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ON A SUBSET OF BAZILEVIČ FUNCTIONS IDENTIFIED BY THE THREE-LEAF FUNCTION, MILLER-ROSS FUNCTION, AND MULTIPLIER OPERATORS

A significant portion of the collection of analytic-univalent functions of the type

$$h(\zeta) = \zeta + \sum_{n=rm+1}^{\infty} a_n \zeta^n$$

whose definition is found in the unit disk

$$\Omega := \{z : |z| < 1\},$$

is investigated in this work. Several subsets of the well-known set of Bazilevič functions are included in this new set. The new set and its findings are developed using the Miller-Ross function, the Schwarz function, some multiplier operators, and some mathematical ideas such as subordination, set theory, infinite series generation, and convolution of some geometric expressions. Among the main achievements are the estimates for the coefficient bounds and the Fekete-Szegö functional. Generally speaking, the new set reduces to a number of known subsets with some supposedly unique results when some parameters are altered inside their declaration intervals.

Key words: analytic function, Miller-Ross function, Schwarz function, Bazilevič function, multiplier operator, three-leaf-type function.

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Үш деңгейлі функция, Миллер-Росс функциясы және көбейткіш операторлары арқылы анықталған Базилевич функцияларының ішкі жиынында

Типтің аналитикалық бір мәнді функцияларын жинаудың маңызды бөлігі

$$h(\zeta) = \zeta + \sum_{n=rm+1}^{\infty} a_n \zeta^n$$

оның анықтамасы

$$\Omega := \{z : |z| < 1\},$$

бірлік дискісінде осы жұмыста зерттеледі. Базилевич функцияларының белгілі жиынының бірнеше ішкі жиындары осы жаңа жиынға енгізілген. Жаңа жиын және оның нәтижелері Миллер-Росс функциясын, Шварц функциясын, кейбір көбейткіш операторларды және бағыну, жиын теориясы, шексіз қатарларды генерациялау және кейбір геометриялық өрнектерді айналдыру сияқты кейбір математикалық идеяларды пайдалана отырып әзірленді. Негізгі жетістіктердің қатарында коэффициенттер мен Фекете-Сего функционалдың байланыстырылған бағалаулар бар.

Жалпы айтқанда, жаңа жиын кейбір параметрлер жариялау аралықтарында әзгерген кезде кейбір болжамды бірегей нәтижелері бар белгілі ішкі жиындар санына дейін азаяды.
Түйін сөздер: аналитикалық функция, Миллер-Росс функциясы, Шварц функциясы, Басилевич функциясы, көбейту операторы, үш валентті функция, қосылу моделі.

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О подмножестве функций Базилевича, идентифицируемых трехлестной функцией, функцией Миллера-Росса и операторами множителей

Значительная часть коллекции аналитически-однозначных функций типа

$$h(\zeta) = \zeta + \sum_{n=rm+1}^{\infty} a_n \zeta^n$$

определение которых находится в единичном круге

$$\Omega := \{z : |z| < 1\},$$

исследуется в этой работе. Несколько подмножеств известного набора функций Базилевича включены в этот новый набор. Новый набор и его результаты разрабатываются с использованием функции Миллера-Росса, функции Шварца, некоторых операторов множителей и некоторых математических идей, таких как подчинение, теория множеств, генерация бесконечных рядов и свертка некоторых геометрических выражений. Среди основных достижений — оценки границ коэффициентов и функционал Фекете-Сего. Вообще говоря, новый набор сводится к ряду известных подмножеств с некоторыми предположительно уникальными результатами, когда некоторые параметры изменяются внутри их интервалов объявления.
Ключевые слова: аналитическая функция, функция Миллера-Росса, функция Шварца, функция Базилевича, оператор умножения, функция трехлистного типа.

1 Preliminary

In this study, the set of analytic functions of the series type

$$h(\zeta) = \zeta + \sum_{n=2}^{\infty} a_n \zeta^n \quad (\zeta \in \Omega := \{\zeta \in \mathbb{C} : |\zeta| < 1\}). \tag{1}$$

is represented by $\mathcal{A}$. The nature of this function agrees with the fact that $h(0) = 0 = h'(0) - 1$. One of the fundamental principles in geometric function theory is the subordination principle. The principle states that if we have two analytic functions $h(\zeta)$ in (1) and

$$H(\zeta) = \zeta + \sum_{n=2}^{\infty} b_n \zeta^n \quad (\zeta \in \Omega), \tag{2}$$

then h is subordinate to H (usually expressed in notational $h \prec H$) if there is another analytic function

$$d(\zeta) = \sum_{n=1}^{\infty} d_n \zeta^n \in \mathit{\Delta} \quad (\zeta \in \Omega) \tag{3}$$

such that $|d(\zeta)| < 1$, $d(0) = 0$, and

$$h(\zeta) = H(d(\zeta)) \quad (\zeta \in \Omega).$$

Suppose H is also univalent in Ω, then the definition improves to say that

$$h \prec H \iff h(0) = H(0) \quad \text{and} \quad h(\Omega) \subset H(\Omega).$$

The Hadamard product (or convolution) of two functions h in (1) and H in (2) is the third analytic function declared as

$$(h \star H)(\zeta) = (H \star h)(\zeta) = z + \sum_{n=2}^{\infty} (a_n \times b_n)\zeta^n \quad (\zeta \in \Omega).$$

1.1 Bazilevič Functions

An analytic functions of the integral type

$$b(z) = \left\{ (\eta + i\gamma) \int_0^z \rho(\tau) s(\tau)^\gamma \tau^{-(1-i\gamma)} d\tau \right\}^{\frac{1}{\eta+i\gamma}} \tag{4}$$

such that $\eta > 0$, γ have real value, s is a starlike function, and $\rho \in \wp$ are called Bazilevič [2] functions. This set was shown to be the 'largest' subset of the set of univalent functions that is currently known. Numerous scholars have examined different properties of the subsets of the set of Bazilevič functions by varying the parameters in (4); for instance, see [8, 9, 17, 18].

Indeed, an important subset of analytic functions are Bazilevič functions. These functions have been thoroughly examined in a large body of research and are distinguished by their geometric features. Olukoya and Oyekan's work in [12] is noteworthy since it offers some polynomial bounds for functions in the set of modified hyperbolic tangent functions. The behavior of the various analytic functions in certain subsets of the Bazilevič functions was better understood as a result of these findings.

Another related study involved some results on Chebyshev polynomial bounds for sets of analytic-univalent functions, presented in [14]. The work extends the understanding of the geometric properties of Bazilevič functions, shedding more light on their analytical characteristics. Additionally, Oyekan and Awolere [15] explored the polynomial bounds for bi-univalent functions associated with the probability of generalized distribution defined by generalized polylogarithms via Chebyshev polynomial.

Gandhi [3] presented the analytic function

$$3\ell(\zeta) = 1 + \frac{4}{5}\zeta + \frac{1}{5}\zeta^4 \tag{5}$$

in 2020 called a '*three-leaf-type function*' and studied the analytical properties of a certain set of starlike functions defined by the conditions

$$z\frac{h'(z)}{h(z)} \prec 3\ell(z) \quad (\zeta \in \Omega)$$

so that by using (3) in (5), we get

$$3\ell(d(\zeta)) = 1 + \frac{4}{5}d(\zeta) + \frac{1}{5}(d(\zeta))^4$$

to give

$$3\ell(d(\zeta)) = 1 + \frac{2}{5}d_1\zeta + \left(\frac{2}{5}d_2 - \frac{1}{5}d_1^2\right)\zeta^2 + \cdots . \tag{6}$$

1.2 Some Analytic Functions and Operators

In 1993, Miller and Ross [11, p. 88] introduced the special function

$$E_{c,v}(\zeta) = \sum_{n=0}^{\infty} \frac{c^n}{\Gamma(n+v+1)}\zeta^{n+v} \quad (c, v, \zeta \in \mathbb{C}).$$

This special function is famously called Miller-Ross function. The Miller-Rose function is a generalization of many special functions, see [6]. The work of Eker and Ece [5, Eq. 2] introduced the normalized form of $E_{c,v}$ defined by

$$\widetilde{E}_{c,v}(\zeta) = \zeta + \sum_{n=2}^{\infty} \frac{c^{n-1}\Gamma(v+1)}{\Gamma(v+n)}\zeta^n \quad (v > -1;\ c, \zeta \in \Omega). \tag{7}$$

For $h \in \mathcal{A}$ of type (1), the multiplier operator $I^{\delta}_{\eta_1,\eta_2}$ that maps $\mathcal{A}$ to $\mathcal{A}$ and introduced by Hameed *et al.* [7] was defined on h as

$$I^{\delta}_{\varsigma_1,\varsigma_2}h(\zeta) = \zeta + \sum_{n=2}^{\infty} \left(\frac{1 + (\varsigma_1 + \varsigma_2)(l-1)}{1 + \varsigma_2(l-1)}\right)^{\delta} a_n\zeta^n \quad (\zeta \in \Omega) \tag{8}$$

for the parameters: $\delta \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}$, $l \geqq 1$, and $0 \leqq \varsigma_1 \leqq \varsigma_2$. More so, Oyekan [13] investigated the operator $\mathcal{D}^{\delta}$ that maps $\mathcal{A}$ to $\mathcal{A}$ by

$$\mathcal{D}^{\delta}h(\zeta) = \zeta + \sum_{n=2}^{\infty} \mathscr{L}_{\delta}^{\delta+n-1} a_n\zeta^n \quad (\zeta \in \Omega) \tag{9}$$

where

$$\mathscr{L}_{\delta}^{\delta+n-1} = \binom{\delta+n-1}{\delta} = \frac{(\delta+1)(\delta+2)\cdots(\delta+n-1)}{(n-1)!} = \frac{(\delta+1)_{n-1}}{(1)_{n-1}}. \tag{10}$$

In 2023, Oyekan [13] modified function $h \in \mathcal{A}$ and showcased the analytic function $\mathfrak{h} \in \mathcal{A}(r, m)$ as

$$\mathfrak{h}(\zeta) = \zeta + \sum_{n=rm+1}^{\infty} a_n\zeta^n \quad (\zeta \in \Omega) \tag{11}$$

where r is fixed and $r, m \in \mathbb{N}$. Observe that if $r = 1 = m$ in (11), then $\mathfrak{h} = h$ in (1) and if $r = 1$ (or $m = 1$), then we will have the function studied by Hameed *et al.* [7]. Now in the likes of (11), we have (7) expressed in the form

$$\mathcal{E}_{c,v}(\zeta) = \zeta + \sum_{n=rm+1}^{\infty} \frac{c^{n-1}\Gamma(v+1)}{\Gamma(v+n)}\zeta^n,$$

the multiplier operator (8) defined as

$$\mathcal{I}^{\delta}_{\varsigma_1,\varsigma_2}\mathfrak{h}(\zeta) = \zeta + \sum_{n=rm+1}^{\infty} \left(\frac{1 + (\varsigma_1 + \varsigma_2)(l-1)}{1 + \varsigma_2(l-1)}\right)^{\delta} a_n\zeta^n \quad (\zeta \in \Omega).$$

and the multiplier operator (9) modified as

$$\mathcal{D}^{\delta}\mathfrak{h}(\zeta) = \zeta + \sum_{n=rm+1}^{\infty} \mathscr{L}_{\delta}^{\delta+n-1} a_n\zeta^n \quad (\zeta \in \Omega)$$

where $\mathscr{L}_{\delta}^{\delta+n-1}$ is as defined by (10). Likewise, for $\mathfrak{h}$ of the series type (11), Hameed *et al.* [7] studied the operator $\mathcal{R}^{\delta}_{\varsigma_1,\varsigma_2}$ that maps $\mathcal{A}(r,m)$ to $\mathcal{A}(r,m)$ and defined by

$$\mathcal{R}^{\delta}_{\varsigma_1,\varsigma_2}\mathfrak{h}(\zeta) = \mathcal{I}^{\delta}_{\varsigma_1,\varsigma_2}\mathfrak{h}(\zeta) \star \mathcal{D}^{\delta}\mathfrak{h}(\zeta) = \zeta + \sum_{n=rm+1}^{\infty} \mathscr{L}_{\delta}^{\delta+n-1}\left(\frac{1 + (\varsigma_1 + \varsigma_2)(l-1)}{1 + \varsigma_2(l-1)}\right)^{\delta} a_n\zeta^n \quad (\zeta \in \Omega).$$

Now, we declare the analytic function

$$\begin{aligned} \mathcal{J}^{\delta,c,v}_{\varsigma_1,\varsigma_2}(\zeta) &= \mathcal{E}_{c,v}(\zeta) \star \mathcal{R}^{\delta}_{\varsigma_1,\varsigma_2}\mathfrak{h}(\zeta) \\ &= \zeta + \sum_{n=rm+1}^{\infty} \frac{c^{n-1}\Gamma(v+1)}{\Gamma(v+n)}\mathscr{L}_{\delta}^{\delta+n-1}\left(\frac{1 + (\varsigma_1 + \varsigma_2)(l-1)}{1 + \varsigma_2(l-1)}\right)^{\delta} a_n\zeta^n \quad (\zeta \in \Omega) \end{aligned} \tag{12}$$

where all parameters are as aforementioned.

2 A Set of Lemmas

Let the function d be as defined in (3), then the following lemmas hold for the main results.

Lemma 2.1 ([4]) *Let $d \in \Delta$, then $|d_n| \leqq 1, \ \forall n \in \mathbb{N}$. Equality occurs for functions $d(\zeta) = e^{i\vartheta}\zeta^n$ $(\vartheta \in [0, 2\pi))$.*

Lemma 2.2 ([1]) *Let $d \in \Delta$, then for complex number ξ,*

$$|d_2 - \xi d_1^2| \leqq \max\{1; \ |\xi|\}.$$

Equality holds for functions $d(z) = \zeta$ or $d(z) = \zeta^2$.

3 Main Results

3.1 A Novel Class of Analytic Functions

Definition 3.1 *A function $\mathfrak{h} \in \mathcal{A}(r,m)$ of the form* (11) *belongs to the set $\nabla^{\delta,c,v}_{\sigma,\varsigma_1,\varsigma_2}(3\ell)$, if it satisfies the subordination condition*

$$\frac{\left(\mathcal{J}^{\delta,c,v}_{\varsigma_1,\varsigma_2}(\zeta)\right)'\left(\mathcal{J}^{\delta,c,v}_{\varsigma_1,\varsigma_2}(\zeta)\right)^{\sigma-1}}{\zeta^{\sigma-1}} \prec 3\ell(\zeta) \tag{13}$$

where we declare the parameters: $c, \zeta \in \Omega$, $\delta \in \mathbb{N}_0$, $v > -1$, $0 \leqq \varsigma_1 \leqq \varsigma_2$, and $0 \leqq \sigma \leqq 1$, for functions $\mathcal{J}^{\delta,c,v}_{\varsigma_1,\varsigma_2}(\zeta)$ and $3\ell(\zeta)$ defined in (12) *and* (5)*, respectively.*

This study aims to explore and analyze a subset of Bazilevič functions characterized by the Miller-Ross function and a certain multiplier operator in the space of a three-leaf function, a nuanced area in geometric function theory. Some achieved results are the upper estimates for $|a_{m+1}|$, $|a_{2m+1}|$, and $|a_{2m+1} - \xi a^2_{m+1}|$ functionals; see [9, 10, 12–14, 16] for some details on bounds. Several (presumably) new results are reported as corollaries and remarks.

3.2 Coefficient Estimates

Theorem 3.2 *Let $\mathfrak{h} \in \mathcal{A}(r,m)$ belong to the set $\nabla^{\delta,c,v}_{\sigma,\varsigma_1,\varsigma_2}(3\ell)$. Then*

$$|a_{m+1}| \leqq \frac{2}{5(\sigma+m)\frac{c^m\Gamma(v+1)}{\Gamma(v+m+1)}\mathscr{L}^{\delta+m}_{\delta}\left(\frac{1+(\varsigma_1+\varsigma_2)(l-1)}{1+\varsigma_2(l-1)}\right)^{\delta}} \tag{14}$$

and

$$\begin{aligned}|a_{2m+1}| \leqq & \frac{1}{5(\sigma+2m)\frac{c^{2m}\Gamma(v+1)}{\Gamma(v+2m+1)}\mathscr{L}^{\delta+2m}_{\delta}\left(\frac{1+(\varsigma_1+\varsigma_2)(l-1)}{1+\varsigma_2(l-1)}\right)^{\delta}} \\ & + \frac{4}{25(\sigma+m)(\sigma+2m)\frac{c^{2m}\Gamma(v+1)}{\Gamma(v+2m+1)}\mathscr{L}^{\delta+2m}_{\delta}\left(\frac{1+(\varsigma_1+\varsigma_2)(l-1)}{1+\varsigma_2(l-1)}\right)^{\delta}} \\ & + \frac{4\sigma(m+1)}{25(\sigma+m)^2(\sigma+2m)\frac{c^{2m}\Gamma(v+1)}{\Gamma(v+2m+1)}\mathscr{L}^{\delta+2m}_{\delta}\left(\frac{1+(\varsigma_1+\varsigma_2)(l-1)}{1+\varsigma_2(l-1)}\right)^{\delta}} \\ & + \frac{2\sigma(\sigma-1)}{25(\sigma+m)^2(\sigma+2m)\frac{c^{2m}\Gamma(v+1)}{\Gamma(v+2m+1)}\mathscr{L}^{\delta+m}_{\delta}\mathscr{L}^{\delta+2m}_{\delta}\left(\frac{1+(\varsigma_1+\varsigma_2)(l-1)}{1+\varsigma_2(l-1)}\right)^{\delta}}.\end{aligned} \tag{15}$$

Proof. Since $\mathfrak{h} \in \nabla^{\delta,c,v}_{\sigma,\varsigma_1,\varsigma_2}(3\ell)$, then by the principle of subordination, we can express (13) such that

$$\frac{\left(\mathcal{J}^{\delta,c,v}_{\varsigma_1,\varsigma_2}(\zeta)\right)'\left(\mathcal{J}^{\delta,c,v}_{\varsigma_1,\varsigma_2}(\zeta)\right)^{\sigma-1}}{\zeta^{\sigma-1}} = 1 + \frac{4}{5}\omega(\zeta) + \frac{1}{5}(\omega(\zeta))^4$$

or in a simplified form

$$\frac{\zeta\left(\mathcal{J}^{\delta,c,v}_{\varsigma_1,\varsigma_2}(\zeta)\right)'\left(\frac{\mathcal{J}^{\delta,c,v}_{\varsigma_1,\varsigma_2}(\zeta)}{\zeta}\right)^{\sigma}}{\mathcal{J}^{\delta,c,v}_{\varsigma_1,\varsigma_2}(\zeta)}=1+\frac{4}{5}\omega(\zeta)+\frac{1}{5}(\omega(\zeta))^4$$

and

$$\zeta\left(\mathcal{J}^{\delta,c,v}_{\varsigma_1,\varsigma_2}(\zeta)\right)'\left(\frac{\mathcal{J}^{\delta,c,v}_{\varsigma_1,\varsigma_2}(\zeta)}{\zeta}\right)^{\sigma}=\left(1+\frac{4}{5}\omega(\zeta)+\frac{1}{5}(\omega(\zeta))^4\right)\left(\mathcal{J}^{\delta,c,v}_{\varsigma_1,\varsigma_2}(\zeta)\right). \tag{16}$$

Putting (3) and (12) into (16) with some simplifications yields

$$\begin{aligned}&\left(\zeta+(rm+1)\frac{c^{rm}\Gamma(v+1)}{\Gamma(v+rm+1)}\mathscr{L}^{\delta+rm}_{\delta}\left(\frac{1+(\varsigma_1+\varsigma_2)(l-1)}{1+\varsigma_2(l-1)}\right)^{\delta}a_{rm+1}\zeta^{rm+1}\right)\\&\quad\times\Bigg\{1+\sigma\frac{c^{rm}\Gamma(v+1)}{\Gamma(v+rm+1)}\mathscr{L}^{\delta+rm}_{\delta}\left(\frac{1+(\varsigma_1+\varsigma_2)(l-1)}{1+\varsigma_2(l-1)}\right)^{\delta}a_{rm+1}\zeta^{rm}\\&\quad+\frac{\sigma(\sigma-1)}{2}\frac{c^{2rm}\left[\Gamma(v+1)\right]^2}{\left[\Gamma(v+rm+1)\right]^2}\mathscr{L}^{\delta+rm}_{2\delta}\left(\frac{1+(\varsigma_1+\varsigma_2)(l-1)}{1+\varsigma_2(l-1)}\right)^{2\delta}b^2_{rm+1}\zeta^{2rm}+\cdots\Bigg\}\\&=\left(1+\frac{2}{5}d_1\zeta+[\frac{2}{5}d_2-\frac{1}{5}d_1^2]\zeta^2+\cdots\right)\\&\quad\times\left(\zeta+\frac{c^{rm}\Gamma(v+1)}{\Gamma(v+rm+1)}\mathscr{L}^{\delta+rm}_{\delta}\left(\frac{1+(\varsigma_1+\varsigma_2)(l-1)}{1+\varsigma_2(l-1)}\right)^{\delta}a_{rm+1}\zeta^{rm+1}\right).\end{aligned}$$

Now, for $r\in\{1,2,3,\ldots\}$ we have a simplified series

$$\begin{aligned}&\zeta+(\sigma+m+1)\frac{c^{m}\Gamma(v+1)}{\Gamma(v+m+1)}\mathscr{L}^{\delta+m}_{\delta}\left(\frac{1+(\varsigma_1+\varsigma_2)(l-1)}{1+\varsigma_2(l-1)}\right)^{\delta}a_{m+1}\zeta^{m+1}\\&\quad+\Bigg[\sigma(m+1)\frac{c^{2m}\left[\Gamma(v+1)\right]^2}{\left[\Gamma(v+m+1)\right]^2}\mathscr{L}^{\delta+m}_{2\delta}\left(\frac{1+(\varsigma_1+\varsigma_2)(l-1)}{1+\varsigma_2(l-1)}\right)^{2\delta}b^2_{m+1}\\&\quad+\frac{\sigma(\sigma-1)}{2}\frac{c^{2m}\left[\Gamma(v+1)\right]^2}{\left[\Gamma(v+m+1)\right]^2}\mathscr{L}^{\phi+m}_{2\phi}\left(\frac{1+(\varsigma_1+\varsigma_2)(l-1)}{1+\varsigma_2(l-1)}\right)^{2\delta}b^2_{m+1}\\&\quad+(\sigma+2m+1)\frac{c^{2m}\Gamma(v+1)}{\Gamma(v+2m+1)}\mathscr{L}^{\delta+2m}_{\delta}\left(\frac{1+(\varsigma_1+\varsigma_2)(l-1)}{1+\varsigma_2(l-1)}\right)^{\delta}a_{2m+1}\Bigg]\zeta^{2m+1}+\cdots\\&=\zeta+\frac{2}{5}d_1\zeta^2+\frac{c^{m}\Gamma(v+1)}{\Gamma(v+m+1)}\mathscr{L}^{\delta+m}_{\delta}\left(\frac{1+(\varsigma_1+\varsigma_2)(l-1)}{1+\varsigma_2(l-1)}\right)^{\delta}a_{m+1}\zeta^{m+1}\\&\quad+\left(\frac{2}{5}d_2-\frac{1}{5}d_1^2\right)\zeta^3+\frac{2}{5}d_1\frac{c^{m}\Gamma(v+1)}{\Gamma(v+m+1)}\mathscr{L}^{\delta+m}_{\delta}\left(\frac{1+(\varsigma_1+\varsigma_2)(l-1)}{1+\varsigma_2(l-1)}\right)^{\delta}a_{m+1}\zeta^{m+2}\\&\quad+\cdots+\frac{c^{2m}\Gamma(v+1)}{\Gamma(v+2m+1)}\mathscr{L}^{\delta+2m}_{\delta}\left(\frac{1+(\varsigma_1+\varsigma_2)(l-1)}{1+\varsigma_2(l-1)}\right)^{\delta}a_{2m+1}\zeta^{2m+1}+\cdots.\end{aligned}\tag{17}$$

Clearly, taking the first corresponding coefficients in (17) shows that

$$(\sigma+m+1)\frac{c^{m}\Gamma(v+1)}{\Gamma(v+m+1)}\mathscr{L}_{\delta}^{\delta+m}\left(\frac{1+(\varsigma_1+\varsigma_2)\,(l-1)}{1+\varsigma_2(l-1)}\right)^{\delta}a_{m+1} = \frac{2}{5}d_1+\frac{c^{m}\Gamma(v+1)}{\Gamma(v+m+1)}\mathscr{L}_{\delta}^{\delta+m}\left(\frac{1+(\varsigma_1+\varsigma_2)\,(l-1)}{1+\varsigma_2(l-1)}\right)^{\delta}a_{m+1}$$

so that

$$a_{m+1}=\frac{2d_1}{5(\sigma+m)\frac{c^{m}\Gamma(v+1)}{\Gamma(v+m+1)}\mathscr{L}_{\delta}^{\delta+m}\left(\frac{1+(\varsigma_1+\varsigma_2)(l-1)}{1+\varsigma_2(l-1)}\right)^{\delta}} \tag{18}$$

and

$$|a_{m+1}|\leqq\frac{2|d_1|}{5(\sigma+m)\frac{c^{m}\Gamma(v+1)}{\Gamma(v+m+1)}\mathscr{L}_{\delta}^{\delta+m}\left(\frac{1+(\varsigma_1+\varsigma_2)(l-1)}{1+\varsigma_2(l-1)}\right)^{\delta}}$$

so that applying Lemma 2.1 gives the result in (14). Next, taking the second corresponding coefficients in (17) shows that

$$\begin{aligned}\sigma(m+1)&\frac{c^{2m}\left[\Gamma(v+1)\right]^2}{\left[\Gamma(v+m+1)\right]^2}\mathscr{L}_{2\delta}^{\delta+m}\left(\frac{1+(\varsigma_1+\varsigma_2)\,(l-1)}{1+\varsigma_2(l-1)}\right)^{2\delta}b_{m+1}^2\\&+\frac{\sigma(\sigma-1)}{2}\frac{c^{2m}\left[\Gamma(v+1)\right]^2}{\left[\Gamma(v+m+1)\right]^2}\mathscr{L}_{2\delta}^{\delta+m}\left(\frac{1+(\varsigma_1+\varsigma_2)\,(l-1)}{1+\varsigma_2(l-1)}\right)^{2\delta}b_{m+1}^2\\&+(\sigma+2m)\frac{c^{2m}\Gamma(v+1)}{\Gamma(v+2m+1)}\mathscr{L}_{\delta}^{\delta+2m}\left(\frac{1+(\varsigma_1+\varsigma_2)\,(l-1)}{1+\varsigma_2(l-1)}\right)^{\delta}a_{2m+1}+\cdots\\=&\left(\frac{2}{5}d_2-\frac{1}{5}d_1^2\right)+\frac{2}{5}d_1\frac{c^{m}\Gamma(v+1)}{\Gamma(v+m+1)}\mathscr{L}_{\phi}^{\delta+m}\left(\frac{1+(\varsigma_1+\varsigma_2)\,(l-1)}{1+\varsigma_2(l-1)}\right)^{\delta}a_{m+1}+\cdots\end{aligned}$$

so that

$$\begin{aligned}a_{2m+1}=&\frac{\frac{2}{5}d_2-\frac{1}{5}d_1^2}{(\sigma+2m)\frac{c^{2m}\Gamma(v+1)}{\Gamma(v+2m+1)}\mathscr{L}_{\delta}^{\phi+2m}\left(\frac{1+(\varsigma_1+\varsigma_2)(l-1)}{1+\varsigma_2(l-1)}\right)^{\delta}}\\&+\frac{4d_1^2}{25(\sigma+m)(\sigma+2m)\frac{c^{2m}\Gamma(v+1)}{\Gamma(v+2m+1)}\mathscr{L}_{\delta}^{\delta+2m}\left(\frac{1+(\varsigma_1+\varsigma_2)(l-1)}{1+\varsigma_2(l-1)}\right)^{\delta}}\\&+\frac{4\sigma(m+1)d_1^2}{25(\sigma+m)^2(\sigma+2m)\frac{c^{2m}\Gamma(v+1)}{\Gamma(v+2m+1)}\mathscr{L}_{\delta}^{\delta+2m}\left(\frac{1+(\varsigma_1+\varsigma_2)(l-1)}{1+\varsigma_2(l-1)}\right)^{\delta}}\\&-\frac{2\sigma(\sigma-1)d_1^2}{25(\sigma+m)^2(\sigma+2m)\frac{c^{2m}\Gamma(v+1)}{\Gamma(v+2m+1)}\mathscr{L}_{\delta}^{\delta+m}\mathscr{L}_{\delta}^{\delta+2m}\left(\frac{1+(\varsigma_1+\varsigma_2)(l-1)}{1+\varsigma_2(l-1)}\right)^{\delta}}\end{aligned} \tag{19}$$

and

$$|a_{2m+1}| \leqq \frac{\frac{2}{5}|d_2 - \frac{1}{2}d_1^2|}{(\sigma+2m)\frac{c^{2m}\Gamma(v+1)}{\Gamma(v+2m+1)}\mathscr{L}_{\delta}^{\phi+2m}\left(\frac{1+(\varsigma_1+\varsigma_2)(l-1)}{1+\varsigma_2(l-1)}\right)^{\delta}}$$
$$+ \frac{4|d_1|^2}{25(\sigma+m)(\sigma+2m)\frac{c^{2m}\Gamma(v+1)}{\Gamma(v+2m+1)}\mathscr{L}_{\delta}^{\delta+2m}\left(\frac{1+(\varsigma_1+\varsigma_2)(l-1)}{1+\varsigma_2(l-1)}\right)^{\delta}}$$
$$+ \frac{4\sigma(m+1)|d_1|^2}{25(\sigma+m)^2(\sigma+2m)\frac{c^{2m}\Gamma(v+1)}{\Gamma(v+2m+1)}\mathscr{L}_{\delta}^{\delta+2m}\left(\frac{1+(\varsigma_1+\varsigma_2)(l-1)}{1+\varsigma_2(l-1)}\right)^{\delta}}$$
$$- \frac{2\sigma(\sigma-1)|d_1|^2}{25(\sigma+m)^2(\sigma+2m)\frac{c^{2m}\Gamma(v+1)}{\Gamma(v+2m+1)}\mathscr{L}_{\delta}^{\phi+m}\mathscr{L}_{\delta}^{\delta+2m}\left(\frac{1+(\varsigma_1+\varsigma_2)(l-1)}{1+\varsigma_2(l-1)}\right)^{\delta}}$$

so that applying Lemmas 2.1 and 2.2 gives the result in (15).

Theorem 3.3 *If $\mathfrak{h} \in \nabla_{\sigma,\varsigma_1,\varsigma_2}^{\delta,c,v}(3\ell)$, then for a complex value ξ,*

$$|a_{2m+1} - \xi a_{m+1}^2| \leqq \frac{2}{5(\sigma+2m)\frac{c^{2m}\Gamma(v+1)}{\Gamma(v+2m+1)}\mathscr{L}_{\delta}^{\delta+2m}\left(\frac{1+(\varsigma_1+\varsigma_2)(l-1)}{1+\varsigma_2(l-1)}\right)^{\delta}} \times \max\{1, \chi\}$$

where

$$\chi = \left| \frac{2}{5(\sigma+m)} - \frac{2\sigma(m+1)}{5(\sigma+m)^2}\frac{\sigma(\sigma-1)}{5(\sigma+m)^2\mathscr{L}^{\phi+m}(\delta)} - \frac{1}{2} - \xi\frac{2(\sigma+2m)\Gamma(v+2m+1)\mathscr{L}_{\delta}^{\delta+2m}}{5(\sigma+m)^2\frac{\Gamma(v+1)}{[\Gamma(v+m+1)]^2}\mathscr{L}_{2\delta}^{\delta+m}\left(\frac{1+(\varsigma_1+\varsigma_2)(l-1)}{1+\varsigma_2(l-1)}\right)^{\delta}} \right|.$$

Proof. Using (18) and (19) means

$$a_{2m+1} - \xi a_{m+1}^2 = \frac{2d_2 - d_1^2}{5(\sigma+2m)\frac{c^{2m}\Gamma(v+1)}{\Gamma(v+2m+1)}\mathscr{L}_{\delta}^{\delta+2m}\left(\frac{1+(\varsigma_1+\varsigma_2)(l-1)}{1+\varsigma_2(l-1)}\right)^{\delta}}$$
$$+ \frac{4d_1^2}{25(\sigma+m)(\sigma+2m)\frac{c^{2m}\Gamma(v+1)}{\Gamma(v+2m+1)}\mathscr{L}_{\delta}^{\delta+2m}\left(\frac{1+(\varsigma_1+\varsigma_2)(l-1)}{1+\varsigma_2(l-1)}\right)^{\delta}}$$
$$- \frac{4\sigma(m+1)d_1^2}{25(\sigma+m)^2(\sigma+2m)\frac{c^{2m}\Gamma(v+1)}{\Gamma(v+2m+1)}\mathscr{L}_{\delta}^{\delta+2m}\left(\frac{1+(\varsigma_1+\varsigma_2)(l-1)}{1+\varsigma_2(l-1)}\right)^{\delta}}$$
$$- \frac{2\sigma(\sigma-1)d_1^2}{25(\sigma+m)^2(\sigma+2m)\frac{c^{2m}\Gamma(v+1)}{\Gamma(v+2m+1)}\mathscr{L}_{\delta}^{\delta+m}\mathscr{L}_{\delta}^{\delta+2m}\left(\frac{1+(\varsigma_1+\varsigma_2)(l-1)}{1+\varsigma_2(l-1)}\right)^{\delta}}$$
$$- \xi\left(\frac{2d_1}{5(\sigma+m)\frac{c^{m}\Gamma(v+1)}{\Gamma(v+m+1)}\mathscr{L}_{\delta}^{\delta+m}\left(\frac{1+(\varsigma_1+\varsigma_2)(l-1)}{1+\varsigma_2(l-1)}\right)^{\delta}}\right)^2$$

where further simplification gives

$$a_{2m+1}-\xi a_{m+1}^2=\frac{2}{5(\sigma+2m)\frac{c^{2m}\Gamma(v+1)}{\Gamma(v+2m+1)}\mathscr{L}_\delta^{\delta+2m}\left(\frac{1+(\varsigma_1+\varsigma_2)(l-1)}{1+\varsigma_2(l-1)}\right)^\delta}$$
$$\times\left\{d_2+\left[\frac{2}{5(\sigma+m)}-\frac{2\sigma(m+1)}{5(\sigma+m)^2}-\frac{\sigma(\sigma-1)}{5(\sigma+m)^2\mathscr{L}_\delta^{\delta+m}}-\frac{1}{2}\right.\right.$$
$$\left.\left.-\xi\frac{2(\sigma+2m)\Gamma(v+2m+1)\mathscr{L}_\delta^{\delta+2m}}{5(\sigma+m)^2\frac{\Gamma(v+1)}{[\Gamma(v+m+1)]^2}\mathscr{L}_{2\delta}^{\delta+m}\left(\frac{1+(\varsigma_1+\varsigma_2)(l-1)}{1+\varsigma_2(l-1)}\right)^\delta}\right]d_1^2\right\}.$$

Therefore,

$$\left|a_{2m+1}-\xi a_{m+1}^2\right|\leqq\frac{2}{5(\sigma+2m)\frac{c^{2m}\Gamma(v+1)}{\Gamma(v+2m+1)}\mathscr{L}_\delta^{\delta+2m}\left(\frac{1+(\varsigma_1+\varsigma_2)(l-1)}{1+\varsigma_2(l-1)}\right)^\delta}$$
$$\times\left|d_2+\left[\frac{2}{5(\sigma+m)}-\frac{2\sigma(m+1)}{5(\sigma+m)^2}-\frac{\sigma(\sigma-1)}{5(\sigma+m)^2\mathscr{L}_\delta^{\delta+m}}-\frac{1}{2}\right.\right.$$
$$\left.\left.-\xi\frac{2(\sigma+2m)\Gamma(v+2m+1)\mathscr{L}_\delta^{\delta+2m}}{5(\sigma+m)^2\frac{\Gamma(v+1)}{[\Gamma(v+m+1)]^2}\mathscr{L}_{2\delta}^{\delta+m}\left(\frac{1+(\varsigma_1+\varsigma_2)(l-1)}{1+\varsigma_2(l-1)}\right)^\delta}\right]d_1^2\right|.$$

so that applying Lemma 2.1 gives the result in the theorem.

Putting $m=1$ in Theorems 3.2 and 3.3 gives the following results.

Corollary 3.4 *If h given by (1) belongs to the set $\nabla_{\sigma,\varsigma_1,\varsigma_2}^{\delta,c,v}(3\ell)$, then*

$$|a_2|\leqq\frac{2}{5(\sigma+1)\frac{\Gamma(v+1)}{\Gamma(v+2)}\mathscr{L}_\delta^{\delta+1}\left(\frac{1+(\varsigma_1+\varsigma_2)(l-1)}{1+\varsigma_2(l-1)}\right)^\delta},$$
$$|a_3|\leqq\frac{1}{5(\sigma+2)\frac{\Gamma(v+1)}{\Gamma(v+3)}\mathscr{L}_\delta^{\delta+2}\left(\frac{1+(\varsigma_1+\varsigma_2)(l-1)}{1+\varsigma_2(l-1)}\right)^\delta}$$
$$+\frac{4}{25(\sigma+1)(\sigma+2)\frac{\Gamma(v+1)}{\Gamma(v+3)}\mathscr{L}_\delta^{\delta+2}\left(\frac{1+(\varsigma_1+\varsigma_2)(l-1)}{1+\varsigma_2(l-1)}\right)^\delta}$$
$$+\frac{8\sigma}{25(\sigma+1)^2(\sigma+2)\frac{\Gamma(v+1)}{\Gamma(v+3)}\mathscr{L}_\delta^{\delta+2}\left(\frac{1+(\varsigma_1+\varsigma_2)(l-1)}{1+\varsigma_2(l-1)}\right)^\delta}$$
$$+\frac{2\sigma(\sigma-1)}{25(\sigma+1)^2(\sigma+2)\frac{\Gamma(v+1)}{\Gamma(v+3)}\mathscr{L}_\delta^{\delta+1}\mathscr{L}_\delta^{\delta+2}\left(\frac{1+(\varsigma_1+\varsigma_2)(l-1)}{1+\varsigma_2(l-1)}\right)^\delta},$$

and

$$|a_3-\xi a_2^2|\leqq\frac{2}{5(\sigma+2)\frac{\Gamma(v+1)}{\Gamma(v+3)}\mathscr{L}_\delta^{\delta+2}\left(\frac{1+(\varsigma_1+\varsigma_2)(l-1)}{1+\varsigma_2(l-1)}\right)^\delta}\times\max\{1,\chi\}$$

where

$$\chi = \left| \frac{2}{5(\sigma+1)} - \frac{4\sigma}{5(\sigma+1)^2} - \frac{\sigma(\sigma-1)}{5(\sigma+m)^2\mathscr{L}_{\delta}^{\delta+1}} - \frac{1}{2} \right.$$
$$\left. - \xi \frac{2(\sigma+2)\Gamma(v+3)\mathscr{L}_{\delta}^{\delta+2}}{5(\sigma+1)^2 \frac{\Gamma(v+1)}{[\Gamma(v+2)]^2} \mathscr{L}_{2\delta}^{\delta+1} \left(\frac{1+(\varsigma_1+\varsigma_2)(l-1)}{1+\varsigma_2(l-1)}\right)^{\delta}} \right|.$$

Remark 3.5 *Corollary 3.4 presumably holds new results.*

Putting $\varsigma_1 = 0 = \varsigma_2$ in Corollary 3.4 gives the following results.

Corollary 3.6 *If h given by (1) belongs to the set $\nabla_{\sigma,0,0}^{\delta,c,v}(3\ell)$, then*

$$|a_2| \leqq \frac{2}{5(\sigma+1)\frac{\Gamma(v+1)}{\Gamma(v+2)}\mathscr{L}_{\delta}^{\delta+1}},$$
$$|a_3| \leqq \frac{1}{5(\sigma+2)\frac{\Gamma(v+1)}{\Gamma(v+3)}\mathscr{L}_{\delta}^{\phi+2}} + \frac{4}{25(\sigma+1)(\sigma+2)\frac{\Gamma(v+1)}{\Gamma(v+3)}\mathscr{L}_{\delta}^{\phi+2}}$$
$$+ \frac{8\sigma}{25(\sigma+1)^2(\sigma+2)\frac{\Gamma(v+1)}{\Gamma(v+3)}\mathscr{L}_{\delta}^{\delta+2}} + \frac{2\sigma(\sigma-1)}{25(\sigma+1)^2(\sigma+2)\frac{\Gamma(v+1)}{\Gamma(v+3)}\mathscr{L}_{\delta}^{\delta+1}\mathscr{L}_{\delta}^{\delta+2}},$$

and

$$\left|a_3 - \xi a_2^2\right| \leqq \frac{2}{5(\sigma+2)\frac{\Gamma(v+1)}{\Gamma(v+3)}\mathscr{L}_{\delta}^{\delta+2}} \times \max\{1, |\chi|\}$$

where

$$\chi = \frac{2}{5(\sigma+1)} - \frac{4\sigma}{5(\sigma+1)^2} - \frac{\sigma(\sigma-1)}{5(\sigma+1)^2\mathscr{L}_{\delta}^{\delta+1}} - \frac{1}{2} - \xi\frac{2(\sigma+2)\Gamma(v+3)\mathscr{L}_{\delta}^{\delta+2}}{5(\sigma+1)^2\frac{\Gamma(v+1)}{[\Gamma(v+2)]^2}\mathscr{L}_{2\delta}^{\delta+1}}.$$

Remark 3.7 *Corollary 3.6 presumably holds new results.*

Putting $v = 0$ in Corollary 3.6 gives the following results.

Corollary 3.8 *If h given by (1) belongs to the set $\nabla_{\sigma,0,0}^{1,1,\delta}(3\ell)$, then*

$$|a_2| \leqq \frac{2}{5(\sigma+1)\mathscr{L}_{\delta}^{\delta+1}},$$
$$|a_3| \leqq \frac{2}{5(\sigma+2)\mathscr{L}_{\delta}^{\delta+2}} + \frac{8}{25(\sigma+1)(\sigma+2)\mathscr{L}_{\delta}^{\delta+2}}$$
$$+ \frac{16\sigma}{25(\sigma+1)^2(\sigma+2)\mathscr{L}_{\delta}^{\delta+2}} + \frac{4\sigma(\sigma-1)}{25(\sigma+1)^2(\sigma+2)\mathscr{L}_{\delta}^{\delta+1}\mathscr{L}_{\delta}^{\delta+2}},$$

and

$$|a_3 - \xi a_2^2| \leqq \frac{4}{5(\sigma+2)\mathscr{L}_{\delta}^{\delta+2}} \times \max\{1, |\chi|\}$$

where

$$\chi = \frac{2}{5(\sigma+1)} - \frac{4\sigma}{5(\sigma+1)^2} - \frac{\sigma(\sigma-1)}{5(\sigma+1)^2\mathscr{L}_\delta^{\delta+1}} - \frac{1}{2} - \xi\frac{4(\sigma+2)\mathscr{L}_\delta^{\delta+2}}{5(\sigma+1)^2\mathscr{L}_{2\delta}^{\delta+1}},$$

$$\frac{\Gamma(1)}{\Gamma(2)} = 1 \qquad \textit{and} \qquad \frac{\Gamma(1)}{\Gamma(3)} = \frac{1}{2}.$$

Remark 3.9 *Corollary 3.8 presumably holds new results.*

Putting $\sigma = 0$ in Corollary 3.8 gives the following results.

Corollary 3.10 *If h given by (1) belongs to the set $\nabla_{0,0,0}^{0,0,\delta}(3\ell)$, then*

$$|a_2| \leqq \frac{2}{5\mathscr{L}_\delta^{\delta+1}},\ |a_3| \leqq \frac{18}{50\mathscr{L}_\delta^{\delta+2}},\ \textit{and}\ |a_3 - \xi a_2^2| \leqq \frac{2}{5\mathscr{L}_\delta^{\delta+2}} \max\left\{1, \left|\frac{1}{10} + \xi\frac{8\mathscr{L}_\delta^{\delta+2}}{5\mathscr{L}_{2\delta}^{\delta+1}}\right|\right\}.$$

Remark 3.11 *Corollary 3.10 presumably holds new results.*

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