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DEVELOPMENT OF A MODEL OF EFFICIENT RESOURCE ALLOCATION IN AN OPEN THREE-SECTOR ECONOMY FOR BALANCED GROWTH

This paper is devoted to the study of efficient resource allocation in an open three-sector economy model, taking into account external and internal constraints. The aim of the study is to develop an algorithm for finding a steady state of the economic system and efficient allocation of labor and investment resources. The model is based on the Cobb-Douglas production functions, which describe output depending on capital and labor. The main attention is paid to issues of resource allocation, investment and compliance with balance equations. The use of the Lagrange multiplier method to maximize production in the consumer sector made it possible to determine the efficient values ??of the control parameters, which contributes to achieving sustainable balanced growth and equilibrium in the economic system. The scientific significance of the study lies in expanding the understanding of stable states of open three-sector economic models, which contributes to improving the allocation of resources in production systems. The article discusses existing analysis methods and algorithms with an emphasis on mathematical modeling and numerical methods for solving nonlinear programming problems. Algorithmic and software aspects of economic system modeling are also described, including software implementation in the Maple language. The work emphasizes the importance of scientific and technological progress and efficient resource allocation to achieve economic development goals.

Key words: Cobb-Douglas function, effective allocation, open three-sector economy, software package, computational algorithm.

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Теңгерімді өсу үшін ашық үш секторлы экономикада ресурстарды тиімді бөлу моделін әзірлеу

Зерттеу жұмысы сыртқы және ішкі шектеулерді ескере отырып, ашық үш секторлы экономика моделіндегі ресурстарды оңтайлы үлестіруді зерттеуге арналған. Зерттеудің мақсаты экономикалық жүйенің тұрақты күйін табу және еңбек және инвестициялық ресурстарды тиімді бөлу алгоритмін әзірлеу болып табылады. Модель капитал мен еңбекке байланысты өнім шығаруды сипаттайтын Кобб-Дугластың өндірістік функцияларына негізделген. Ресурстарды бөлуде, инвестициялар және баланстық теңдеулерді сақтау мәселелеріне баса назар аударылады. Тұтыну секторындағы өндіріс көлемін ұлғайту үшін Лагранж көбейткіштер әдісін қолдану экономикалық жүйеде тұрақты теңдестірілген өсу мен тепе-теңдікке қол жеткізуге ықпал ететін басқару параметрлерінің тиімді мәндерін анықтауға мүмкіндік берді.
Зерттеудің ғылыми маңыздылығы өндірістік жүйелердегі ресурстарды үлестіруді жетілдіруге ықпал ететін ашық үш секторлы экономикалық модельдердің тұрақты күйлерін түсінуді кеңейту болып табылады. Мақалада сызықтық емес бағдарламалау есептерін шешуге арналған математикалық модельдеуге және сандық әдістерге баса назар аудара отырып, қолданыстағы талдау әдістері мен алгоритмдері қарастырылады. Экономикалық жүйені модельдеудің алгоритмдік және бағдарламалық аспектілері, соның ішінде Maple тілінде бағдарламалық қамтамасыз етуді енгізу сипатталған. Жұмыс экономикалық даму мақсаттарына жету үшін ғылыми-техникалық прогресс пен ресурстарды тиімді үлестірудің маңыздылығын көрсетеді.

Кілттік сөздер: Кобб-Дуглас өндірістік функциясы, ашық үш секторлы экономика, бағдарламалық қамтамасыз ету, есептеу алгоритмі.

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Разработка модели эффективного распределения ресурсов в открытой трехсекторной экономике для сбалансированного роста

Данная работа посвящена исследованию эффективного распределения ресурсами в модели открытой трехсекторной экономики с учетом внешних и внутренних ограничений. Целью исследования является разработка алгоритма для нахождения стационарного состояния экономической системы и эффективного распределения трудовых и инвестиционных ресурсов. Модель основывается на производственных функциях Кобба-Дугласа, описывающих выпуск продукции в зависимости от капитала и труда. Основное внимание уделено вопросам распределения ресурсов, инвестициям и соблюдению балансовых уравнений. Применение метода множителей Лагранжа для максимизации объема производства в потребительском секторе позволило определить эффективные значения управляющих параметров, что способствует достижению устойчивого сбалансированного роста и равновесия в экономической системе. Научная значимость исследования состоит в расширении понимания стабильных состояний открытых трехсекторных экономических моделей, что способствует совершенствованию распределения ресурсов в производственных системах. В статье рассматриваются существующие методы анализа и алгоритмы с акцентом на математическое моделирование и численные методы для решения задач нелинейного программирования. Также описаны алгоритмические и программные аспекты моделирования экономической системы, включая программную реализацию на языке Maple. Работа подчеркивает важность научно-технического прогресса и эффективного распределения ресурсами для достижения целей экономического развития.

Ключевые слова: функция Кобба-Дугласа, эффективное распределение, открытая трехсекторная экономика, программный комплекс, вычислительный алгоритм.

1 Introduction

Modern economic systems are characterized by a high degree of complexity and dynamism, which requires the use of effective modeling and management methods. Optimization of resource allocation in an economy with many interconnected sectors is an important task for ensuring sustainable economic growth of nonlinear continuous systems with control constraints. Despite the existing theoretical developments, many aspects of optimal control remain insufficiently studied, which creates gaps in understanding the impact of management decisions on economic dynamics.

Numerical methods are widely used to solve optimal control problems related to cost minimization and maintaining sustainable development. Achieving success in this area requires the use of modern methods of economic modeling and the use of the latest information technologies.

The object of the study is a mathematical model of an open economic system, and the subject is algorithms for finding a stationary state of an economic system. The purpose of the work is to create an effective tool for minimizing economic costs and optimizing resource allocation between sectors. To achieve this goal, the following tasks were set: analysis of existing methods, development of a numerical algorithm, as well as its software implementation using the Maple programming language. Research methods include numerical modeling and the application of mathematical methods to solve nonlinear systems of equations.

2 Literature review

For a long time, the Cobb-Douglas production function [1] remained the main tool for assessing the production potential of both the country as a whole and its individual regions. This function occupies a significant place in economic theory, as it allows one to analyze the relationship between production factors and the volume of output. It also serves as a basis for more complex models, such as the function with constant elasticity of substitution (CES function). Today, there is an active development of the methodology for applying the Cobb-Douglas production function in two directions: by including additional factors and improving the methods for assessing its dynamic parameters [2].

The creation of economic and mathematical models for analyzing the dynamic development of industrial enterprises is becoming an increasingly urgent task of modern economic theory [3] - [5]. A successful solution to this problem provides the possibility of a more accurate analysis of the activities of enterprises, allows one to calculate the limits of the use of production resources, predict the volume of output, profits and costs, and assess the interchangeability of production factors. The paper [6] proposes a mathematical model of enterprise development taking into account both internal and external investments, with an emphasis on the development of a three-factor economic and mathematical model for analyzing dynamic development. The article [7] examines a decision support system for optimal resource management, including data collection and analysis, development of mathematical models for resource allocation and optimization of management processes. Particular attention is paid to the importance of feedback and improvement of these systems.

Previously, the authors studied a model of a three-sector closed-type economy, determined its optimal equilibrium state [8] and solved optimal control problems for this system [9] - [12].

Also, one of the pressing problems in this area is the impossibility of finding an analytical solution to optimal control problems in nonlinear economic models. This requires the use of computational methods and specialized software. An important area of research is the development of effective algorithms, the creation of software systems for their implementation and the study of optimal decision-making methods in dynamic controlled systems. The article [13] examines the features and difficulties of building software for solving optimal control problems.

3 Mathematical model of open economy

Let us consider a mathematical model of a three-sector economy, presented in absolute terms, proposed by V. A. Kolemaev [14]. According to this model, the output of each i-th sector is described by the Cobb-Douglas production function [1]:

$$X_i = F_i(K_i, L_i) = A_i K_i^{\alpha_i} L_i^{1-\alpha_i},\ i = (0, 1, 2) \tag{1}$$

(X_i is the volume of output, K_i is the volume of fixed assets, L_i is the volume of labor resources, A_i is the coefficient of neutral scientific and technological progress, α_i is the coefficient of elasticity of assets, $(1 - \alpha_i)$ is the coefficient of elasticity of labor resources).

Using some notations, we will derive a system of equations presented in the form of relative indicators for a model of an economy with three sectors:

$$\dot{k}_i = -\lambda_i k_i + \frac{s_i}{\theta_i}(x_1 + y_1), \quad k_i(0) = k_i^0, \lambda_i > 0, \quad (i = 0,\ 1,\ 2); \tag{2}$$

$$x_i = \theta_i A_i k_i^{\alpha_i}, \quad A_i > 0, \quad 0 < \alpha_i < 1 \quad (i = 0,\ 1,\ 2). \tag{3}$$

with balance equations:

$$s_0 + s_1 + s_2 = 1, \quad s_i > 0, \quad (i = 0,\ 1,\ 2), \tag{4}$$

$$\theta_0 + \theta_1 + \theta_2 = 1, \quad \theta_i > 0, \quad (i = 0,\ 1,\ 2), \tag{5}$$

$$(1 - \beta_0)x_0 = \beta_1 x_1 + \beta_2 x_2 + y_0, \quad 0 < \beta_i < 1, \quad (i = 0, 1, 2). \tag{6}$$

$$q_0 y_0 = q_1 y_1 + q_2 y_2. \tag{7}$$

$$y_1 \leq \gamma_1 x_1 \tag{8}$$

(k_0, k_1, k_2) - vectors describing the current state of the system, $(s_0, s_1, s_2, \theta_0, \theta_1, \theta_2)$ - is the vector of control, (k_0^0, k_1^0, k_2^0) - initial state of the system, y_i - specific import $i = 1, 2, y_0$ - specific export, q_0 - world price of exported materials, q_1, q_2- world prices of imported goods, γ_1- coefficient of quotas for import of investment goods.

4 Statement of the problem of finding a steady state

The purpose of the existence of the consumer sector of an open economy is to maximize the volume of production of consumer goods, since this sector receives profit exclusively from their sale. To achieve this goal, all available resources shut be distributed equally.

Thus, the problem of finding a steady state is reduced to determining the point at which the system of a three-sector economy is stabilized, and the specific output of the consumer sector $x_2 = \theta_2 A_2 k_2^{\alpha_2}$ reaches a maximum within the permissible values of the control parameters - the distribution of labor and investment resources. This leads to a nonlinear programming problem aimed at refining the model and finding a steady state of the system.

In the article, industrial safety is considered as compliance with the conditions that ensure equilibrium and stability of the system in the production environment.

$$y_1 = \gamma_1 x_1 \tag{9}$$

Then we will represent the formulation of the problem of finding the efficient distribution of resources of the economic system as follows:

$$x_2 \longrightarrow max \tag{10}$$

$$x_0 = \theta_0 A_0 k_0^{\alpha_0}, \quad x_1 = \theta_1 A_1 k_1^{\alpha_1}, \quad x_2 = \theta_2 A_2 k_2^{\alpha_2} \tag{11}$$

under the conditions:

$$s_0 + s_1 + s_2 = 1, \tag{12}$$

$$\theta_0 + \theta_1 + \theta_2 = 1, \tag{13}$$

$$(1-\beta_0)\theta_0 A_0 k_0^{\alpha_0} = (\beta_1 + \frac{q_1}{q_0}\gamma_1)\theta_1 A_1 k_1^{\alpha_1} + \beta_2\theta_2 A_2 k_2^{\alpha_2} + z_0 \tag{14}$$

where $z_0 = \dfrac{q_2}{q_0} y_2$.

Thus, the resulting simplified model allows for a more effective analysis of the stability of the economic system, since it identifies only those conditions and constraints that directly affect equilibrium and stability in the production environment.

The investment and labor balances are maintained even if the sum of the corresponding shares is slightly less than one, which may be the result of rounding or calculation errors.

5 Solution of the problem

Since we are looking for a stationary solution of the system, the capital-labor ratio of the sectors remains unchanged over time. This implies that the rate of change of the capital-labor ratio in equations (2) is zero. As a result, we obtain the following nonlinear system:

$$-\lambda_0 k_0 + \frac{s_0}{\theta_0}(1+\gamma_1)\theta_1 A_1 k_1^{\alpha_1} = 0, \tag{15}$$

$$-\lambda_1 k_1 + s_1(1+\gamma_1) A_1 k_1^{\alpha_1} = 0, \tag{16}$$

$$-\lambda_2 k_2 + \frac{s_2}{\theta_2}(1+\gamma_1)\theta_1 A_1 k_1^{\alpha_1} = 0, \tag{17}$$

From (15) - (17) we derive the values of the parameters $k_0, \quad k_1, \quad k_2$, which play a critical role in determining the dynamics of the model and take into account the limitations and dependencies between sectors:

$$k_0 = \frac{A_1}{\lambda_0}(1+\gamma_1)(\frac{A_1}{\lambda_1}(1+\gamma_1))^{\dfrac{\alpha_1}{1-\alpha_1}} \frac{s_0\theta_1}{\theta_0 s_1} s_1^{\dfrac{1}{1-\alpha_1}}, \tag{18}$$

$$k_1 = (\frac{A_1}{\lambda_1}(1+\gamma_1))^{\frac{1}{1-\alpha_1}} s_1^{\frac{1}{1-\alpha_1}}, \tag{19}$$

$$k_2 = \frac{A_1}{\lambda_2}(1+\gamma_1)(\frac{A_1}{\lambda_1}(1+\gamma_1))^{\frac{\alpha_1}{1-\alpha_1}} \frac{s_2\theta_1}{\theta_2 s_1} s_1^{\frac{1}{1-\alpha_1}}, \tag{20}$$

Let us introduce the following notation:

$$\phi_0 = \frac{\lambda_0 k_0}{(1+\gamma_1)A_1 k_1^{\alpha_1}}, \quad \phi_1 = \frac{\lambda_1 k_1^{1-\alpha_1}}{(1+\gamma_1)A_1}, \quad \phi_2 = \frac{\lambda_2 k_2}{(1+\gamma_1)A_1 k_1^{\alpha_1}}.$$

$$\sigma_0 = (1-\beta_0)A_0 k_0^{\alpha_0}, \quad \sigma_1 = (\beta_1 + \frac{q_1}{q_0}\gamma_1)A_1 k_1^{\alpha_1}, \quad \sigma_2 = \beta_2 A_2 k_2^{\alpha_2}.$$

From balance relation (14) we find θ_0:

$$\theta_0 = \frac{\sigma_1}{\sigma_0}\theta_1 + \frac{\sigma_2}{\sigma_0}\theta_2 + \frac{z_0}{\sigma_0}. \tag{21}$$

Substituting the found θ_0 into (13), we find θ_1 in the form:

$$\theta_1 = \frac{\sigma_0 - z_0 - (\sigma_2 + \sigma_0)}{(\sigma_1 + \sigma_0)\theta_2}, \tag{22}$$

then

$$\theta_0 = \frac{\sigma_1 + z_0 + (\sigma_2 - \sigma_1)\theta_2}{\sigma_0 + \sigma_1}. \tag{23}$$

Using equation (15) and the found values of θ_0 and θ_1, determine s_0:

$$s_0 = \phi_0 \frac{\sigma_1 + z_0 + (\sigma_2 - \sigma_1)\theta_2}{\sigma_0 - z_0 - (\sigma_2 + \sigma_0)\theta_2}. \tag{24}$$

Moving on to the investment balance equation (16), we can determine the expression for s_2:

$$s_2 = 1 - \phi_1 - \phi_0 \frac{\sigma_1 + z_0 + (\sigma_2 - \sigma_1)\theta_2}{\sigma_0 - z_0 - (\sigma_2 + \sigma_0)\theta_2}. \tag{25}$$

Next, substituting the found parameters s_2, θ_1 and equation (17), we express θ_2:

$$\theta_2 = \frac{(1-\phi_1)(\sigma_0 - z_0) - \phi_0(\sigma_1 + z_0)}{\phi_2(\sigma_0 + \sigma_1) + (1-\phi_1)(\sigma_0 + \sigma_2) + \phi_0(\sigma_2 + \sigma_1)}. \tag{26}$$

Maximization of the specific output of the consumer sector (10) $x_2 = \theta_2 A_2 k_2^{\alpha_2}$, we will represent as follows:

$$f(s_1) = \frac{((1-\phi_1)(\sigma_0 - z_0) - \phi_0(\sigma_1 + z_0))A_2 k_2^{\alpha_2}}{\phi_2(\sigma_0+\sigma_1) + (1-\phi_1)(\sigma_0+\sigma_2) + \phi_0(\sigma_2+\sigma_1)} \longrightarrow max \tag{27}$$

Then

$$\theta_2 = \frac{x_2}{A_2 k_2^{\alpha_2}}. \tag{28}$$

The process of finding the efficient steady state is implemented through the construction of a nonlinear function $f(s_1)$, which serves as the basis for determining all dependent parameters. The solution algorithm begins with finding the value of the parameter s_1 from the nonlinear algebraic function $f(s_1)$. The parameter s_1 varies in the range from 0 to 1 , which is illustrated in the graph (Figure 1).

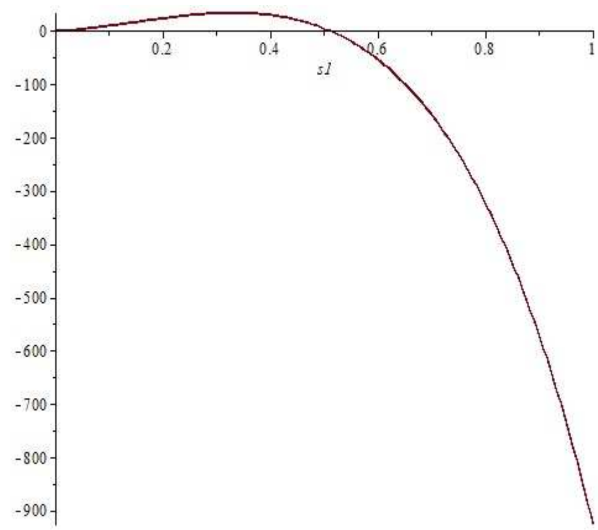

Figure 1: Graph of functions $f(s_1)$

At efficient growth for control parameters θ_i, s_i the equality relations are satisfied:

$$\frac{s_0\theta_1}{s_1\theta_0} = \frac{\alpha_0(1-\alpha_1)}{\alpha_1(1-\alpha_0)}; \qquad \frac{s_0\theta_2}{s_2\theta_0} = \frac{\alpha_0(1-\alpha_2)}{\alpha_2(1-\alpha_0)}; \qquad \frac{s_2\theta_1}{s_1\theta_2} = \frac{\alpha_2(1-\alpha_1)}{\alpha_1(1-\alpha_2)}.$$

To prove the fulfillment of efficient relations between the model parameters the Lagrange multiplier method is used. This allows taking into account the restrictions on the distribution of labor and investment resources, as well as to determine the conditions of stationarity of the system. We express the parameters θ_0, θ_1 through θ_2, and s_0, s_2 through s_1:

$$\theta_1 = h(1-\theta_2), \quad \theta_0 = (1-h)(1-\theta_2), \tag{29}$$

$$s_0 = m(1-s_1), \quad s_2 = (1-m)(1-s_1). \tag{30}$$

Control parameters: $(0 < h < 1, 0 < \theta_2 < 1, 0 < m < 1, 0 < s_1 < 1)$.

The equations describing the capital-labor ratio of sectors can be represented as follows:

$$x_0 - y_0(1-h)(1-\theta_2)(\frac{m(1-s_1)h}{(1-h)s_1})^{\alpha_0} s_1^{\overline{\frac{\alpha_0}{1-\alpha_1}}} = 0, \tag{31}$$

$$x_1 - y_1 h(1-\theta_2) s_1^{\overline{\frac{\alpha_1}{1-\alpha_1}}} = 0, \tag{32}$$

$$x_2 - y_2\theta_2(\frac{(1-m)(1-s_1)h(1-\theta_2)}{\theta_2 s_1})^{\alpha_2} s_1^{\overline{\frac{\alpha_2}{1-\alpha_1}}} = 0. \tag{33}$$

where

$$b_0 = \frac{A_0(A_1(1+\gamma_1))^{\overline{\frac{\alpha_0}{1-\alpha_1}}}}{\lambda_0^{\alpha_0}\lambda_1^{\overline{\frac{\alpha_0\alpha_1}{1-\alpha_1}}}}, \quad b_1 = \frac{(A_1(1+\gamma_1))^{\overline{\frac{1}{1-\alpha_1}}}}{\lambda_1^{\overline{\frac{1}{1-\alpha_1}}}}, \quad b_2 = \frac{A_2(A_1(1+\gamma_1))^{\overline{\frac{\alpha_2}{1-\alpha_1}}}}{\lambda_2^{\alpha_2}\lambda_1^{\overline{\frac{\alpha_2\alpha_1}{1-\alpha_1}}}}.$$

The classical the Lagrange function for this problem is written as follows:

$$L(\theta, s, x, c) = -x_2 + c_1((1-\beta_0)x_0 - (\beta_1 + \frac{q_1}{q_0}\gamma_1)x_1 - \beta_2 x_2 - z_0) +$$

$$+c_2(x_0 - y_0(1-h)(1-\theta_2)(\frac{m(1-s_1)h}{(1-h)s_1})^{\alpha_0} s_1^{\overline{\frac{\alpha_0}{1-\alpha_1}}}) + c_3(x_1 - y_1 h(1-\theta_2)s_1^{\overline{\frac{\alpha_1}{1-\alpha_1}}}) +$$

$$+c_4(x_2 - y_2\theta_2(\frac{(1-m)(1-s_1)h(1-\theta_2)}{\theta_2 s_1})^{\alpha_2} s_1^{\overline{\frac{\alpha_2}{1-\alpha_1}}}) \tag{34}$$

where, c_1, c_2, c_3, c_4 are the Lagrange multipliers. Applying the first-order conditions for the extremum of the function, we formulate the stationarity condition:

$$\frac{\partial L}{(\partial\theta_2)} = 0, \frac{\partial L}{(\partial s_1)} = 0, \frac{\partial L}{\partial h} = 0, \frac{\partial L}{\partial m} = 0.$$

$$\frac{\partial L}{(\partial\theta_2)} = c_2 x_0 + c_3 x_1 - c_4 x_2 \frac{1-\theta_2-\alpha_2}{\theta_2} = 0, \tag{35}$$

$$\frac{\partial L}{(\partial s_1)} = -(\frac{1}{1-\alpha_1} - \frac{1}{1-s_1})(\alpha_0 c_2 x_0 + \alpha_2 c_4 x_2) - \frac{\alpha_1}{1-\alpha_1} c_3 x_1 = 0, \tag{36}$$

$$\frac{\partial L}{\partial h} = \frac{h-\alpha_0}{1-h}c_2x_0 - c_3x_1 - \alpha_2c_4x_2 = 0, \tag{37}$$

$$\frac{\partial L}{\partial m} = -\frac{\alpha_0}{m}c_2x_0 + \frac{\alpha_2}{1-m}c_4x_2 = 0. \tag{38}$$

In equations (35) - (38), the parameters obtained from equations (31) - (33) are taken into account.

$$y_0\left(\frac{m(1-s_1)h}{(1-h)s_1}\right)^{\alpha_0} s_1^{\frac{\alpha_0}{1-\alpha_1}} = \frac{x_0}{(1-h)(1-\theta_2)},$$

$$y_1 s_1^{\frac{\alpha_1}{1-\alpha_1}} = \frac{x_1}{h(1-\theta_2)},$$

$$y_2\left(\frac{(1-m)(1-s_1)h(1-\theta_2)}{\theta_2 s_1}\right)^{\alpha_2} s_1^{\frac{\alpha_2}{1-\alpha_1}} = \frac{x_2}{\theta_2}.$$

From (35) - (38) we find s_1, θ_2, h, m:

$$\theta_2 = \frac{(1-\alpha_2)c_4x_2}{c_3x_1 + c_2x_0 + c_4x_2},$$

$$s_1 = \frac{(c_3x_1 + \alpha_0c_2x_0 + \alpha_2c_4x_2)\alpha_1}{\alpha_0c_2x_0 + \alpha_1c_3x_1 + \alpha_2c_4x_2},$$

$$h = \frac{c_3x_1 + \alpha_0c_2x_0 + \alpha_2c_4x_2}{c_3x_1 + c_2x_0 + \alpha_2c_4x_2},$$

$$m = \frac{\alpha_0c_2x_0}{\alpha_0c_2x_0 + \alpha_2c_4x_2}.$$

From the indicated relations

$$\frac{s_0\theta_1}{\theta_0 s_1} = \frac{m(1-s_1)h}{(1-h)s_1} \quad \textit{and} \quad \frac{s_2\theta_1}{\theta_2 s_1} = \frac{(1-m)(1-s_1)h(1-\theta_2}{\theta_2 s_1}.$$

we can obtain the following expressions:

$$\frac{m(1-s_1)h}{(1-h)s_1} = \frac{\alpha_0c_2x_0(1-\alpha_1)(\alpha_0c_2x_0+\alpha_2c_4x_2)(c_3x_1+\alpha_0c_2x_0+\alpha_2c_4x_2)}{(\alpha_0c_2x_0+\alpha_2c_4x_2)(c_3x_1+\alpha_0c_2x_0+\alpha_2c_4x_2)\alpha_1(1-\alpha_0)c_2x_0} = \frac{\alpha_0(1-\alpha_1)}{\alpha_1(1-\alpha_0)},$$

$$\frac{(1-m)(1-s_1)h(1-\theta_2}{\theta_2 s_1} =$$

$$= \frac{\alpha_2c_4x_2(1-\alpha_1)(\alpha_0c_2x_0+\alpha_2c_4x_2)(c_3x_1+c_2x_0+\alpha_2c_4x_2)(c_3x_1+\alpha_0c_2x_0+\alpha_2c_4x_2)}{(\alpha_0c_2x_0+\alpha_2c_4x_2)(1-\alpha_2)c_4x_2(c_3x_1+\alpha_0c_2x_0+\alpha_2c_4x_2)\alpha_1(c_3x_1+c_2x_0+\alpha_2c_4x_2)} =$$

$$= \frac{\alpha_2(1-\alpha_1)}{\alpha_1(1-\alpha_2)}.$$

6 Results and discussion

The efficient stationary state in the context of the open economy problem was studied by A.V. Kolemaev [14]. In his study, he proposed a three-sector model, with the help of which he identified the stationary distribution of resources in the economy. In the work [14], the classical the Lagrange method was used to find the efficient distribution of labor resources. Table 1 presents the parameters of the model on which the solution obtained by A.V. Kolemaev is based.

Table 1: Parameter data and solutions found by V.A. Kolemaev

i	0	1	2
Model coefficients			
β_i	0,46	0,68	0,49
α_i	0,39	0,29	0,52
λ_i	0.05	0.05	0.05
A_i	6,19	1,35	2,71
γ_i	-	0,5	-
Solution			
θ_i	0,37	0,08	0,55
s_i	0,32	0,16	0,52
k_i	98,3	337	108
x_i	22,1	4,0	17,5

Based on the described method, a solution was obtained for the problem presented in Table 1 that determines the efficient distribution of resources and the stationary state of the economic system. The results obtained are summarized in Table 2.

Table 2: Results of calculating the effective steady state of an open economic system using the proposed method

i	0	1	2
θ_i	0,49	0,17	0,34
s_i	0,37	0,33	0,29
k_i	1361.26	3395.74	1535.33
x_i	82.95	59.16	33,68

Let us compare the results presented in Table 1 with the results obtained by the proposed method, which are reflected in Table 2. It is important to note that the distribution of labor resources found by this method has a high degree of correspondence with the results of Kolemaev. This confirms the correctness of the approach, showing that it allows us to determine with high accuracy the efficient distribution of labor resources for the given conditions.

Differences in the distribution of investment resources and capital-labor ratio by sector may be associated with differences in approaches to accounting for the investment balance and the capital-labor ratio model.

7 Conclusion

The article studies the problem of efficient resource allocation for a nonlinear system representing an economic model. The paper proposes an original approach based on the classical the Lagrange method, which takes into account constraints on control parameters. This method allows for a more accurate analysis of resource allocation in a changing investment climate, expanding the capabilities of the model for forecasting the dynamics of an economic system.

The results obtained using the proposed approach confirm its effectiveness for finding the efficient resource allocation and determining the steady state of the system. The computational experiments demonstrated the efficiency and reliability of the algorithm.

In a further study, we consider the control problem for a three-sector economic structure, where the dynamic equations are described by a system of three ordinary differential equations with state-dependent coefficients. The goal of the problem is to transfer a nonlinear system from a given initial state to a small neighborhood of the equilibrium state on an infinite time interval. A model will be constructed that describes the control of a nonlinear system, linear in state and control, with state-dependent coefficients. The second Lyapunov method will be used to establish the asymptotic stability of the closed system.

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