2]Қазақ ұлттық қыздар педагогикалық университеті, Қазақстан, Алматы қ.
[3]Шымкент университеті, Қазақстан, Шымкент қ.
*e-mail: zhkadirbayeva@gmail.com

Дифференциалдық теңдеулер жүйесінің жиынын құру мен бағдарламалық көпбейне маңайындағы орнықтылық

Берілген бағдарламалық көпбейне бойынша дифференциалдық теңдеулер жүйесінің барлық жиынын құру мәселесі шешілді. Бағдарламалық көпбейненің құрылып жатқан жүйе үшін интегралдық болуының қажетті және жеткілікті шарттары құрылды. Бұл шарттар қажетті функцияларға қатысты тікбұрышты сызықты алгебралық жүйесін құрайды. Тікбұрышты матрицалардың қасиетін пайдалана отырып, дифференциалдық теңдеулер жүйесінің жиыны құрылады. Сонымен бірге, қатаң кері байланысты тура емес автоматтық басқару жүйелерін құру есебі қарастырылады. Алғашқы және тұрақты түрткілер болуы себепті берілген бағдарлама әрдайым дәл орындала бермейді. Сондықтан бағдарламалық көпбейненің өзінің орнықтылығы талабы орынды болып табылады. Бұдан қарастырылып отырған есеп бағдарламалық көпбейненің орнықтылығын зерттеуге келтіріледі. Ляпунов функцияларын құру арқылы бағдарламалық көпбейненің абсолюттік орнықтылығының жеткілікті шарттары алынады. Алынған нәтижелер орнықты тура емес автоматтық басқару жүйелерін құруда қолданыла алады.

Түйін сөздер: абсолют орнықтылық, бағдарламалық көпбейне, жанама автоматты басқару жүйелері, Ляпунов функциясы, Лурье-Летов типті жүйе.

С.С. Жуматов[1], Ж.М. Кадирбаева[1,2*], З.С. Кобеева[3]
[1]Институт математики и математического моделирования, Казахстан, г. Алматы
[2]Казахский национальный женский педагогический университет, Казахстан, г. Алматы
[3]Шымкентский университет, Казахстан, г. Шымкент
*e-mail: zhkadirbayeva@gmail.com

Построение множества систем дифференциальных уравнений и устойчивость в окрестности программного многообразия

Решена задача построения всего множества систем дифференциальных уравнений для заданного программного многообразия. Сформулированы необходимые и достаточные условия, при которых программное многообразие является интегральным для разрабатываемой системы. Эти условия образуют прямоугольную линейную алгебраическую систему относительно требуемых функций. С использованием свойства прямоугольной матрицы было построено множество систем дифференциальных уравнений. Рассмотрена также задача построения автоматических систем непрямого управления с жесткими обратными связями. Заданная программа не всегда выполняется точно из-за существующих начальных и постоянных возмущений. Поэтому требование устойчивости самого программного многообразия является необходимым. Тогда наша задача сводится к исследованию устойчивости программного многообразия. С помощью построения функций Ляпунова для системы в канонической форме выводятся достаточные условия абсолютной устойчивости программного многообразия. Эти результаты могут быть применены для проектирования устойчивых систем непрямого автоматического управления.
Ключевые слова: абсолютная устойчивость, программное многообразие, непрямые системы автоматического регулирования, функция Ляпунова, система типа Лурье-Летова.

1 Introduction

Inverse problems in ordinary differential equations dates back to the fifties of the last century. In this direction N.P.Erugin's article is fundamental [1]. In his work, Erugin formulated and solved the problem of forming a system of differential equations corresponding to a given integral curve. His approach, in particular, generalized problems related to determining the forces and moments that result in motion with specified characteristics, making it one of the possible motions of a mechanical system. This line of inquiry later evolved into broader problems, such as constructing differential equations systems, automatic control systems designed for a specific manifold, inverse dynamics problems, and the formulation of systems for programmed motion. A comprehensive review of these investigations can be found in [2]–[5].

In this article, we address the more general problem of developing a complete set of differential equation systems for a specified program manifold and explore the construction of indirect control systems based on this manifold.

In the phase space X^n, we choose a simply connected closed domain $G(\rho)$:

$$G(\rho) = \{(t,x) : t \in I \wedge \|\omega(t,x)\| \leq \rho < \infty\}, \quad I = [0,\infty). \tag{1}$$

It is important to mention that the domain $G(\rho)$ can also be multi-connected consisting of finite simply connected continuous sets, and a manifold located at $t = t_0$ in one of these domains remains in it for all $t \geq t_0$.

Within this domain (1), we address the problem of formulating a set of ordinary differential equations' systems for a given smooth program manifold $\Omega(t)$, expressed in the following form:

$$\dot{x} = f\left(t, x\right), \tag{2}$$

where f, x are n-dimensional vectors, $f \in R^n$ is continuous with respect to all variables and the existence conditions of the solution $x(t) = 0$ are satisfied; and the program manifold $\Omega(t)$ defined by the following equations

$$\Omega\left(t\right) \equiv \omega(t, x) = 0, \tag{3}$$

would be integral for them, where an s-dimensional vector ω ($s \leq n$) is continuous in the domain (1) including the manifold $\Omega(t)$, along with its partial derivatives.

Definition 1. The manifold $\Omega(t)$ would be called a program (integral) manifold of the system (2) if from $x(t_0) \in \Omega(t_0)$ follows $x(t) \in \Omega(t)$ for all $t > t_0$.

To address the problem, we will apply N.P. Yerugin's method [1]. Using this approach, necessary and sufficient conditions are established to ensure that the given integrals indeed form a program manifold for the system of equations being constructed. These conditions are later utilized as equations to define the required functions for constructing the corresponding set of differential equation systems.

According to a given program manifold a set of systems differential equations describing the motion of material systems is constructed. For the systems under construction, the necessary and sufficient conditions are composed the program manifold is integral.

2 Constructing a set of systems of ordinary differential equations according to a given program manifold.

Necessary and sufficient conditions that the given manifolds $\Omega(t)$ will be integral for a system of differential equations (2) we get

$$\dot{\omega} = \frac{\partial \omega}{\partial t} + Hf\left(t, x\right) = F(t, x, \omega), \tag{4}$$

here the vector-functions $f(t, x)$ are the right-hand parts of the equations (2), $H = \dfrac{\partial \omega}{\partial x}$ is the Jacobi matrix, $F(t, x, \omega)$ are arbitrary functions that vanish on a program manifold (3). From these conditions, the right parts of the desired system of differential equations are determined. At the same time, arbitrary functions $F(t, x, \omega)$ and the undefined right-hand sides $(n-s)$-dimensional vector function $f_2(t)$ are subject to conditions for the presence of a solution to the constructed system of equations (2).

The Jacobi matrix is

$$H = \begin{Vmatrix} \frac{\partial \omega_1}{\partial x_1} & \cdots & \frac{\partial \omega_1}{\partial x_n} \\ \cdots & \ddots & \cdots \\ \frac{\partial \omega_s}{\partial x_1} & \cdots & \frac{\partial \omega_s}{\partial x_n} \end{Vmatrix}.$$

In the future the s-dimensional vector function $F(t, x, \omega)$ will be called the Erugin function, satisfying conditions $F(t, x, 0) \equiv 0$ [2].

Suppose that the rank of the rectangular functional matrix $H \in R^{(s\times n)}$ has $s, s < n$ at the all points of the manifold $\Omega(t)$ and introducing notation through $y_1, \dots, y_s$, some s components of x and through $z_1, ..., z_{n-s}$, the remaining components of x, we presented the matrix H in the following form:

$$H(t,x) = H_1(t,y) + H_2(t,z),$$

where H_1 is non-degenerate $(s \times s)$ matrix, and H_2 is a rectangular $[s \times (n-s)]$ matrix:

$$H_1 = \begin{Vmatrix} \frac{\partial \omega_1}{\partial y_1} & \cdots & \frac{\partial \omega_1}{\partial y_s} \\ \dots & \ddots & \dots \\ \frac{\partial \omega_s}{\partial y_1} & \cdots & \frac{\partial \omega_s}{\partial y_s} \end{Vmatrix}, \quad H_2 = \begin{Vmatrix} \frac{\partial \omega_1}{\partial z_1} & \cdots & \frac{\partial \omega_1}{\partial z_{n-s}} \\ \dots & \ddots & \dots \\ \frac{\partial \omega_s}{\partial z_1} & \cdots & \frac{\partial \omega_s}{\partial z_{n-s}} \end{Vmatrix}.$$

In this case, equation (2) will be written as follows:

$$\dot{y} = f_1(t,y,z), \quad \dot{z} = f_2(t,y,z) \tag{5}$$

where $f_1(t,y,z)$ is s-dimensional, $f_2(t,y,z)$ is $(n-s)$-dimensional vectors satisfying the condition for the existence of solutions $y(t) = 0, z(t) = 0$ and conditions (4) will be written in the following form

$$\dot{\omega} = \frac{\partial \omega}{\partial t} + H_1(t,y) f_1(t,y,z) + H_2(t,z) f_2(t,y,z) = 0. \tag{6}$$

The problem of constructing is solved by finding the entire set of right-hand sides of the systems (5) fulfilling equality (6).

Let's choose the right part of equality (4) as follows

$$\dot{\omega} = F(t,y,z,\omega), \tag{7}$$

where $F(t,y,z,\omega)$ is some continuous the Yerugin vector-function, satisfying a condition

$$F(t,y,z,0) \equiv 0. \tag{8}$$

Then equating the right sides of equations (6) and (7), we get a system of linear algebraic equations with respect to $f_1(t,y,z)$ and $f_2(t,y,z)$:

$$H_1 f_1(t,y,z) + H_2 f_2(t,y,z) = F(t,y,z,\omega) - \frac{\partial \omega}{\partial t}. \tag{9}$$

the solution of which contains an $\forall$ $(n-s)$-dimensional vector-function $f_2(t,y,z)$ and depends on the expression $F(t,y,z,\omega) - \dfrac{\partial \omega}{\partial t}$.

Choosing $f_2(t,y,z)$ as an arbitrary function we find the desired function $f_1(t,y,z)$:

From equation (9) we find

$$f_1(t,y,z) = H_1^{-1}\Big[F(t,y,z,\omega) - H_2 f_2(t,y,z) - \frac{\partial \omega}{\partial t}\Big].$$

Therefore, we obtain the required set of differential equations systems in next form

$$\begin{cases} \dot{y} = H_1^{-1}\Big[F(t,y,z,\omega) - H_2 f_2\left(t,y,z\right) - \dfrac{\partial \omega}{\partial t}\Big] \\ \dot{z} = f_2\left(t,y,z\right) \end{cases} \tag{10}$$

In general, the problem under consideration has an ambiguous solution, since $s < n$ and arbitrary functions F and f_2 are present. Ambiguity can be utilized to solve inverse problems in combination with stability and optimality problems of a given program manifold. We will discuss stability issues in detail henceforward.

Thus,the right-hand system (2) is constructed in the form like (10). Here an arbitrary function $f_2(t,y,z)$ satisfies existence condition of the solution $z = 0$, the Yerugin function $F(t,y,z,\omega)$ has property (8). The function $F(t,y,z,\omega)$is such that the trivial solution $\omega = 0$ from (7) is stable for all x from the domain (1). This implies that the program manifold $\Omega(t)$ will be stable with respect to the vector function ω. Subsequently, in numerous cases, the problem of constructing equations of motion for which the integral manifold $\Omega(t)$ has the property of stability is reduced to choosing $F(t,y,z,\omega)$ as follows that the trivial solution $\omega = 0$ from (7) will be stable.

3 The program manifold's absolute stability for systems automatic indirect control with rigid feedbacks.

Now let's consider the problem of constructing automatic indirect control systems with feedbacks over a given manifold. Together with the system (2), we consider the indirect control system with feedback on the coordinate ξ of the control organ of the following structure [2]:

$$\begin{aligned} &\dot{x} = f\left(t,x\right) - B_1\xi, \quad t \in I = [0,\ \infty)\,, \\ &\dot{\xi} = \varphi\left(\sigma\right), \quad \sigma = P^T\omega - Q\xi, \end{aligned} \tag{11}$$

here $x \in R^n$ is a state vector, a vector-function $f \in R^n$ to be defined, satisfying the Lipschitz condition of existence of a solution $x(t) = 0$, and $B_1 \in R^{n\times r}$, $P \in R^{s\times r}$ are constant matrices, $Q \in R^{r\times r}$ is constant matrix of rigid feedback, $\varphi(\sigma)$ is function differentiable with respect to σ, satisfies next conditions

$$\varphi(0) = 0 \wedge 0 < \sigma^T\varphi(\sigma) < \sigma^T K\sigma \quad \forall\sigma \neq 0. \tag{12}$$

Here $K = K^T > 0, K \in R^{r\times r}$.

If a condition $\xi = 0$ is fulfilled on the manifold $\omega = 0$, then for the system (11), (12) the manifold $\Omega(t)$ will be also integral. To do this, it is necessary and sufficient to satisfy the $Q \neq 0$.

If we choose the Yerugin function linear with respect to the vector-function ω,then taking into account the necessary and sufficient condition that the manifold $\Omega(t)$ is integral to the system (3), we will arrive at the Lurye-Letov type system in coordinates (ω,ξ):

$$\begin{aligned} &\dot{\omega} = -A\omega - B\xi, \quad t \in I = [0,\ \infty)\,, \\ &\dot{\xi} = \varphi\left(\sigma\right), \quad \sigma = P^T\omega - Q\xi, \end{aligned} \tag{13}$$

where nonlinearity satisfies conditions (12), and $-A$ is Hurwitz matrix, $A \in R^{s\times s}$, $H = \dfrac{\partial\omega}{\partial x}$, $B = HB_1$,

$$F(t,x,\omega) = -A\omega. \tag{14}$$

Definition 2. If a program manifold $\Omega(t)$ of an indirect control system, with rigid feedback is globally stable on solutions of system (11) for any $\omega(t_0, x_0)$ and nonlinear function of control $\varphi(\sigma)$, satisfying conditions (12), then it is called absolutely stable with respect to the vector-function ω.

Statement of the problem. For any $\omega(t_0, x_0)$ and nonlinear function of control $\varphi(\sigma)$ determine a sufficient condition of absolute stability for the program manifold $\Omega(t)$ of the indirect control system with rigid feedback, relative to the vector function ω.

For system (13) we construct a definitely positive Lyapunov function of the following form

$$V = \omega^T L_0\omega + 2\omega^T L_1\xi + \xi^T L_2\xi + \int\limits_0^\sigma \varphi^T(\sigma)\beta d\sigma, \tag{15}$$

under the conditions

$$L = \left\| \begin{array}{cc} L_0 & L_1 \\ L_1^T & L_2 \end{array} \right\| > 0, \quad \beta = diag(\beta_1, ..., \beta_r) > 0, \tag{16}$$

which are equivalent to the following inequalities:

$$L_0 > 0 \wedge L_2 - L_1^T L_0^{-1} L_1 > 0.$$

The derivative of the function (15) in time t by virtue of the system (13) has the form

$$-\dot{V} = \omega^T C_0\omega + 2\omega^T C_1\xi + \xi^T g\xi + 2\omega^T C_2\varphi(\sigma) + 2\xi^T C_3\varphi(\sigma) + \varphi^T \sigma)\rho\varphi(\sigma) > 0,$$

where

$$C_0 = A^T L_0 + L_0 A, \quad C_1 = L_0 B + A^T L_1, g = BL_1 + L_0 B^T,$$
$$C_2 = -L_1 + \frac{1}{2}A^T P\beta, C_3 = -L_2 + B^T P\beta, \rho = \beta Q.$$

In order for it to be $-\dot{V} > 0$, it is sufficient to fulfill Sylvester's conditions:

$$\left\| \begin{array}{ccc} C_0 & C_1 & C_2 \\ C_!^T & g & C_3 \\ C_2^T & C_3^T & \rho \end{array} \right\| > 0, \tag{17}$$

these conditions are satisfied when if one of the groups the following inequalities are true

$$C_0 > 0 \wedge g - C_1^T C_0^{-1} C_1 > 0 \wedge \left| \begin{array}{ccc} C_0 & C_1 & C_2 \\ C_!^T & g & C_3 \\ C_2^T & C_3^T & \rho \end{array} \right| > 0,$$

$$g > 0 \wedge \rho - C_3^T g^{-1} C_3 > 0 \wedge \begin{vmatrix} C_0 & C_1 & C_2 \\ C_!^T & g & C_3 \\ C_2^T & C_3^T & \rho \end{vmatrix} > 0.$$

Theorem 1. Let there be matrices $L > 0$, $\beta > 0$ satisfying condition (16), the nonlinear control function $\varphi(\sigma)$ satisfies conditions (12) and the Yerugin function is chosen in the form (14). Then, for absolute stability of the program manifold of Lurye-Letov type systems with respect to vector-function ω, it is sufficient fulfillment of the generalized Sylvester's conditions (17).

Now, using a non-degenerate transformation

$$\varpi = -A\omega - B\xi, \quad \sigma = P^T\omega - Q\xi,$$

we present the system (13) to new coordinates (ϖ, σ), which are convenient for conducting research in applied problems:

$$\begin{aligned} &\dot{\varpi} = -A\varpi - B\varphi(\sigma), \quad t \in I = [0, \infty), \\ &\dot{\xi} = \varphi(\sigma), \quad \dot{\sigma} = P^T\varpi - Q\varphi(\sigma). \end{aligned} \tag{18}$$

Here the nonlinearity $\varphi(\sigma)$ satisfies the following conditions, which are equivalent to the conditions (12):

$$\varphi(0) = 0 \wedge 0 < \varphi^T(\sigma)K^{(-1)}\varphi(\sigma) < \varphi^T(\sigma)\sigma \quad \forall \sigma \neq 0. \tag{19}$$

For system (18) we construct a Lyapunov function of the following form

$$V = \varpi^T L \varpi + 2\varpi^T L_1 \sigma + \sigma^T L_2 \sigma > 0. \tag{20}$$

The Lyapunov function will be definitely positive if the following inequalities are fulfilled

$$L > 0 \wedge L_2 - L_1^T L^{-1} L_1 > 0. \tag{21}$$

Differentiating the expression (20) in time t by virtue of the system (18), we get

$$-\dot{V} = \varpi^T C \varpi + 2\varpi^T C_1 \varphi + 2\varpi^T(A^T L_1 - PL_2)\sigma + 2\varphi^T(B^T L_1 + QL_2)\sigma. \tag{22}$$

Applying the s-procedure [6], when performing the equalities

$$PL_2 = A^T L_1, \quad 2B^T L_1 + Q^T L_2 = E. \tag{23}$$

from (22) we get

$$-\dot{V} = \omega^T C \omega + 2\omega^T C_1 \varphi + \varphi^T C_2 \varphi + S > 0,$$

where

$$C = LA + A^T L - 2L_1 P^T, \quad C_1 = LB + L_1 Q, \quad C_2 = K^{(-1)}$$

Here S is defined by the following formula

$$S = (\varphi^T \sigma - \varphi^T K^{(-1)} \varphi) > 0.$$

In order for it to be $-\dot{V} > 0$, it is sufficient to fulfill Sylvester's conditions:

$$\left\| \begin{array}{cc} C & C_1 \\ C_1^T & C_2 \end{array} \right\| > 0, \qquad C > 0 \wedge C_2 - C_1^T C^{-1} C_1 > 0. \tag{24}$$

Theorem 2. Let the Yerugin function was chosen in the form (14), the nonlinearity $\varphi(\sigma)$ satisfy conditions (19) and the conditions (21) are valid. Then, for absolute stability of the program manifold of Lurye-Letov type systems with respect to vector-function ϖ, it is sufficient to satisfy Sylvester's conditions (24).

Comment 1. The equalities (23) play an important role in the construction of Lurye-Letov-type systems to define the matrices P and Q.

Over the past decade, work has been underway to the construction various of autonomous and non-autonomous basic and indirect systems of automatic control on the given program manifold possessing of quality properties and solving of different inverse problems of dynamical systems, to study of stochastic stability (see [7]– [17]).

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References

[1] Erugin N.P., "Construction of the entire set of systems of differential equations that have a given integral manifold", *Prikladnaya Matematika i Mecanika,* 10:6, (1952): 659–670.

[2] Maygarin B.G., "Stability and quality of process of nonlinear automatic control system", Nauka, Alma-Ata (1981)

[3] Zhumatov S.S., Krementulo B.B., Maygarin B.G., "Lyapunov's second method in the problems of stability and control by motion", Gylym, Almaty (1999)

[4] Galiullin A.S., Mukhametzyanov I.A., Mukharlyamov R.G., "Review of researches on the analytical construction of the systems programmatic motions", *Vestnik RUDN,* 1, (1994): 5–21.

[5] Llibre J., Ramirez R., "Inverse Problems in Ordinary Differential Equations and Applications", Springer International Publishing Switzerland(2016)

[6] Aizerman M.A., Gantmacher F.R., "Absolute stability of regulated systems", M. Publishing House of the USSR Academy of Sciences, (1963)

[7] Zhumatov S.S., "Frequently conditions of convergence of control systems in the neighborhoods of program manifold", *Nelineinye kolebania,* 28:3, (2016): 367–375.

[8] Zhumatov S.S., "Absolute stability of a program manifold of non-autonomous basic control systems", *News NAS RK. Series physico-mathematical,* 322:6, (2018): 37–43.

[9] Mukharlyamov R.G., "Controlling the dynamics of a system with differential connections", *News of the Russian Academy of Sciences. Theory and control systems,* 3, (2019): 22–33.

[10] Kaspirovich I.E., Mukharlyamov R.G., "On methods for constructing dynamic equations taking into account the stabilization of connections", *News of the Russian Academy of Sciences. MTT,* 3, (2019): 123–134.

[11] Tleubergenov M.I., Ibraeva G.T., "On the restoration problem with degenerated diffusion", *TWMS Journal of Pure and Applied Mathematics,* 6:1, (2015): 93–99.

[12] Vassilina G.K., Tleubergenov M.I., "Solution of the problem of stochastic stability of an integral manifold by the second Lyapunov method", *Ukrainian Mathematical Journal,* 68:1, (2016): 14–28. https://doi.org/10.1007/s 11253-016-1205-6

[13] Zhumatov S.S., "On the stability of a program manifold of control systems with variable coefficients" , *Mathematical Journal,* 71:8, (2020): 1202–1213.

[14] Zhumatov S.S., "On the absolute stability of a program manifold of non-autonomous control systems with non-stationary nonlinearities" , *Kazakh Mathematical Journal,* 19:4, (2020): 35–46.

[15] Zhumatov S.S., "Stability of the program manifold of different automatic indirect control systems" , *News Of the Khoja Akhmet Yassawi Kazakh-Turkish International University. Mathematics, physics, computer science series,* 16:1, (2021): 69–82.

[16] Zhumatov S.S., Vasilina G., "The Absolute Stability of Program Manifold of Control Systems with Rigid and Tachometric Feedbacks" , *Lobachevskii Journal of Mathematics,* 43:11, (2022): 3344–3351.

[17] Zhumatov S.S., Mynbayeva S.T., "Stability of the program manifold of automatic indirect control systems taking into account the external loads" , *Advances in the Theory of Nonlinear Analysis and its Applications,* 7:2, (2023): 405–412.

Information about authors: *Zhumatov Sailaubay – Doctor of Physical and Mathematical Sciences, Chief Researcher of the Institute of Mathematics and Mathematical Modeling SC MSHE RK (Almaty, Kazakhstan, email: sailau.math@mail.ru,);*

Kadirbayeva Zhazira (corresponding author) – Candidate of Physical and Mathematical Sciences, Leading Researcher of the Institute of Mathematics and Mathematical Modeling SC MSHE RK; Associate Professor of the Mathematics Department at the Kazakh National Women's Teacher Training University (Almaty, Kazakhstan, email: zhkadirbayeva@gmail.com);

Kobeyeva Zagira – PhD, head of the Mathematics and Computer Science department at the Shymkent University (Shymkent, Kazakhstan, email: kobeebazagi82@mail.ru).

Авторлар туралы мәлімет: *Жұматов Сайлаубай – Физика-математика ғылымдарының докторы, ҚР ҒЖБМ ҒК Математика және математикалық модельдеу институтының жетекші ғылыми қызметкері (Алматы, Қазақстан, электрондық пошта: sailau.math@mail.ru,);*

Қадырбаева Жазира (корреспондент автор) – Физика-математика ғылымдарының кандидаты, ҚР ҒЖБМ ҒК Математика және математикалық модельдеу институтының бас ғылыми қызметкері; Қазақ ұлттық қыздар педагогикалық университеті Математика кафедрасының кауымдастырылган профессоры (Алматы, Қазақстан, электрондық пошта: zhkadirbayeva@gmail.com);

Көбеева Загира – PhD, Шымкент университеті Математика және информатика кафедрасының меңгерушісі (Шымкент, Қазақстан, электрондық пошта: kobeebazagi82@mail.ru).

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