1-бөлім **Раздел 1** **Section 1**

Математика **Математика** **Mathematics**

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A. Kairatkyzy[1], **L. Zhapsarbayeva**[1*], **A.Tleulessova**[1,3],
S. Kabdrakhova[2,3], **S. Burgumbayeva**[1]
[1]L.N. Gumilyov Eurasian National University, Kazakhstan, Astana
[2]Al-Farabi Kazakh National University, Kazakhstan, Almaty
[3]Institute of Mathematics and Mathematical Modeling, Kazakhstan, Almaty
*e-mail: leylazhk67@yandex.kz

RECONSTRUCTION THE DOMAIN OF DEFINITION OF SOME DIFFERENTIAL OPERATOR ON A DIRECTED GRAPH

In this work it is proposed to study differential operators on a graph as an operator composed of differential operators on one-dimensional arcs and matrix operators on interior vertices of the graph. The work explores some questions concerning the theoretical side of ordinary differential equations with integro-differential conditions on stratified sets like graph. The attention will be paid to reconstruction of the domain of differential operator on directed graph. The reconstruction of the domain of differential operator means a simple specifying the boundary conditions from a known differential equations and its known eigenvalues. The paper studies the case of the second order differential equations with irregular boundary conditions on the vertices of directed graph. To achieve our goal we use the fact that finite set of eigenvalues serves as additional information for reconstruction of the domain of the differential operator on stratified set. The constructive algorithms for reconstructing the domain of definition of differential operator on directed graph are developed. All boundary functions from the spectral data are uniquely restored.

Key words: boundary functions, differential operator, stratified set, eigenvalues, Fourier coefficients.

А. Қайратқызы[1], Л.Қ. Жапсарбаева[1*], А.Б. Тлеулесова[1,3],
С.С. Кабдрахова[2,3], С.Қ. Бұргұмбаева[1]
[1]Л.Н. Гумилев атындағы Еуразия ұлттық университеті, Қазақстан, Астана қ.
[2]Әл-Фараби атындағы Қазақ ұлттық университеті, Қазақстан, Алматы қ.
[3]Математика және Математикалық модельдеу институты, Қазақстан, Алматы қ.
*e-mail: leylazhk67@yandex.kz

Бағытталған графтағы дифференциалдық оператордың анықталу облысын қалпына келтіру

Бұл жұмыста бір өлшемді доғалардағы дифференциалдық операторлардан және графтың ішкі төбелеріндегі матрицалық операторлардан тұратын оператор ретінде графтағы дифференциалдық операторларды зерттеу ұсынылады. Жұмыс граф тәріздес стратификацияланған жиындардағы интегро-дифференциалдық шарттары бар қарапайым дифференциалдық теңдеулердің теориялық жағына қатысты кейбір сұрақтарды зерттейді. Бағытталған графтағы дифференциалдық оператордың анықталу облысын қалпына келтіруге назар аударылады. Дифференциалдық оператордың анықталу облысын қалпына келтіру үшін белгілі дифференциалдық теңдеулерден және оның белгілі меншікті мәндерінен шекаралық шарттарды анықтауды білдіреді. Бұл жұмыс бағытталған графтың төбелеріндегі регулярлы емес шекаралық шарттары бар екінші ретті дифференциалдық теңдеулердің жағдайын зерттейді.

Мақсатымызға жету үшін меншікті мәндердің ақырлы жиыны стратификацияланған жиында дифференциалдық оператордың анықталу облысын қалпына келтіру үшін қосымша ақпарат ретінде қызмет ететінін қолданамыз. Бағытталған графтағы дифференциалдық оператордың анықталу облысын қалпына келтірудің конструктивті алгоритмі әзірленді. Барлық шекаралық функциялар спектрлік деректерден бірмәнді түрде қалпына келтірілген.

Түйін сөздер: шекаралық шарттар, дифференциалдық оператор, стратификацияланған жиын, меншікті мәндер, Фурье коэффициенттері.

А. Кайраткызы[1], Л.К. Жапсарбаева[1*], А.Б. Тлеулесова[1,3],
С.С. Кабдрахова[2,3], С.К. Бургумбаева[1]
[1]Евразийский национальный университет имени Л.Н.Гумилева, Казахстан, г. Астана
[2]Казахский национальный университет имени аль-Фараби, Казахстан, г. Алматы
[3]Институт Математики и Математического Моделирования, Казахстан, г. Алматы
*e-mail: leylazhk67@yandex.kz

Восстановление области определения одного дифференциального оператора на ориентированном графе

В данной работе предлагается изучать дифференциальные операторы на графах как оператор, составленный из дифференциальных операторов на одномерных дугах и матричных операторов на внутренних вершинах графа. В работе исследуются некоторые вопросы, касающиеся теоретической стороны обыкновенных дифференциальных уравнений с интегро-дифференциальными условиями на стратифицированных множествах типа граф. Особое внимание будет уделено восстановлению области определения дифференциального оператора на направленном графе. Восстановление области определения дифференциального оператора означает простое определение граничные функции из известного дифференциального уравнения и его известных собственных значений. В статье изучается случай дифференциальных уравнений второго порядка с нерегулярными граничными условиями на вершинах ориентированного графа. Для достижения нашей цели мы используем тот факт, что конечный набор собственных значений служит дополнительной информацией для восстановления области определения дифференциального оператора на стратифицированном множестве. Разработаны конструктивные алгоритмы восстановления области определения дифференциального оператора на ориентированном графе. Все граничные функции из спектральных данных восстанавливаются однозначно.
Ключевые слова: граничные условия, дифференциальный оператор, стратифицированное множество, собственные значения, коэффициенты Фурье.

1 Introduction

In this paper we want to develop a general theory to analyze the reconstruction the domain of definition of differential operators in stratified sets. Boundary value problems for differential equations on network-like sets (geometric graphs, trees) are often presented in natural science and technology (see [1–4] and the literature therein). Therefore, the analysis of boundary value problems for system of differential equations is of interest beyond the field of mathematics. Graph-like spaces have also been used as models in mathematical biology, for example, for a human lung, neural networks etc. [5]. Penner and et al applied graph-like space with a large number of vertex and edge to encode the structure of a protein in [6]. Recently, quantum models have successfully been used to carbon nano-structures [1, 7].

The possibility of recovering differential operators on graphs from the Weyl functions, from the system of spectra was investigated in [8–10]. Thus, various boundary value problems for differential equations have been investigated on graphs as stratified sets. However, the range of unsolved problems arising by analogy with the problems for differential operators on standard domains (segments, rectangles, multidimensional segments) is far from exhausted and it requires additional research.

The differential operator acts as a system of differential equations on each edge and the system is coupled via boundary conditions at the boundary vertices in order to assure the self-adjointness. One of the main questions is: what conditions at the vertices of the graph are the most "natural"? The standard answer is the Kirchhoff conditions. In monograph [11], the vertices of graphs are divided into two types: boundary vertices and interior vertices. If we assume that the Kirchhoff conditions or conditions from [12, 13] hold at the interior vertices, there is still a problem of determining conditions at boundary vertices.

From a mathematical point of view, graphs are interesting in that they are a good model for studying the properties of systems depending on the geometry and topology of space. Graphs are composed of zero-dimensional and one-dimensional manifolds, and in this sense it is interesting how the mixed dimension affects to certain properties of mathematical objects defined on graphs.

This work presents methods for determining spectral data to identify the boundary conditions of boundary value problems for second order differential equations on directed graphs.

The traces of the initial operator regularized in the sense of V.A. Sadovnichy [14] are used as additional information used to identify the boundary coefficients for one-dimensional differential operators on an interval. The some result in this regard can be found in [15, 16]. However, this paper highlights that not every finite set of eigenvalues is suitable for unique reconstruction of the domain of the second order differential operator on stratified set.

In this work, differential equations on stratified sets can be proposed as a crossbreed of matrix and differential operators. Thus, the analysis of above mentioned differential equations is an urgent problem.

Our goal is to investigate the unique reconstruction of the domain of definition of differential operators on the directed graph. The method for reconstruction of the domain of definition of second order differential operator on a graph are developed. We have uniquely reconstructed all boundary functions by the spectral data. The results presented here is based on the method developed by Kanguzhin B.E.

2 Unique reconstruction of the domain of differential operator on an interval

Let $b < \infty$ be given the differential expression

$$l(y) \equiv y^{(n)}(x) + \sum_{k=0}^{n-1} p_k(x) y^{(k)}(x),\ 0 < x < b$$

with regular coefficients

$$p_k \in C^k[0, b], k = 0, 1, \dots, n-1.$$

We consider the following differential equation

$$l(y) = f(x),\ 0 < x < b. \tag{1}$$

First we state the direct problem: what additional conditions must satisfy the solution of equation (1) so that equation (1) with any right-hand side $f(x) \in L^2(0,b)$ has a unique solution with a priori estimate

$$\|y\|_{L^2(0,b)} \leq C\|f(x)\|_{L^2(0,b)},\ 0 < x < b, \tag{2}$$

where C does not depend on f?

This question is answered in the following theorem, which follows from the work of M. Otelbaev [17].

Теорема 1 (Otelbayev theorem.) *A)Equation (1) with integro-differential conditions of the following form*

$$y^{(\nu-1)}(0) - \int_0^b l(y)\overline{\sigma_\nu(x)}dx = 0, \nu = 1, ..., n \tag{3}$$

for arbitrary set of boundary functions

$$(\sigma_1(x), ..., \sigma_n(x)) \in L^2(0,b) \otimes ... \otimes L^2(0,b)$$

has a unique solution $y(x)$ for any right-hand side $f(x)$ from $L^2(0,b)$, moreover inequality (2) holds.

B) The inverse statement also holds. If equation (1) with some additional linear conditions has a unique solution with requirement (2) for any right-hand side $f(x)$ from $L^2(0,b)$, then there exists the set of boundary functions $(\sigma_1(x), ..., \sigma_n(x)) \in L^2(0,b) \otimes ... \otimes L^2(0,b)$ such that the additional conditions are equivalent to the conditions of the form (3).

So, specifying coefficients $p_0(\cdot), p_1(\cdot), ..., p_{n-1}(\cdot)$ of equation (1) and boundary functions $\sigma_0(\cdot), \sigma_1(\cdot), ..., \sigma_{n-1}(\cdot)$ from the spectral data leads to unique reconstruction of the BVP (1)-(3).

1st BVP.

$$l(y) = f(x),\ 0 < x < b,$$

$$y(0) - \int_0^b l(y)\overline{\sigma_1(x)}dx = 0,$$

$$y^{(\nu-1)}(0) = 0, \nu = 2, ..., n.$$

2d BVP.

$$l(y) = f(x),\ 0 < x < b,$$

$$y^{(\nu-1)} - \int_0^b l(y)\overline{\sigma_\nu(x)}dx = 0, \nu = 1, 2,$$

$$y^{(\nu-1)}(0) = 0, \nu = 3, ..., n.$$

Furthermore, in a similar way third and so on $(n-1)$-th boundary value problems can be defined. Consequently, the BVP on n-th step synchronizes with the 1st BVP (1)- (3). Hence, all boundary functions can be uniquely reconstructed by the the spectra of the given BVP.

3 Reconstruction of the domain of differential operators on directed graph

Let $\Im = \{\mathcal{V}, \epsilon\}$ be a directed graph [18]. The set of vertices, we denote by $\mathcal{V}$. The set of arcs, we denote by ε. We represent below the directed graph $\Im$ with four vertices and three arcs, taking vertices 1, 2, 3 as exterior and vertex 4 as a interior (Fig. 1). We denote arc by $a = [l, m], a \in \epsilon$, with vertices l and m. The arc $a = [l, m]$ has orientation from l to m. We take arcs with the length 2π.

We consider the space

$$L_2(\Im) := \prod_{a\in\epsilon} L_2(a)$$

with the elements

$$(\rightarrow) := [y_a(x_a), a \in \epsilon]^T$$

(where $\rightarrow = (x_a, a \in \epsilon)$, and $\prod_{a\in\epsilon}$ is the Cartesian product of the spaces) and with a finite norm

$$\|\overrightarrow{Y}\|_{L_2(\Im)} = \sqrt{\sum_{a\in\epsilon} \int_a |y_a(x_a)|^2 dx_a}.$$

In the standard way we introduce the space

$$W_2^2(\Im) := \sum_{a\in\epsilon} W_2^2(a).$$

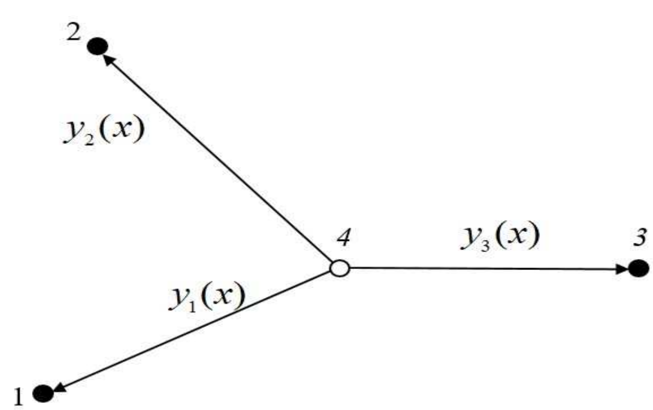

Figure 1: Distribution of the solutions on the graph-star (boundary vertices are painted)

We consider operator B defined by the following differential expressions

$$y_j''(x_j) = \lambda y_j(x_j),\ 0 < x_j < 2\pi,\ j = \overline{1,3} \tag{4}$$

in the domain $D(B) = W_2^2(\Im)$ and with the Kifchhoff low at the interior vertex

$$\begin{aligned} &y_1(0) = y_2(0) = y_3(0), \\ &y_1'(0) = y_2'(0) + y_3'(0) \end{aligned} \tag{5}$$

and with boundary forms

$$\begin{cases} U_{11}(y_1) = 0, \\ U_{21}(y_1) = 0, \end{cases} \tag{6}$$

$$\begin{cases} U_{12}(y_2) = 0, \\ U_{22}(y_2) = 0, \end{cases} \tag{7}$$

$$\begin{cases} U_{13}(y_3) = 0, \\ U_{23}(y_3) = 0, \end{cases} \tag{8}$$

at the boundary vertices, where

$$\begin{cases} U_{11}(y_1) = y_1(2\pi) - \int\limits_0^{2\pi} y_1''(x_1)\overline{\sigma}_{11}(x_1)dx_1, \\ U_{21}(y_1) = y_1'(2\pi) - \int\limits_0^{2\pi} y_1''(x_1)\overline{\sigma}_{21}(x_1)dx_1, \end{cases}$$

$$\begin{cases} U_{12}(y_2) = y_2(2\pi) - \int\limits_0^{2\pi} y_2''(x_2)\overline{\sigma}_{12}(x_2)dx_2, \\ U_{22}(y_2) = y_2'(2\pi) - \int\limits_0^{2\pi} y_2''(x_2)\overline{\sigma}_{22}(x_2)dx_2, \end{cases}$$

$$\begin{cases} U_{13}(y_3) = y_3(2\pi) - \int\limits_0^{2\pi} y_3''(x_3)\overline{\sigma}_{13}(x_3)dx_3, \\ U_{23}(y_3) = y_3'(2\pi) - \int\limits_0^{2\pi} y_3''(x_3)\overline{\sigma}_{23}(x_3)dx_3. \end{cases}$$

Here the set of functions $\sigma_{1j}, \sigma_{2j}\,(j = \overline{1,3})$ are called boundary functions.

Теорема 2 *Conditions (6)- (8) for differential operator B on the graph-star $\Im$ is uniquely identified by the spectral data.*

Proof. We need to identify the unknown boundary functions

$$\sigma_{1j}, \sigma_{2j}\,(j = \overline{1,3})$$

by using sets of eigenfunctions

$$\lambda_{1j} \leq \lambda_{2j} \leq ..., \tag{9}$$

$$\mu_{1j} \leq \mu_{2j} \leq .... \tag{10}$$

One set of eigenvalues $\lambda_{1j} \leq \lambda_{2j} \leq ...$ is given. In any case as the spectral data we consider eigenvalues (9) of the initial operator, namely of problem (4), (5), (6), (7), (8). Boundary conditions of the problem (4)- (8) cannot be identified uniquely by the spectra (9). Therefore we need some auxiliary spectral data. As auxiliary spectral data (10) we can take the spectral characteristics of the similar operators.

We construct the following algorithm to solve the problem.

Step 1. In this step we construct the second (10) spectral data. Here we take $\sigma_{21} = 0$. And we consider the following auxiliary problem

$$y_1'' = \lambda y_1, \ 0 < x_1 < 2\pi, \tag{11}$$

$$y_1(2\pi) - \int_0^{2\pi} y_1''(x_1)\overline{\sigma}_{11}(x_1)dx_1 = 0, \tag{12}$$

$$y_1'(2\pi) = 0. \tag{13}$$

By Otelbayev theorem (Theorem 1), the problem (11)- (13) has a solution. Then the set of eigenvalues of the the problem (11)- (13) we denote by $\mu_{11} \leq \mu_{21} \leq ...$.

In a similar way, by taking $\sigma_{22} = 0$, we establish that the following problem

$$y_2'' = \lambda y_2, \ 0 < x_2 < 2\pi, \tag{14}$$

$$y_2(2\pi) - \int_0^{2\pi} y_2''(x_2)\overline{\sigma}_{12}(x_2)dx_2 = 0, \tag{15}$$

$$y_2'(2\pi) = 0 \tag{16}$$

has a solution by Theorem 1. The set of eigenvalues of the the problem (14)- (16) we denote by $\mu_{12} \leq \mu_{22} \leq ....$

In a similar way, by taking $\sigma_{23} = 0$, we establish that the following problem

$$y_3'' = \lambda y_3, \ 0 < x_3 < 2\pi, \tag{17}$$

$$y_3(2\pi) - \int_0^{2\pi} y_3''(x_3)\overline{\sigma}_{13}(x_3)dx_3 = 0, \tag{18}$$

$$y_3'(2\pi) = 0 \tag{19}$$

has a solution by Theorem 1. The set of eigenvalues of the the problem (17)- (19) we denote by $\mu_{13} \leq \mu_{23} \leq ...$.

Step 2. Now we will restore the boundary functions σ_{11}, σ_{12}, σ_{13} of the problems (11)- (13), (14)- (16), (17)- (19), respectively, on the graph $L_2(\Im)$.Then we denote by $\psi_{n1}(x_1) = y_1(x_1, \mu_{n1})$, $\psi_{n2}(x_2) = y_2(x_2, \mu_{n2})$, $\psi_{n3}(x_3) = y_3(x_3, \mu_{n3})$, the eigenfunctions of the problems (11)- (13), (14)- (16), (17)- (19), respectively. Moreover it is known that $\psi'_{nj}(2\pi) = 0 \, (j = \overline{1,3})$ by conditions (13), (16), (19).

Namely

$$\begin{cases} \psi_{nj}(x_j) = y_j(x_j, \mu_{nj}) \neq 0, \\ \psi'_{nj}(2\pi) = 0. \end{cases}$$

Hence it follows that $\psi_{nj}(2\pi) \neq 0\,(j = \overline{1,3})$. Then we can take as follows $\psi_{nj}(2\pi) = 1\,(j = \overline{1,3})$. Namely

$$y_j(x_j, \mu_{nj}) = \cos\sqrt{\mu_{nj}}x_j,\ 0 < x_j < 2\pi\,(j = \overline{1,3}). \tag{20}$$

At the interior vertex 4 the functions (20) satisfy (5). By substituting the solution (20) into the condition (12), (15), (18), we have

$$1 - \int_0^{2\pi} \mu_{nj}\cos\sqrt{\mu_{mj}}x_j\overline{\sigma}_{1j}(x_j)dx_j = 0,\ j = 1,2,3.$$

Consequently,

$$\int_0^{2\pi} \cos\sqrt{\mu_{nj}}x_j\overline{\sigma}_{1j}(x_j)dx_j = \frac{1}{\mu_{nj}},\ j = 1,2,3. \tag{21}$$

The expression on the left-hand side of the equation (21) are the Fourier coefficients of expansion of the function $\sigma_{1j}(x_j), 0 < x_j < 2\pi\,(j = \overline{1,3})$ into the Fourier series. So, we have identified the boundary functions $\sigma_{1j}(x_j), 0 < x_j < 2\pi\,(j = \overline{1,3})$.

Thus, we found $\sigma_{11}(\cdot), \sigma_{12}(\cdot), \sigma_{13}(\cdot)$.

Step 3. In this step we restore unknown boundary functions $\sigma_{2j}(x_j), 0 < x_j < 2\pi\,(j = \overline{1,3})$ by given eigenvalues $\lambda_{1j} \leq \lambda_{2j} \leq ...$ and boundary functions $\sigma_{1j}(x_j), 0 < x_j < 2\pi\,(j = \overline{1,3})$ from previous step. For this we represent the auxiliary functions $\chi_{1j}, \chi_{2j}\,(j = \overline{1,3})$ in the following form

$$\chi_{1j}(x_j, \lambda_j) = \cos\sqrt{\lambda_j}x_j\ \ (j = \overline{1,3}).$$

Hence we have the conditions $\chi_{1j}(2\pi, \lambda_j) = 1\,(j = \overline{1,3})$, then for its derivative is $\chi'_{1j}(2\pi, \lambda_j) = 0\,(j = \overline{1,3})$.

Now the function $\chi_{2j}(x_j, \lambda_j)$ can be chosen so that to be a solution of the problem

$$\chi''_{2j}(x_j, \lambda_j) = \lambda_j\chi_{2j}(x_j, \lambda_j),\ 0 < x_j < 2\pi\,(j = \overline{1,3}), \tag{22}$$

$$\chi_{2j}(2\pi) - \int_0^{2\pi} \chi''_{2j}(x_j)\overline{\sigma}_{1j}(x_j)dx_j = 0, \tag{23}$$

$$\chi'_{2j}(2\pi) = 1. \tag{24}$$

Then the functions $\chi_{1j}(x_j, \lambda_j), \chi_{2j}(x_j, \lambda_j),\ 0 < x_j < 2\pi\,(j = \overline{1,3})$ form a fundamental set of solutions, since they are linear independent functions

$$\chi'_{1j}(2\pi, \lambda_j) = 0, \chi'_{2j}(2\pi, \lambda_j) = 1\,(j = \overline{1,3}).$$

Now we return to the initial problem (4), (5), (6), (7), (8). If $\lambda_j = \lambda_{nj}\,(j = \overline{1,3})$, then the general solution of the equation (4) can be expressed via fundamental set of solutions χ_{1j} and $\chi_{2j}(j = \overline{1,3})$:

$$U_1(y_j(x_j, \lambda_{nj})) = C_{1j}U_1(\chi_{1j}(x_j, \lambda_{nj})) + C_{2j}U_1(\chi_{2j}(x_j, \lambda_{nj})) = 0,\ (j = \overline{1,3}).$$

Hence

$$C_{1j}U_1(\chi_{1j}(x_j, \lambda_{nj})) = 0\ (j = \overline{1,3}), \tag{25}$$

since equality (23) implies that

$$C_{2j}U_1(\chi_{2j}(x_j, \lambda_{nj})) = 0\ (j = \overline{1,3}).$$

Now the substitution into the conditions (6), (7), (8), gives

$$U_2(y_j(x_j, \lambda_{nj})) = C_{1j}U_2(\chi_{1j}(x_j, \lambda_{nj})) + C_{2j}U_2(\chi_{2j}(x_j, \lambda_{nj})) = 0\ (j = \overline{1,3}).$$

Consequently, we can choose

$$C_{2j} = -C_{1j}\frac{U_2(\chi_{1j}(x_j, \lambda_{nj}))}{U_2(\chi_{2j}(x_j, \lambda_{nj}))}\ (j = \overline{1,3}),$$

or

$$C_{1j} = U_2(\chi_{2j}), C_{2j} = -U_2(\chi_{1j})\ (j = \overline{1,3}).$$

So, we obtain the following equations

$$y_j(x_j, \lambda_{nj}) = U_2(\chi_{2j}(x_j, \lambda_{nj}))\chi_{1j}(x_j, \lambda_{nj}) - U_2(\chi_{1j}(x_j, \lambda_{nj}))\chi_{2j}(x_j, \lambda_{nj})\ (j = \overline{1,3}).$$

Since $\chi_{2j}(x_j, \lambda_n)$ is the solution of the problem (22), (23), (24), then $U_{1j}(\chi_{2j}) = 0$, hence

$$U_{1j}(\chi_{1j}(x_j, \lambda_n))U_{2j}(\chi_{2j}(x_j, \lambda_n)) = 0\ (j = \overline{1,3}).$$

Thus

$$U_{2j}(\chi_{2j}(x_j, \lambda)) \mid_{\lambda=\lambda_n} = 0\ (j = \overline{1,3}).$$

If $U_{1j}(\chi_{1j}) = 0$, then the functions $\chi_{1j}(x_j, \lambda_{nj})$ are the eigenfunctions of the problems (11), (12), (13); (14), (15), (16); (17), (18), (19). Then the values λ_n will be eigenfunctions of the problems (11), (12), (13); (14), (15), (16); (17), (18), (19). Hence $\lambda_{nj} = \mu_{nj}$, then

$$U_{1j}(\chi_{1j}) \neq 0(j = \overline{1,3}).$$

Therefore we require that $\lambda_{nj} \neq \mu_{nj}$. Then

$$U_{2j}(\chi_{2j}(x_j, \lambda)) \mid_{\lambda=\lambda_{nj}} = 0\ (j = \overline{1,3}).$$

We rewrite the last equality

$$\chi'_{2j}(2\pi,\lambda_n)-\lambda_{nj}\int\limits_0^{2\pi}\chi_{2j}(x_j,\lambda nj)\overline{\sigma}_{2j}(x_j)dx_j=0\ (j=\overline{1,3}).$$

And by (24) it is known that $\chi'_{2j}(0,\lambda_{nj})=1$, then

$$\int\limits_0^{2\pi}\chi_{2j}(x_j,\lambda_{nj})\overline{\sigma}_{2j}(x_j)dx_j=\frac{1}{\lambda_{nj}}\ (j=\overline{1,3}).$$

The functions $\sigma_{2j}(x_j),\ 0<x_j<2\pi\ (j=\overline{1,3})$ can be identified from the last equation, since the eigenfunctions $\frac{1}{\lambda_{nj}}\ (j=\overline{1,3})$ serve as the Fourier coefficients of these functions. The proof the of the theorem is complete.

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Information about authors:

Kairatkyzy Agnur – PhD student of the Department of Fundamental Mathematics of L.N. Gumilyov Eurasian National University (Astana, Kazakhstan, email: kairatkyzy_agnura@mail.ru);

Zhapsarbayeva Lyailya (corresponding author) – PhD, c.ph.-m.sc., Associate Professor of the Department of Fundamental Mathematics of L.N. Gumilyov Eurasian National University (Astana, Kazakhstan, email: leylazhk67@yandex.kz);

Tleulessova Agila – c.ph.-m.sc., Associate Professor of the Department of Fundamental Mathematics of L.N. Gumilyov Eurasian National University (Astana, Kazakhstan, email: agila_72@mail.ru);

Kabdrakhova Symbat – c.ph.-m.sc., Associate Professor of Al-Farabi Kazakh National University, Leading Researcher of Institute of Mathematics and Mathematical Modeling (Almaty, Kazakhstan, Symbat2909.sks@gmail.com);

Burgumbayeva Saule – PhD, Assistant Professor of the Department of Higher Mathematics of L.N. Gumilyov Eurasian National University (Astana, Kazakhstan, e-mail: burgumbayeva_sk@enu.kz).

Авторлар туралы мәлімет:

Қайратқызы Агнур – Л.Н. Гумилев атындағы Еуразия ұлттық университетінің Іргелі математика кафедрасының PhD студенті (Астана қ., Қазақстан, электрондық пошта: kairatkyzy_agnura@mail.ru);

Жапсарбаева Ляйля (корреспондент автор) – PhD, ф.-м.ғ.к., Л.Н. Гумилев атындағы Еуразия ұлттық университетінің Іргелі математика кафедрасының қауымдастырылған профессоры (Астана қ., Қазақстан, электрондық пошта: leylazhk67@yandex.kz);

Тлеулесова Агила – ф.-м.ғ.к., Л.Н. Гумилев атындағы Еуразия ұлттық университетінің Іргелі математика кафедрасының қауымдастырылған профессоры (Астана қ., Қазақстан), Математика және Математикалық модельдеу институтының жетекші ғылыми қызметкері (Алматы қ., Қазақстан, электрондық пошта: agila_72@mail.ru);

Кабдрахова Сымбат – ф.-м.ғ.к., әл-Фараби атындағы ҚазҰУ ақпараттық технологиялар факультетінің қауымдастырылған профессоры, Математика және Математикалық модельдеу институтының жетекші ғылыми қызметкері (Алматы қ., Қазақстан, электрондық пошта: Symbat2909.sks@gmail.com);

Burgumbayeva Saule – PhD, Л.Н. Гумилев атындағы Еуразия ұлттық университетінің Жоғары математика кафедрасының аға оқытушысы (Астана қ., Қазақстан, электрондық пошта: burgumbayeva_sk@enu.kz).

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