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ILL-POSEDNESS OF A MIXED PROBLEM IN A CYLINDRICAL DOMAIN FOR THE MULTIDIMENSIONAL LAVRENTIEV-BITSADZE EQUATION

Studies of well-posed and ill-posed problems in mathematical physics, including inverse problems and their practical applications, are of considerable interest, where the key issue is the correct formulation of the direct problem. Hyperbolic and elliptic equations are widely used in biomedical modeling, including to describe tumor growth and deformations of biological tissues. Analogies between membrane oscillations and tissue dynamics are widely used in biomechanics and mathematical medicine. For example, the spatial oscillations of elastic membranes are described by partial differential equations. When the membrane deflection is specified by a function $u(x,t)$, $x \in R^m$, $m \geq 2$, application of Hamilton's principle leads to a multidimensional wave equation, and in the case of equilibrium, to the Laplace equation. Consequently, the dynamics of elastic membranes can be described by the multidimensional Lavrentiev-Bitsadze equation. The problems considered in the article are ill-posed problems. The proof of non-unique solvability and the construction of an explicit solution is in fact a regularization of an ill-posed problem through the spectral method and integral representations, etc. In this article, the ambiguity of the solution is proven and an explicit form of the classical solution of a mixed problem for the multidimensional Lavrentiev-Bitsadze equation, is presented.

Key words: Ill-posedness, mixed problem, cylindrical domain, Bessel function, boundary conditions.

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Көп өлшемді Лаврентьев-Бицадзе теңдеуі үшін цилиндрлік облыстағы аралас есептің қисынды қойылмауы

Математикалық физиканың қисынды және қисынды емес қойылған есептерді, соның ішінде кері есептерді және олардың практикалық қолданылуын зерттеу айтарлықтай қызығушылық тудырады, мұндағы негізгі мәселе тура есептің қисынды қойылуы болып табылады. Гиперболалық және эллиптикалық теңдеулер биомедициналық модельдеуде, соның ішінде ісіктің ісуі мен биологиялық тіндердің деформацияларын сипаттау үшін кеңінен қолданылады. Мембрана тербелістері мен тіннің өзгеріс динамикасы биомеханика мен математикалық медицинада кеңінен қолданылады. Мысалы, серпімді мембраналардың кеңістіктік тербелістері дербес дифференциалдық теңдеулермен сипатталады. Мембрананың ауытқуы $u(x,t)$, $x \in R^m$, $m \geq 2$, функциясымен анықталған кезде, Гамильтон принципін қолдану көп өлшемді толқындық теңдеуге, ал тепе-теңдік жағдайында - Лаплас теңдеуіне әкеледі. Демек, серпімді мембраналардың динамикасын көп өлшемді Лаврентьев-Бицадзе теңдеуімен сипаттауға болады. Бұл мақалада қарастырылатын есептер қисынды емес есептер болып табылады.

Бірегей шешілімділікті дәлелдеу және айқын шешімді құруда спектрлік әдіс және интегралдық бейнелеулер арқылы қисынды емес есепті регуляризациялау болып табылады. Мақалада теориялық нәтижелер келтірілген – көп өлшемді Лаврентьев-Бицадзе теңдеуі үшін аралас есептің бірегей шешілетіндігін дәлелдеп және классикалық шешімі үшін айқын түрін алған.
Түйін сөздер: қисындылық, аралас есеп, цилиндрлік облыс, Бессель функциясы, шекаралық шарттар.

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Некорректность смешанной задачи в цилиндрической области для многомерного уравнения Лаврентьева-Бицадзе

Исследования корректных и некорректных задач математической физики, включая обратные задачи и их практические применения, представляют значительный интерес, где ключевым моментом выступает корректная постановка прямой задачи. Гиперболические и эллиптические уравнения широко используются в биомедицинском моделировании, в том числе для описания роста опухолевых образований и деформаций биологических тканей. Аналогии между колебаниями мембран и динамикой поведения тканей широко применяются в биомеханике и математической медицине. Так колебания упругих мембран в пространстве описываются уравнениями в частных производных. При задании прогиба мембраны функцией $u(x,t)$, $x \in R^m$, $m \geq 2$, применение принципа Гамильтона приводит к многомерному волновому уравнению, а в случае равновесного состояния - к уравнению Лапласа. Следовательно, динамика упругих мембран может быть описана многомерным уравнением Лаврентьева-Бицадзе. Рассматриваемые в статье задачи являются некорректными. Доказательство неоднозначности решения и построение явного решения фактически представляет собой регуляризацию некорректной задачи с помощью спектрального метода, интегральных представлений и т.д. В статье доказана неоднозначность решения и представлен явный вид классического решения смешанной задачи для многомерного уравнения Лаврентьева-Бицадзе.
Ключевые слова: корректность, смешанная задача, цилиндрическая область, функция Бесселя, граничные условия.

1 Introduction

The team of authors has maintained scientific cooperation for many years, based on the results of research the field of well-posedness and ill-posed of problems in mathematical physics and related inverse problems, as well as their applications. When studying inverse and ill-posed problems, the formulation of the direct problem and its well-posedness conditions are of great importance, such a connection is presented [1–3]. As is known, hyperbolic and elliptic equations are used in biomedical models, including tumor growth and deformation of biological tissues. In [4], a linear stability analysis of growing tissues is discussed. The equations for small perturbations are reduced to mixed-type systems. In particular, wave-like and diffusion-like regimes (hyperbolic and elliptic, respectively) are described, which may coexist due to heterogeneous growth. Indeed, analogies between membrane vibrations and tissue dynamics are actively used in biomechanics and mathematical medicine. For example, this includes predicting brain tumor growth, where mechanical pressure on the surrounding tissues is taken into account, as well as analyzing tissue deformation during tumor invasion (such as glioblastoma), where the deformation follows wave-type equations with viscoelastic terms. The above problems have been studied in detail in [5–8], but for multidimensional hyperbolic-elliptic equations these problems have not yet been studied.

2 Problem statement and result

Let $\Omega_{\alpha\beta}$ be a finite domain of the Euclidean space E_{m+1} of points $(x_1,\ldots,x_m,t)$, bounded at $t>0$ by the cylinder $\Gamma_\beta=\{(x,t):|x|=1\}$ and the plane $t=\beta>0$, and for $t<0$ the cylinder $\Gamma_\alpha=\{(x,t):|x|=1\}$ and the plane $t=\alpha<0$, where $|x|$ is the length of the vector $x=(x_1,\ldots,x_m)$, $m\geq 2$. Denote by Ω_β^+ and Ω_α^- parts of the domain $\Omega_{\alpha\beta}$, lying in the half-spaces $t>0$ and $t<0$; σ_β – the upper base of the domain Ω_β^+,a σ_α – the lower base of the domain Ω_α^-.

Let S be the common part of the boundaries of the domains Ω_β^+ and Ω_α^-, representing the set of $\{t=0, 0<|x|<1\}$ points from E_m [9].

In the domain of $\Omega_{\alpha\beta}$ we consider

$$(\mathrm{sgn}t)\Delta_x u-u_{tt}=0, \tag{1}$$

where Δ_x is the Laplace operator for variables $x_1,\ldots,x_m$ [9].

Next, it is convenient [9] for us to move from Cartesian coordinates $x_1,\ldots,x_m,t$ to spherical $r,\theta_1,\ldots,\theta_{m-1},t$, $r\geq 0$, $0\leq\theta_1<2\pi$, $0\leq\theta_i\leq\pi$, $i=\overline{2,m-1}$.

As a multidimensional mixed problem, consider the following problem

Problem 1 *Find the solution of the equation* (1) *in the domain* $\Omega_{\alpha\beta}$ *for* $t\neq 0$ *from the class* $C(\overline{\Omega}_{\alpha\beta})\cap C^1(\Omega_{\alpha\beta})\cap C^2(\Omega_\beta^+\cup\Omega_\alpha^-)$ *satisfying the boundary conditions*

$$u\Big|_{\Gamma_\alpha}=\psi_1(t,\theta), \tag{2}$$

$$u\Big|_{\Gamma_\alpha}=\psi_2(t,\theta),\quad u\Big|_{\sigma_\alpha}=\varphi(r,\theta), \tag{3}$$

at the same time, $\psi_1(0,\theta)=\psi_2(0,\theta)$, $\psi_2(\alpha,\theta)=\varphi(r,\theta)$.

Let $\{Y_{n,m}^k(\theta)\}$ be a system of linearly independent spherical functions of the order n, $1\leq k\leq k_n$, $(m-2)!n!k_n=(n+m-3)!(2n+m-2)$ and $W_2^l(S)$, $l=0,1,\ldots$ are Sobolev spaces [9].

It takes place ([10], p. 142-144)

Lemma 1 *Let* $f(r,\theta)\in W_2^l(S)$*. If* $l\geq m-1$*, then the series*

$$f(r,\theta)=\sum_{n=0}^{\infty}\sum_{k=1}^{k_n}f_n^k(r)Y_{n,m}^k(\theta), \tag{4}$$

and also the series obtained from it by differentiation of order $p\leq l-m+1$ *converge absolutely and uniformly [9, 10].*

Lemma 2 *In order for* $f(r,\theta)\in W_2^l(S)$ *it is necessary and sufficient that the coefficients of the series* (4) *satisfy the inequalities [9, 10]*

$$|f_0^1(r)|\leq c_1,\quad \sum_{n=1}^{\infty}\sum_{k=1}^{k_n}n^{2l}|f_n^k(r)|^2\leq c_2,\quad c_1,c_2=const.$$

By $\psi_{2n}^k(t)$, $\overline{\varphi}_n^k(r)$, we denote the expansion coefficients of the series (4), respectively, of the functions $\psi_2(t,\theta)$, $\varphi(r,\theta)$ [9].

Then the following is true

Theorem 1 *If $\psi_1(t,\theta) \in W_2^l(\Gamma_\beta)$, $\psi_2(t,\theta) \in W_2^l(\Gamma_\alpha)$, $\varphi(r,\theta) \in W_2^l(\sigma_\alpha)$, $l > \dfrac{3m}{2}$, then Problem 1 is solvable and not unique.*

Proof. In spherical coordinates the equation (1) in the domain Ω_α^- has the form [10,11]

$$u_{rr} + \frac{m-1}{r}u_r - \frac{1}{r^2}\delta u + u_{tt} = 0, \tag{5}$$

where is

$$\delta \equiv -\sum_{j=1}^{m-1} \frac{1}{g_j \sin^{m-j-1}\theta_j} \frac{\partial}{\partial\theta_j}\left(\sin^{m-j-1}\theta_j \frac{\partial}{\partial\theta_j}\right), \; g_1 = 1, \; g_j = (\sin\theta_1 \ldots \sin\theta_{j-1})^2, \; j > 1.$$

It is known ([9], [10], p. 239) that the spectrum of the operator δ consists of eigenvalues $\lambda_n = n(n+m-2)$, $n = 0, 1, \ldots$, each of which corresponds to $k_n$orthonormal functions $Y_{n,m}^k(\theta)$.

Since the desired solution to Problem 1 in the domain $\overline{\Omega}_\alpha$ belongs to the class $C(\overline{\Omega}_\alpha^-) \cap C^2(\Omega_\alpha^-)$, it can be sought in the form as

$$u(r,\theta,t) = \sum_{n=0}^{\infty}\sum_{k=1}^{k_n} \overline{u}_n^k(r,t) Y_{n,m}^k(\theta), \tag{6}$$

where $\overline{u}_n^k(r,t)$ are the functions to be defined.

Substituting (6) into (5), using the orthogonality of spherical functions [9,10], we arrive at the equation

$$\overline{u}_{nrr}^k + \frac{m-1}{r}\overline{u}_{nr}^k + \overline{u}_{ntt}^k - \frac{\lambda_n}{r^2}\overline{u}_n^k = 0, \quad k = \overline{1,k_n}, \; n = 0, 1, ..., \tag{7}$$

Moreover, from the boundary condition (3), taking into account Lemma 1, we have

$$\overline{u}_n^k(1,t) = \psi_{2n}^k(t), \quad \overline{u}_n^k(r,\alpha) = \overline{\varphi}_n^k(r), \quad k = \overline{1,k_n}, \quad n = 0, 1, \ldots. \tag{8}$$

In (7), (8), replacing $\overline{v}_n^k(r,t) = \overline{u}_n^k(r,t) - \psi_n^k(t)$, we get

$$\overline{v}_{nrr}^k + \frac{m-1}{r}\overline{v}_{nr}^k - \frac{\lambda_n}{r^2}\overline{v}_n^k + \overline{v}_{ntt}^k = \overline{f}_n^k(r,t). \tag{9}$$

$$\begin{gathered} \overline{v}_n^k(1,t) = 0, \quad \overline{v}_n^k(r,\alpha) = \varphi_n^k(r), \quad k = \overline{1,k_n}, \quad n = 0, 1, \ldots. \\ \overline{f}_n^k(r,t) = \frac{\lambda_n}{r^2}\psi_{2n}^k(t) - \psi_{2ntt}^k, \quad \varphi_n^k(r) = \overline{\varphi}_n^k(r) - \psi_n^k(\alpha). \end{gathered} \tag{10}$$

By replacing $\overline{v}_n^k(r,t) = r^{(1-m)/2} v_n^k(r,t)$, the problem (9), (10) is reduced to the following problem

$$Lv_n^k \equiv v_{nrr}^k + \frac{\lambda_n}{r^2} v_n^k + v_{ntt}^k = \widetilde{f}_n^k(r,t), \tag{11}$$

$$\begin{gathered} v_n^k(1,t) = 0, \quad v_n^k(r,\alpha) = \widetilde{\varphi}_n^k(r), \quad k = \overline{1,k_n}, \quad n = 0,1,\ldots, \\ \lambda_n = \frac{[(m-1)(3-m) - 4\lambda_n]}{2}, \quad \widetilde{f}_n^k(r,t) = r^{(m-1)/2}\overline{f}_n^k(r,t), \quad \widetilde{\varphi}_n^k(r) = r^{(m-1)/2}\varphi_n^k(r). \end{gathered} \tag{12}$$

We seek the solution to problem (11), (12) in the form $v_n^k(r,t) = v_{1n}^k(r,t) + v_{2n}^k(r)$, where $v_{1n}^k(r,t)$ – is the solution to the problem

$$Lv_{1n}^k = \widetilde{f}_n^k(r,t), \quad v_{1n}^k(1,t) = 0, \quad v_{1n}^k(r,\alpha) = 0, \tag{13}$$

and $v_{2n}^k(r,t)$ – solving the problem

$$Lv_{2n}^k = 0, \quad v_{2n}^k(1,t) = 0, \quad v_{2n}^k(r,\alpha) = \widetilde{\varphi}_n^k(r). \tag{14}$$

We will consider the solution of the problems in the form

$$v_n^k(r,t) = \sum_{s=1}^{\infty} R_s(r) T_s(t). \tag{15}$$

At the same time, let

$$\widetilde{f}_n^k(r,t) = \sum_{s=1}^{\infty} a_{s,n}^k(t) R_s(r), \quad \widetilde{\varphi}_{2n}^k(r) = \sum_{s=1}^{\infty} b_{s,n}^k R_s(r). \tag{16}$$

Substituting (15) into (13), taking into account (16), we get

$$R_{srr} + \left(\frac{\overline{\lambda}_n}{r^2} + \mu\right) R_s = 0, \quad 0 < r < 1, \quad R_s(1) = 0, \quad |R_s(0)| < \infty, \tag{17}$$

$$T_{stt} - \mu T_s(t) = a_{s,n}^k(t), \quad \alpha < t < 0, \tag{18}$$

$$T_s(\alpha) = 0. \tag{19}$$

A limited solution to the (17) problem is [12]

$$R_s(r) = \sqrt{r} J_\nu(\mu_{s,n} r), \quad \mu = \mu_{s,n}^2. \tag{20}$$

$\nu = n + (m-2)/2, \mu_{s,n}$ – zeros of Bessel functions of the first kind $J_\nu(z), \mu = \mu_{s,n}^2$.

The general solution of the equation (18) is represented as [12]

$$T_{s,n}(t) = c_{1s}\cosh\mu_{s,n}t + c_{2s}\sinh\mu_{s,n}t - \frac{\cosh\mu_{s,n}t}{\mu_{s,n}}\int\limits_t^0 a^k_{s,n}(\xi)\sinh\mu_{s,n}\xi d\xi +$$

$$+\frac{\sinh\mu_{s,n}t}{\mu_{s,n}}\int\limits_t^0 a^k_{s,n}(\xi)\cosh\mu_{s,n}\xi d\xi,$$

c_{1s}, c_{2s} – arbitrary constants, satisfying the condition (19), we will have

$$\begin{gathered}\mu_{s,n}T_{s,n}(t) = c_{1s}\mu_{s,n}[\cosh\mu_{s,n}t - (\coth\mu_{s,n}\alpha)\sinh\mu_{s,n}t] + \\ + \left[(\coth\mu_{s,n}\alpha)\int\limits_\alpha^0 a^k_{s,n}(\xi)(\sinh\mu_{s,n}\xi)d\xi - \int\limits_\alpha^0 a^k_{s,n}(\xi)(\cosh\mu_{s,n}\xi)d\xi\right]\sinh\mu_{s,n}t - \\ -(\cosh\mu_{s,n}t)\int\limits_t^0 a^k_{s,n}(\xi)\sinh\mu_{s,n}\xi d\xi + (\sinh\mu_{s,n}t)\int\limits_t^0 a^k_{s,n}(\xi)\cosh\mu_{s,n}\xi d\xi.\end{gathered} \tag{21}$$

Substituting (20) into (16), we get

$$r^{-\frac{1}{2}}\widetilde{f}^k_n(r,t) = \sum_{s=1}^{\infty} a^k_{s,n}(t)J_\nu(\mu_{s,n}r), \quad r^{-\frac{1}{2}}\widetilde{\varphi}^k_n(r) = \sum_{s=1}^{\infty} b^k_{s,n}J_\nu(\mu_{s,n}r), \quad 0 < r < 1. \tag{22}$$

Series (22) – expansions into Fourier-Bessel series [12], if

$$a^k_{s,n}(t) = 2[J_{\nu+1}(\mu_{s,n})]^{-2}\int\limits_0^1 \sqrt{\xi}\widetilde{f}^k_n(\xi,t)J_\nu(\mu_{s,n}\xi)d\xi, \tag{23}$$

$$b^k_{s,n} = 2[J_{\nu+1}(\mu_{s,n})]^{-2}\int\limits_0^1 \sqrt{\xi}\widetilde{\varphi}^k_n(\xi)J_\nu(\mu_{s,n}\xi)d\xi, \tag{24}$$

where $\mu_{s,n}$, $s = 1, 2, \dots$ are the positive zeros of the Bessel functions $J_\nu(z)$, arranged in order of increasing magnitude.

From (20), (21) we obtain the solution to the problem (13) in the form

$$v^k_{1n}(r,t) = \sum_{s=1}^{\infty} \sqrt{r}T_{s,n}(t)J_\nu(\mu_{s,n}r), \tag{25}$$

where $T_{s,n}(t)$ – are determined from (21), and $a^k_{s,n}(t)$ – from (23).

Next, substituting (15) into (14), taking into account (16) we will have

$$V_{stt} - \mu^2_{s,n}V_s = 0, \quad \alpha < t < 0, \tag{26}$$

$$V_s(\alpha) = b_{s,n}^k. \tag{27}$$

The general solution of the equation (26) has the form

$$V_{s,n}(t) = c'_{1s} \cosh \mu_{s,n} t + c'_{2s} \sinh \mu_{s,n} t,$$

where c'_{1s}, c'_{2s} – arbitrary constants, satisfying which condition (27) we get

$$V_{s,n}(t) = c'_{1s}[\cosh \mu_{s,n} t - (\coth \mu_{s,n}\alpha) \sinh \mu_{s,n} t] + \frac{b_{s,n}^k \sinh \mu_{s,n} t}{\sinh \mu_{s,n}\alpha}. \tag{28}$$

From (20), (28) we get the solution to the problem (14) by the formula

$$v_{2n}^k(r,t) = \sum_{s=1}^{\infty} \sqrt{r} V_{s,n}(t) J_\nu(\mu_{s,n} r), \tag{29}$$

where $V_{s,n}(t)$ are from (28), and $b_{s,n}^k$ – are from (24).

Thus, the boundary value problem for the equation (5) with data

$$u\Big|_{\Gamma_\alpha} = \psi_2(t,\theta), u\Big|_{\sigma_\alpha} = \varphi_2(r,\theta)$$

in the domain of Ω_α^- has countless solutions of the type

$$u(r,\theta,t) = \sum_{n=1}^{\infty} \sum_{k=1}^{k_n} \{\psi_n^k(t) + r^{(1-m)/2}[v_{1n}^k(r,t) + v_{2n}^k(r,t)]\} Y_{n,m}^k(\theta), \tag{30}$$

where $v_{1n}^k(r,t)$, $v_{2n}^k(r,t)$ are defined from (25), (29).

Using the formula [13] $2J'_\nu(z) = J_{\nu-1}(z) - J_{\nu+1}(z)$, estimates [10, 13]

$$|J_\nu(z)| \le \frac{1}{\Gamma(1+\nu)} \left(\frac{z}{2}\right)^\nu, \quad |k_n| \le c_1 n^{m-2},$$

$$\left| \frac{\partial^q}{\partial \theta_j^q} Y_{n,m}^k(\theta) \right| \le c_2 n^{\frac{m}{2}-1+q}, \quad c_1, c_2 = const, \quad j = \overline{1, m-1}, \quad q = 0, 1, \ldots,$$

$\Gamma(z)$ – gamma function, as well as lemmas, constraints on given functions $\psi_2(t,\theta)$, $\varphi(r,\theta)$, as in [14], [15] it can be shown that the resulting solution (30) belongs to the class $C(\overline{\Omega_\alpha^-}) \cap C(\Omega_\alpha)$.

Next, from (30) at $t \to -0$ it will have

$$u(r,\theta,0) = \tau(r,\theta) = \sum_{n=1}^{\infty} \sum_{k=1}^{k_n} \sum_{s=1}^{\infty} \{\psi_{2n}^k(0) + r^{\frac{(2-m)}{2}}(c_{1s} + c'_{1s})\} J_{n+\frac{(m-2)}{2}}(\mu_{s,n} r) Y_{n,m}^k(\theta),$$

$$u_t(r,\theta,0) = \nu(r,\theta) = \sum_{n=1}^{\infty} \sum_{k=1}^{k_n} \sum_{s=1}^{\infty} \Big\{ \psi_{2nt}^k(0) + r^{\frac{(2-m)}{2}} \Big[-(c_{1s} + c'_{1s}) \mu_{s,n} \coth \mu_{s,n}\alpha +$$
$$+ (\coth \mu_{s,n}\alpha) \int_\alpha^0 a_{s,n}^k(\xi)(\sinh \mu_{s,n}\xi) d\xi - \int_\alpha^0 a_{s,n}^k(\xi)(\cosh \mu_{s,n}\xi) d\xi + \tag{31}$$
$$+ \frac{\mu_{s,n} b_{s,n}^k}{\cosh \mu_{s,n}\alpha} \Big] \Big\} J_{n+\frac{(m-2)}{2}}(\mu_{s,n} r) Y_{n,m}^k(\theta)$$

with $\tau(r,\theta)$,$\nu(r,\theta) \in W_2^l(S)$, $l > \frac{3m}{2}$.

Now we will study Problem 1 in the domain of Ω_β^+, which, by virtue of (2) and (31) is reduced to a mixed problem for the multidimensional wave equation [9]

$$u_{rr} + \frac{(m-1)}{r}u_r - \frac{1}{r^2}\delta u - u_{tt} = 0 \tag{32}$$

with conditions

$$u\Big|_S = \tau(r,\theta), \quad u_t\Big|_S = \nu(r,\theta), \quad u\Big|_{\Gamma_\beta} = \psi_1(t,\theta). \tag{33}$$

The following is shown in [7]

Theorem 2 *The problem* (32), (33) *is uniquely solvable in the class* $C(\overline{\Omega}_\beta^+) \cap C^2(\Omega_\beta^+)$.

From representation (31), and also from Theorem 2 it follows that Problem 1 has countless classical solutions.

Theorem 1 has been proven.

Since in [7, 9] an explicit form of solutions to problem (32), (33) was obtained, then it is possible to write an explicit representation of the solution for Problem 1.

3 Conclusion and discussion

It has been established that the mixed problem for the multidimensional Lavrentiev-Bitsadze equation admits an ambiguous solution, and its explicit classical form has been obtained. This ill-posedness, manifested in the solution's high sensitivity to small data changes, is directly related to the problems of tumor modeling, where parameter instability leads to significant variability in growth predictions and treatment response.It has been established that the mixed problem for the multidimensional Lavrentiev-Bitsadze equation admits an ambiguous solution, and its explicit classical form has been obtained. This ill-posedness, manifested in the solution's high sensitivity to small data changes, is directly related to the problems of tumor modeling, where parameter instability leads to significant variability in growth predictions and treatment response.

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