2-бөлім **Раздел 2** **Section 2**

Механика **Механика** **Mechanics**

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STUDYING FORCED OSCILLATIONS OF A SINGLE-SPAN CANTILEVER BEAM USING THE ANALYTICAL METHOD

This article deals with the "dynamics"of a rod system (a two-support beam with two cantilevers) with three point masses located along its length that simulate the distributed mass of the structure under the action of a vibratory dynamic load applied to the corresponding mass; this load models the kinematic disturbance that occurs in the chassis structure of vehicles when they move along uneven road surfaces. Dynamic forces and displacements were obtained by an accurate analytical method of forces based on modal and vibration analysis of the structure operation. Unit and load coefficients of canonical equations were determined by the Mohr formula using the Vereshchagin rule. A comparative analysis of the parameters of the beam stress-strain state was carried out for absolutely rigid and elastic-pliable support points of the chassis structure. As a result of the study, the necessary unit and final moment and deflection diagrams were presented. For the comparative analysis of the results of dynamic and static calculations, the values of dynamic coefficients for bending moments in characteristic nodes along the beam length were calculated.
Key words:two-support cantilever beam; point masses; external vibration load; kinematic disturbance; dynamic displacements and forces; force method; displacement diagrams; elastic pliability of supports; dynamic coefficient; circular frequencies; frequency spectrum.

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Біраралықты консольды арқалықтың мәжбүрлі тербелісін аналитикалық әдіспен зерттеу

Мақалада өзек жүйесінің "динамикасы"зерттеледі, оның ұзындығы бойынша үш нүктелі массасы бар, тиісті массаға қолданылатын діріл динамикалық жүктеме кезінде құрылымның үлестірілген массасын имитациялайды; бұл жүктеме автомобильдер шассиінің констирукцияларында олардың жол төсемініңсоққыларымен қозғалуы кезінде пайда болатын кинематикалық бұзылуды модельдейді. Динамикалық күштер мен қозғалыстар құрылымның модальды және діріл талдауына негізделген күштердіұ дәл аналитикалық әдісімен алынады. Канондық теңдеулердің бірлік және жүк коэффициенттері Верещагин ережесін қолдана отырып Мор формуласымен анықталады. Шасси конструкциясының абсолютті қатты және серпімді-икемді тірек нүктелерінде арқалықтың кернеулі-деформацияланған күйінің параметрлеріне салыстырмалы талдау жасалды. Зерттеу нәтижелері ретінде моменттердің, иілудің қажетті жекелеген және қорытынды диаграммалары келтірілген. Сондай-ақ, динамикалық және статикалық есептеулердің нәтижелерін салыстырмалы талдау үшін арқалықтың ұзындығына тән түйіндердегі иілу моменттері үшін динамикалық коэффициенттердің мәндері алынды.

Түйін сөздер: консольдері бар қос тіректі арқалық; нүктелік массалар; сыртқы діріл жүктемесі; кинематикалық ұйытқу; динамикалық қозғалыстар мен күштер; күштер әдісі; тіректердің серпімді икемділігі; динамикалық коэффициент; айналмалы жиіліктер; жиілік спектрі.

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Исследование вынужденных колебаний однопролетной балки с консолями аналитическим методом

В данной статье исследуется «динамика» стержневой системы (двухопорной балки с двумя консолями) с тремя точечно расположенными по ее длине массами, имитирующими распределенную массу конструкции при действии вибрационной динамической нагрузки, приложенной к соответствующим масса; эта нагрузка моделирует кинематическое возмущение, возникающее в конструкции шасси автомобилей при движении их по неровностям дорожного полотно. Динамические усилия и перемещения получены точным аналитическим методом сил на основе модального и вибрационного анализа работы конструкции. Единичные и грузовые коэффициенты канонических уравнений определены по формуле Мор с применением правила Верещагина. Выполнен сравнительный анализ параметров напряженно-деформированного состояния балки при абсолютно жестких и упруго-податливах точках опирания конструкции шасси. В качестве результатов исследования приведены необходимые единичные и окончательные эпюры моментов, прогибов. Также, для сравнительного анализа результатов динамического и статического расчетов выполнены значения динамических коэффициентов для изгибающих моментов в характерных узлах по длине балки.

Ключевые слова: вухопорная балка с консолями; точечные массы; внешняя вибрационная нагрузка; кинематическое возмущение; динамические перемещения и усилия; метод сил; упругая податливость опор; динамический коэффициент; круговые частоты; спектр частот.

1 The study purpose and tasks

A study was conducted and a method of dynamic calculation of a rod system in the form of a single-span beam with two cantilevers was developed for the action of a kinematic disturbance of its supports from the vehicle movement on uneven road surfaces using the analytical method of forces in order to determine the parameters of the stress-strain state: dynamic forces and displacements.

The following tasks were solved:

1) developing a calculated dynamic pattern of a beam with the arrangement of point masses, values of amplitudes and frequencies of the disturbing external load for real designs of a truck chassis;

2) carrying out the modal analysis: determining the frequency spectrum and forms of natural vibrations in the form of "standing"waves for a system with three finite degrees of freedom;

3) carrying out the vibration analysis: determining dynamic forces and displacements using the exact method of forces;

4) carrying out the comparative analysis of the impact of the support rigidity degrees (rigid and elastic-pliable options) on the parameters of the stress-strain state for the purpose

of designing a vehicle suspension system to reduce the magnitude of dynamic movements and forces during its movement over uneven road surfaces.

2 Introduction

When designing supporting structures of the chassis of various vehicles, it is necessary to know the parameters of their stress-strain state: internal forces, displacements, deformations, stresses in cross sections.

Chassis support points that are elastic-pliable devices, provide soft suspension of the vehicle body. In this regard, the calculation of chassis beam structures must be performed taking into account their rigidity coefficients.

To assess the impact of the support points of the chassis beam system elasticity, in this work they are calculated as rigid supports (without taking into account their elastic compliance).

Comparison of the obtained results will allow establishing the degree of the support elasticity impact and taking this factor into account when designing the supporting structures of the chassis.

A review of the scientific literature available today on the subject of the proposed article showed the following.

In solving applied problems of mechanics, studying the behavior of beam structures under large deformations is of considerable interest. A number of structures and their elements (high-rise buildings, large bridge spans) can be modeled as beams. In works [1–11], partial differential equations with various boundary conditions of the problem are presented for the corresponding beam oscillations. The equations of motion of most known models describing the nonlinear dynamics of flexible beams contain nonlinear terms of the third order in their structure. To increase the accuracy of calculations, it is necessary to develop equations of motion that contain nonlinear terms of a higher order. In this case, the authors used various classical methods of solving nonlinear differential equations describing nonlinear oscillations in the presence of many parameters: the Laplace transform [2], the variation method [12, 13], the Hamilton method [14], the Navier and Levy solutions [15, 16]. Using the parameter expansion method [17], a large class of nonlinear problems is solved quite accurately and simply. As a rule, to achieve high accuracy, it is sufficient to use several successive approximations. Work [4] shows the advantages of the parameter expansion method, the Hamilton method, and the energy balance method of solving the equation of the problem of transverse beam oscillations. In [12-14], the parameter expansion method was used to study cantilever oscillations for various types of loading. The efficiency of using the parameter expansion method in analyzing the nonlinear problem of stability and free oscillations of beams is also shown in [18]. The well-known Galerkin-Bubnov method is used to obtain a nonlinear ordinary differential equation from the original partial differential equation. Despite sufficient interest in the issues of beam system dynamics [19, 20], the authors were unable to find publications with solutions similar to those proposed in this article.

3 Theoretical provisions and calculation methods

The object of the study is a single-span beam with two cantilevers (Fig. 1, a).

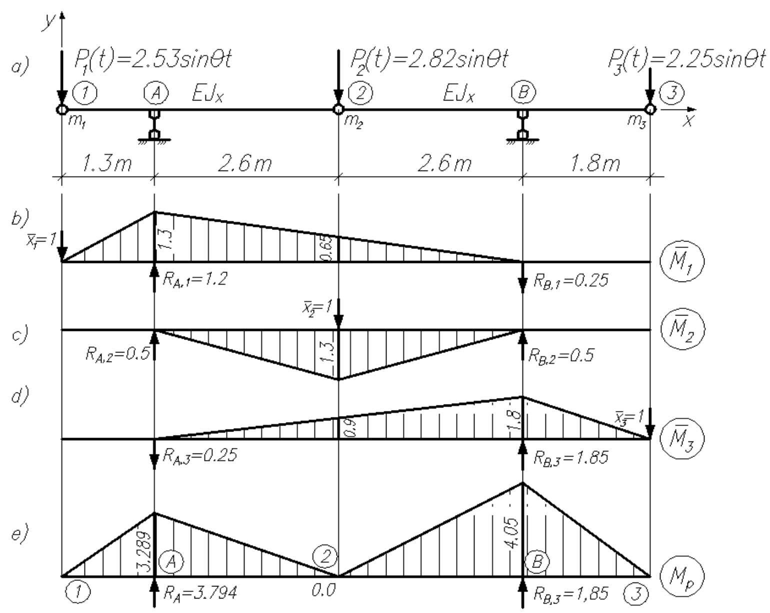

Figure 1: To the dynamic calculation of a single-span cantilever beam using the force method: Calculation scheme (a); single moment diagrams $\overline{M}_1,\ \ \overline{M}_2,\ \ \overline{M}_3$ (b, c, d); load diagram of the M_P (e) moments

For the dynamic study of the beam, the force method is adopted [21-24]. We apply unit inertial forces $\overline{x}_1 = \overline{x}_2 = \overline{x}_3 = 1$, to the given system (Fig. 1, a), and construct unit moment diagrams $\overline{M}_1,\ \ \overline{M}_2,\ \ \overline{M}_3$ from their action (Fig. 1, b, c, d).

1) Calculating the beam for free oscillations (modal analysis)

Write the canonical equations of the force method for the mass amplitudes as for a system with three degrees of freedom from displacements $y_1,\ y_2,\ y_3$.

$$\begin{cases} (\delta_{11} - \lambda_i)\, y_{1,i} + \delta_{12} y_{2,i} + \delta_{13} y_{3,i} = 0 \\ \delta_{21} y_{1,i} + (\delta_{22} - \lambda_i)\, y_{2,i} + \delta_{23} y_{3,i} = 0 \\ \delta_{31} y_{1,i} + \delta_{32} y_{2,i} + (\delta_{33} - \lambda_i)\, y_{3,i} = 0, \end{cases} \tag{1}$$

Here

$$\lambda_i = 1/\omega_i^2 m. \tag{2}$$

(2) is the frequency parameter ($i = 1,\ 2,\ 3$) of free vibrations; m=0.25 (ts^2/m) is the point mass on the beam (Fig. 1, a); ω_i is the circular frequency of the i-th tone of free vibrations.

Using the value of λ_i from (2), there is determined

$$\omega_i = \sqrt{\frac{1}{\lambda_i m}}, (i = 1,\ 2,\ 3). \tag{3}$$

By multiplying the unit moment diagrams $\overline{M}_i\,(i = 1,\ 2,\ 3)$, according to the

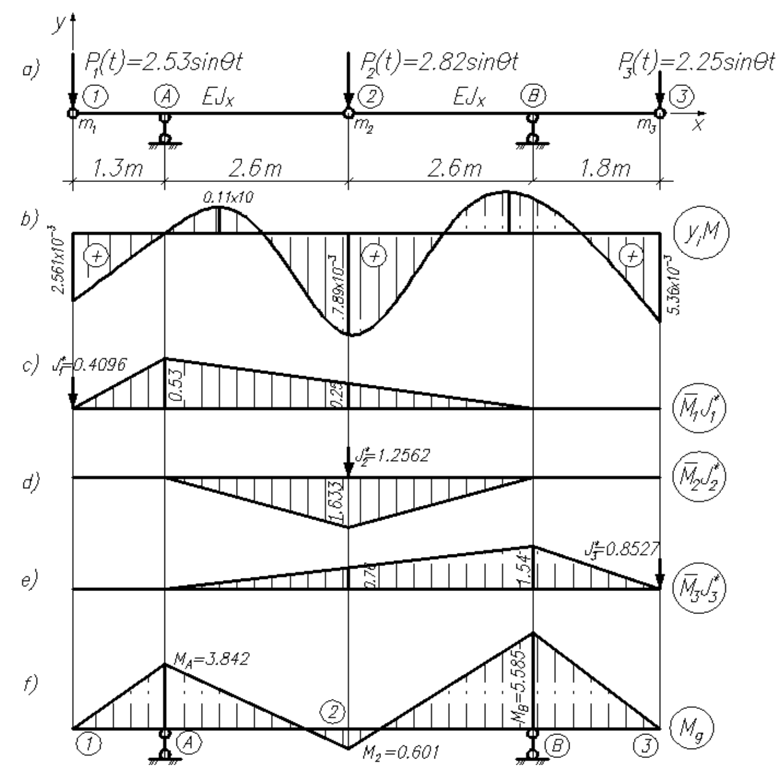

Figure 2: To the dynamic calculation of a single-span cantilever beam using the force method:Calculation scheme (a); resulting diagram of dynamic deflections (b); moment diagrams taking into account the inertial forces $J_1,\ J_2,\ J_3$ (c, d, e); resulting moment diagram (f)

Vereshchagin rule, there are calculated the unit coefficients of canonical equations (1) $(EJ_x = 3.8 \cdot 10^3 \mathrm{tm}^2)$

$$\begin{gathered}\delta_{11} = 0.964\left(10^{-3}\right);\ \delta_{22} = 0.771\left(10^{-3}\right);\ \delta_{33} = 1.989\left(10^{-3}\right);\\ \delta_{12} = \delta_{21} = -0.578\left(10^{-3}\right);\ \delta_{13} = \delta_{31} = 0.534\left(10^{-3}\right);\ \delta_{23} = \delta_{32} = -0.800\left(10^{-3}\right).\end{gathered} \tag{4}$$

Substituting the values of (4) in system (1), there is obtained the secular equation for free oscillations:

$$D = 10^{-3}\begin{bmatrix} \left[0.964 - \lambda(10^3)\right];\ -0.578; 0.534 \\ -0.578;\ \left[0.771 - \lambda\left(10^3\right)\right];\ -0.800 \\ 0.534;\ -0.800;\ \left[1.989 - \lambda(10^3)\right] \end{bmatrix} = 0. \tag{5}$$

Expanding determinant (5), there are determined three eigenvalues and the corresponding three eigenvectors of the matrix $(m = 0.25\ (\mathrm{tf}^2/\mathrm{m}))$

$$\begin{gathered}\lambda_3 = 2.164 \cdot 10^{-4}\\ \omega_3 = \sqrt{\frac{1}{\lambda_1 \cdot 0.25m}} = 135.96(\mathrm{c}^{-1})\end{gathered} \qquad \left\{\begin{aligned} y_{1,3} &= 4.982 \cdot 10^{-9}\\ y_{2,3} &= 0.589\\ y_{3,3} &= 2.486\end{aligned}\right.$$

$$\begin{matrix} \lambda_2 = 8.052 \cdot 10^{-4} \\ \omega_2 = \sqrt{\frac{1}{\lambda_2 \cdot 0.25m}} = 70.486(\text{c}^{-1}) \end{matrix} \qquad \left\{ \begin{matrix} y_{1,2} = -0.589 \\ y_{2,2} = -6.528 \cdot 10^{-9} \\ y_{3,2} = 1.897 \end{matrix} \right. \tag{6}$$

$$\begin{matrix} \lambda_1 = 2.702 \cdot 10^{-3} \\ \omega_1 = \sqrt{\frac{1}{\lambda_3 \cdot 0.25m}} = 38.476(\text{c}^{-1}) \end{matrix} \qquad \left\{ \begin{matrix} y_{1,1} = -2.486 \\ y_{2,1} = -1.897 \\ y_{3,1} = 1.546 \cdot 10^{-9} \end{matrix} \right.$$

Based on the values of eigenvectors (6), construct the corresponding forms of natural oscillations in the form of "standing" waves (Fig. 3).

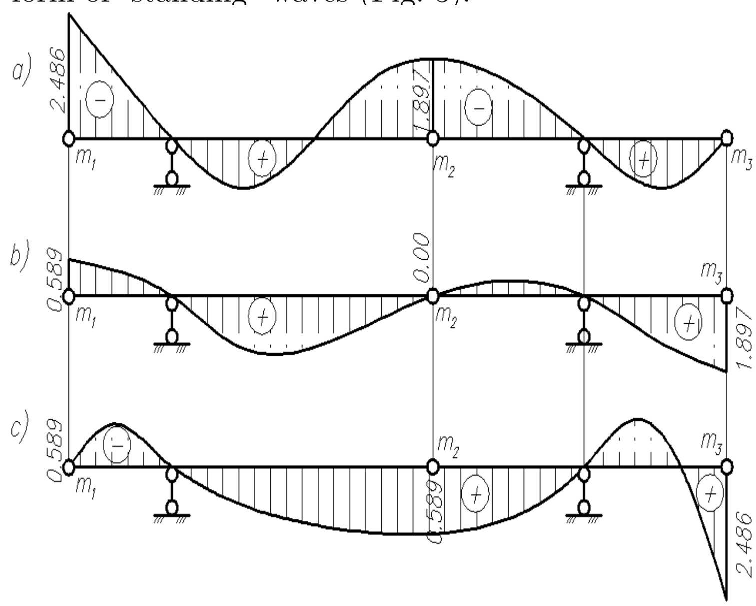

Figure 3: Natural vibration modes of a two-support beam a) 1st mode; b) 2nd mode; c) 3rd mode

2) Forced oscillations of the beam (vibration analysis)

The canonical equations of the force method for the amplitude values of forces and inertia are as follows;

$$\left\{ \begin{matrix} \delta_{11}^* J_1^* + \delta_{12} J_2^* + \delta_{13} J_3^* + \Delta_{1p} \sin\theta\ t = 0; \\ \delta_{21} J_1^* + \delta_{22}^* J_2^* + \delta_{23} J_3^* + \Delta_{2p} \sin\theta\ t = 0; \\ \delta_{31} J_1^* + \delta_{32} J_2^* + \delta_{33}^* J_3^* + \Delta_{3p} \sin\theta\ t = 0. \end{matrix} \right. \tag{7}$$

Calculate the main unit and load coefficients of equations (7) ($\theta = 25.23$ (s^{-1}) taking into account the kinematic disturbance (displacement) of the masses, at $m = 0.25$ (tf^2/m)).

$$\begin{matrix} \delta_{11}^* = \delta_{11} - \frac{1}{m\theta^2} = 10^{-3} \left[0.964 - 1/0.25 \cdot (25.23)^2 \right] = -5.32 \cdot 10^{-3}; \\ \delta_{22}^* = \delta_{22} - \frac{1}{m\theta^2} = 10^{-3} \left(0.771 - 6.284 \right) = -5.513 \cdot 10^{-3}; \\ \delta_{33}^* = \delta_{33} - \frac{1}{m\theta^2} = 10^{-3} \left(1.989 - 6.284 \right) = -4.295 \cdot 10^{-3}. \end{matrix} \tag{8}$$

$$\begin{aligned}\Delta_{1p} &= P_1\delta_{11} = 2.53 \cdot 0.964 \left(10^{-3}\right) = 2.439 \cdot 10^{-3};\\ \Delta_{2p} &= P_2\delta_{22} = 2.82 \cdot 0.771 \left(10^{-3}\right) = 2.174 \cdot 10^{-3};\\ \Delta_{3p} &= P_3\delta_{33} = 2.25 \cdot 1.989 \left(10^{-3}\right) = 4.475 \cdot 10^{-3};\end{aligned} \tag{9}$$

Substituting values (8, 9) in system (7), there is obtained:

$$J_1^* = 0.4076\,(\sin\theta\ t)\,; \quad J_2^* = 1.2562\,(\sin\theta\ t)\,; \quad J_3^* = 0.8527(\sin\theta\ t). \tag{10}$$

Determine dynamic displacements based on formula 3.19 in [21]:

$$y_i\,(t) = J_i^*/(m_i\theta^2). \tag{11}$$

Based on formula (11)

$$y_1 = \frac{(\sin\theta\ t) \cdot 0.4076}{\left[0.25 \cdot (25.23)^2\right]} = 0.002561\,(\sin\theta\ t)\ \ (m)\,;$$

$$y_2 = 0.00789\,(\sin\theta\ t)\ \ (m)\,; y_3 = 0.00536\,(\sin\theta\ t)\ \ (m)\,. \tag{12}$$

4 Research results

Using values (12) (at sinθ t=1), construct the diagram of the amplitude displacements of the beam (Fig. 2, c, d, e).

Using values (10), construct the corrected diagrams $\overline{M}_i J_i^*$ at (at sinθt=1) (Fig. 1, b, c, d).

Construct the calculated dynamic diagram using the following formula (at sinθt=1):

$$M_g = \overline{M}_1 J_1^* + \overline{M}_2 J_2^* + \overline{M}_3 J_3^* + M_P. \tag{13}$$

According to (13), geometrically add the diagrams in Figures 1, e and 2, c, d, e. The dynamic diagram (M_g) is shown in Figure 2, d. The dynamic coefficients of the structure shown in Figure 2, d ($M_{g,i}$ in Figure 1, e; $M_{p,i}$ in Figure 1, d).

$$\mu_i = \frac{M_{g,i}}{M_{p,i}}; \tag{14}$$

a) on the A support: $\mu_{\mathrm{A}} = 1.17$;

b) on the B support: $\mu_{\mathrm{B}} = 1.38$.

When designing the structure in question, to ensure its safety, it is necessary to take internal forces not under static loading but under dynamic loading, when the dynamic coefficient (μ_i) exceeds the value of 1.0.

According to Figure 1,e, there is obtained an interpolation polynomial ([24], p. 13, m=5).

$$y\,(x) = 10^{-3}\left(2.561 - 5.5x + 3.188x^2 - 0.36233x^3 - 0.0011x^4\right);$$

1. with (x = 1):

$$y_4 = 10^{-3}\,(2.561 - 5.5 + 3.188 - 0.36233 - 0.0011) = -0.11443 \cdot 10^{-3} \text{ m};$$

2. with (x = 6.0): $y_B = 0.00$ (checking);
3. with (x = 6.5): $y_5 = -0.042 \cdot 10^{-3}$ m.

5 Conclusions

1. In this work, there was studied a two-support single-span cantilever beam under the action of a harmonic vibration load ($P(t) = P\sin\theta\ t$ with three point masses ($m = 0.25$ (ts^2/m)).
2. The method of calculating dynamic effects is the exact analytical method of forces; both free and forced oscillations of the beam are considered without taking into account the attenuation of the energy of motion at the circular frequency of the disturbing force ($\theta = 25.23$ (s^{-1}).
3. In a closed numerical form, a spectrum of circular frequencies of free oscillations is obtained as a system with three degrees of freedom and three forms of natural oscillations in the form of "standing"waves was constructed; the ordinates of dynamic displacements (Fig. 1, e) and bending moments (Fig. 2, d) were determined.
4. To assess the effect of elasticity of the beam supports A and B, a comparison of the results of two variants of elasticity of the supports was performed:
5. supports A and B were absolutely rigid;
6. supports A and B were elastic-pliable with the rigidity coefficient ($k = 0.87 \cdot 10^{-3} (m/t)$), which corresponds to the spring suspension of the two-axle truck (KaMAZ).
7. Comparison of the data of the two calculation options showed the following:

   1) by the nature of the diagrams, the diagrams y_i) and (M_g) differ sharply: in the elastic-pliable A and B there are deflections; with rigid supports the deflections in these supports are zero; at point 2 of the beam (Fig. 1, a) with absolutely rigid supports the bending moment is positive; with elastic-pliable supports it is negative;

   2) the difference in absolute maximum values of the ordinates of the corresponding diagrams (a beam with rigid supports relative to a beam with elastic-compliant supports):

   - by deflections (at point 2) it is 4.12 times greater;
   - by bending moments (at support B) it is 1.45 times lower;
   - by circular frequencies of free vibrations by ω_1 it is 1.13 times; by ω_2 it is 1.72 times; by ω_3 it is 2.28 times;

- for all three forms of natural vibrations, the presence of 4 half-waves with alternating signs of ordinates with the presence of zero displacement of one of the three point masses is characteristic (Fig. 3, a, b, c).

8. The theoretical provisions and applied results presented in this study can serve as a technical basis for designing and constructing load-bearing structures of the chassis of various vehicles from the point of view of their supporting devices elasticity impact on the nature of the stress-strain state of similar designs.

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