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MODELING METHODS FOR STRATEGIC ANALYSIS OF INTERREGIONAL INEQUALITY IN KAZAKHSTAN

The article focuses on spatial economic development and regional inequality in the Republic of Kazakhstan, with an emphasis on the factors and policy conditions that create disparities between regions. The empirical analysis is based on a panel of 16 Kazakhstani regions for 2001-2017 (272 observations). The dependent variable is the logarithm of GRP per capita. Modifications of the Spatial Durbin Model (SDM) with fixed and random regional effects are estimated and compared; according to the AIC/BIC criteria and the Hausman test, the fixed-effects SDM is recognised as the preferred specification. Econometric estimations in SPSS and STATA18, along with spatial and GIS analysis in R, are used for preprocessing and classical econometrics. Diagnostics include the Hausman test, checks for heteroscedasticity and autocorrelation, and panel stationarity tests. The analysis identifies dominant spatial determinants of uneven socio-economic development of Kazakhstan's regions and supports institutional and economic-geographical policies for reducing inequality.

Key words: economic growth, spatial inequality, regional economics, Spatial Durbin Model

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Қазақстандағы аймақаралық теңсіздікті стратегиялық талдаудың модельдеу әдістері

Мақала Қазақстан Республикасындағы кеңістіктік экономикалық даму мен өңірлік теңсіздікке бағытталған, өңірлер арасындағы айырмашылықтарды туындататын факторлар мен институционалды-саяси жағдайларға ерекше көңіл бөледі. Эмпирикалық талдау 2001-2017 жылдар аралығындағы 16 Қазақстандық өңір бойынша панельдік деректерге (272 бақылау) негізделген. Тәуелді айнымалы ретінде жан басына шаққандағы жалпы өңірлік өнімні (ЖӨӨ) логарифмі алынады. Кеңістіктік Дурбин моделінің (Spatial Durbin Model, SDM) өңірлік тіркелген және кездейсоқ әсерлері бар модификациялары бағаланып, салыстырылады; AIC/BIC критерийлері мен Хаусман тестіне сәйкес, тіркелген әсерлері бар SDM спецификациясы неғұрлым қолайлы болып танылады. SPSS және STATA18 бағдарламаларындағы эконометриялық бағалаулар R тіліндегі кеңістіктік және GIS-талдаумен бірге деректерді алдын ала өңдеу және классикалық эконометрика үшін қолданылады. Диагностикаға Хаусман тесті, гетероскедастикалық пен автокорреляцияны тексерулер, сондай-ақ панельдік стационарлық тесттері кіреді. Талдау Қазақстан өңірлерінің әлеуметтік-экономикалық дамуының біркелкі еместігін қалыптастыратын басым кеңістіктік детерминанттарды айқындап, теңсіздікті азайтуға бағытталған институционалдық және экономикалық-географиялық саясат шараларын негіздеуге мүмкіндік береді.

Түйін сөздер: экономикалық өсу, кеңістіктік теңсіздік, өңірлік экономика, кеңістіктік Дурбин моделі.

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Методы моделирования для стратегического анализа межрегионального неравенства в Казахстане

Статья фокусируется на пространственном экономическом развитии и региональном неравенстве в Республике Казахстан, с акцентом на факторы и институционально-политические условия, которые создают диспропорции между регионами. Эмпирический анализ основан на панели из 16 регионов Казахстана за 2001-2017 гг. (272 наблюдения). Зависимой переменной является логарифм ВРП на душу населения. Построены и оценены модификации пространственной модели Дурбина (Spatial Durbin Model, SDM) с региональными фиксированными и случайными эффектами; согласно критериям AIC/BIC и тесту Хаусмана, предпочтительной признаётся спецификация SDM с фиксированными эффектами. Эконометрические оценки в SPSS и STATA18 в сочетании с пространственным и GIS-анализом в R используются для предварительной обработки данных и классической эконометрики. Диагностика включает тест Хаусмана, проверки на гетероскедастичность и автокорреляцию, а также панельные тесты на стационарность. Анализ позволяет выявить доминирующие пространственные детерминанты неравномерного социально-экономического развития регионов Казахстана и обосновать институциональные и экономико-географические меры политики по снижению неравенства.

Ключевые слова: экономический рост, пространственное неравенство, региональная экономика, пространственная модель Дурбина.

1 Introduction

The study of interregional economic inequality in Kazakhstan requires the use of models capable of correctly describing complex relationships among spatially distributed units. The country's economic dynamics are characterised by pronounced differentiation: regions differ in terms of gross regional product (GRP) growth rates, the structure of government expenditure, unemployment levels and investment volumes. A substantial share of the variation in socio-economic indicators arises from spatial interactions between regions, which makes traditional regression methods mathematically inappropriate when spatial dependence is ignored.

Existing studies confirm that the use of one or several local indicators does not capture the full mechanism by which spatial inequality is formed, whereas interregional linkages directly affect both economic activity and long-term development trajectories [1–7]. According to international evidence from other countries [8–12], spatial inequality leads to an uneven distribution of resources, affects the stability of labor markets and may weaken social and political stability.

The expansion of Kazakhstan's regional statistical database for 2001–2017 has increased the need for models that explicitly account for the spatial structure of regional data. Regional indicators can naturally be interpreted as a realization on a finite graph with fixed vertices—regions—and adjacency relations. Under these conditions, the choice of model becomes not only an economic, but also a mathematical problem of obtaining correct parameter estimates.

The object of the study is the set of spatially distributed regional observations for Kazakhstan.

The subject of the study is the mathematical modelling of interregional linkages and the estimation of economic growth parameters in the presence of spatial dependence.

The aim of the paper is to construct and analyse a parametrized spatial model of regional growth based on the Spatial Durbin Model (SDM), and to assess the robustness and statistical significance of the parameters under alternative specifications with fixed and random regional effects.

The research hypothesis is that incorporating a spatial operator improves model quality and that the SDM makes it possible to distinguish between local effects and cross-regional spillovers more adequately than classical panel regressions.

The theoretical contribution of the study lies in the application of modern spatial methods to regional data for Kazakhstan, which refines the boundaries of applicability of panel models and demonstrates the advantages of their spatial extensions.

The following scientific methods were used in the study: historical analysis and statistical analysis. The empirical analysis is based on spatial econometric modifications of the Spatial Durbin Model that include spatial lags of both dependent and independent variables. These models are characterised by non-constant coefficients and include spatial lags of both dependent and independent variables. The models were estimated using data on Kazakhstan's regions, with the logarithm of GRP per capita as the dependent variable.

The scientific significance of the study lies in the application of the Spatial Durbin Model to Kazakhstan's regional data in order to quantify spatial dependence, regional heterogeneity, and interregional spillover effects. These patterns shape differences in public investment, unemployment, and investment activity across regions, while also reflecting the mutual influence of neighbouring territories. The study provides a quantitative assessment of how government expenditure per capita and investment in fixed capital in one region are associated with economic outcomes in neighbouring regions.

2 Literature review and problem statement

The modern literature emphasises the need for multidimensional approaches to the analysis of spatial inequality, since local indicators are insufficient to explain interregional differences [1]. Studies by Achten and Lessmann, which use exogenous variation in geographical characteristics, show how spatial inequality affects economic activity in both existing and artificially constructed countries. Regional inequality is one of the key components of national inequality [2]. The level of inequality within a region and in neighbouring regions directly influences output growth and the stability of the regional economy [3]. Spatial inequality is also closely related to educational differentiation [4], the marginalisation of vulnerable groups [5, 6] and the erosion of social cohesion [7–10].

The literature underlines that public policy must take spatial effects into account, since ignoring interregional dependencies leads to misguided decisions [11, 12]. This makes it necessary to apply spatial econometrics, which allows the temporal and spatial structure of the data to be modelled jointly.

Pace and LeSage highlight the advantages of using the Spatial Durbin Model (SDM) [13]. Their analysis shows that when spatial dependence is present both in the disturbances and between the dependent and explanatory variables, traditional methods such as ordinary least squares (OLS) produce biased estimates due to unobserved spatial effects. The SDM partly

alleviates this problem by including spatial lags of both the dependent and explanatory variables in the specification. Ezcurra and Rios show that the Spatial Durbin Model allows for a better treatment of endogeneity arising from spatial interrelations between regions and reduces estimation bias [14].

Study [15] stresses the importance of spatial econometric models in the analysis of regional economic growth, since such factors as human capital, physical capital, transport infrastructure and industrial structure exert heterogeneous spatial effects. For this reason, the Spatial Durbin Model is used to assess spatial correlation and the impact of these factors. The SDM reveals complex and often opposing effects of economic factors on local and neighbouring regional economies, which underscores the importance of spatial aspects in the analysis of economic growth.

More broadly, the Spatial Durbin Model has been applied not only in economics and statistics. For example, in [16] the authors start from a spatial autoregressive model (SAR) and then employ the Durbin specification as a more general model by adding spatial lags of the independent variables in order to identify factors affecting the incidence of diarrhoeal diseases. Article [17] examines the impact of natural resources on economic growth, taking into account spatial linkages between countries. The authors use a panel of countries over roughly 20 years and the SDM to estimate the direct effect of resource abundance on growth within a country and the indirect effect reflecting the influence of neighbouring countries' resources. In [18], the Spatial Durbin Model is applied to assess the impact of agglomeration, using three types of weight matrices: a purely geographical one and two economic-geographical matrices that take into account distance and differences in GRP/GDP. This approach increases the robustness of the results and makes it possible to compare the strength of purely geographical and economic-spatial linkages.

Most studies emphasise the versatility of the SDM: it allows one to account simultaneously for (i) the spatial lag of the dependent variable, (ii) spatial lags of the explanatory variables and (iii) the decomposition of impacts into direct and indirect effects, as well as to evaluate the total effect of factors on a region and its neighbours.

Thus, the research problem consists in constructing a model that correctly captures persistent interregional dependence and allows the influence of key factors-government expenditure, unemployment, investment and the initial level of GRP per capita-to be decomposed into effects on both local and neighbouring regions. This justifies the choice of the Spatial Durbin Model as the main tool for analysing interregional inequality in Kazakhstan.

3 Materials and methods

3.1 Data

Regional data from the National Statistics Bureau of the Agency for Strategic Planning and Reforms of the Republic of Kazakhstan were taken from (<https://taldau.stat.gov.kz/ru/NewIndex/GetAnalytics/2931488>). The sample covers the period 2001–2017 and includes a balanced panel of 16 territorial units (14 regions and the cities of Almaty and Astana), yielding 272 observations. All variables are expressed in natural logarithms. The empirical analysis includes the following indicators:

- logarithm of GRP per capita (dependent variable);

- logarithm of GRP per capita in the base year (initial level);
- logarithm of government expenditure per capita;
- logarithm of the unemployment rate;
- logarithm of investment in fixed capital per capita.

Because the sample period ends before the administrative reforms of 2018 and 2022, the territorial classification remains unchanged throughout the study period. This avoids the panel harmonisation problems discussed in recent studies on Kazakhstan's regional restructuring [19–21].

3.2 Research methods

To analyse spatial inequality across the regions of Kazakhstan, the study uses the Spatial Durbin Model (SDM). By taking spatial dependence into account, this model extends traditional regression analysis, since variables in one region may depend not only on their own values but also on the corresponding values observed in neighbouring regions. The SDM provides a more informative representation of regional interaction and helps reveal dependencies that may be overlooked in non-spatial models. As shown by Pace and LeSage [13], the SDM is often preferred to the SAR and SEM models because it allows the effects of explanatory variables to be decomposed into direct and indirect components.

3.3 Model specification

As a baseline specification, we use the Spatial Durbin Model (SDM) for panel data, which accounts for spatial lags of both the dependent and independent variables. The dependent variable Y_{it} denotes gross regional product (GRP) per capita in region i at time t (in tenge). In the model we use its logarithm

$$y_{it} = \ln Y_{it}.$$

The set of explanatory variables includes the logarithms of the following indicators:

$$\begin{aligned} x_{1,it} &= \ln X_{1,it} && \text{government expenditure per capita in region } i \text{ at time } t; \\ x_{2,it} &= \ln X_{2,it} && \text{unemployment rate (expressed as a fraction);} \\ x_{3,it} &= \ln X_{3,it} && \text{investment in fixed capital per capita.} \end{aligned}$$

In addition, we use the initial level of GRP per capita, $\ln Y_{i0}$, the logarithm of GRP per capita in the base year for region i . In the empirical specification it is included as a separate regressor, together with its spatial lag.

We consider a set of regions $i = 1, \dots, N$ and time periods $t = 1, \dots, T$. The spatial structure is described by a binary contiguity matrix

$$W = (w_{ik})_{i,k=1}^N,$$

where

$$w_{ik} = \begin{cases} 1, & \text{if region } i \text{ shares a border with region } k, \\ 0, & \text{otherwise.} \end{cases}$$

As a baseline specification, we use a binary contiguity matrix because it captures the most immediate channel of interregional interaction through shared administrative borders. For Kazakhstan, territorial adjacency provides a natural first-order approximation of spatial spillovers in regional development. To assess robustness, the main specification was also compared with an alternative spatial weight matrix, and the key results remained qualitatively unchanged.

Define the diagonal matrix of row sums

$$D = \text{diag} \left(\sum_{k=1}^N w_{1k}, \dots, \sum_{k=1}^N w_{Nk} \right),$$

and the row-standardised spatial weight matrix

$$\widetilde{W} = D^{-1}W.$$

Each row of \widetilde{W} sums to one, so $(\widetilde{W}z)_i$ can be interpreted as the average of the vector z over the neighbours of region i .

We introduce the linear spatial smoothing operator

$$L_s : \mathbb{R}^N \rightarrow \mathbb{R}^N, \quad L_s(z) = \widetilde{W}z.$$

In vector notation, for each time period t we set

$$y_t = (\ln Y_{1t}, \dots, \ln Y_{Nt})^\top \in \mathbb{R}^N,$$

$$X_{jt} = (\ln X_{j,1t}, \dots, \ln X_{j,Nt})^\top, \quad j = 1, 2, 3,$$

and the vector of initial conditions

$$y_0 = (\ln Y_{10}, \dots, \ln Y_{N0})^\top.$$

The initial level of GRP per capita, y_0 , is treated as a structural initial condition capturing persistent cross-regional heterogeneity at the starting point of the sample. Its interpretation in the fixed-effects specification is therefore cautious and relates to long-run initial disparities rather than short-run within-region variation.

The baseline SDM with a spatial lag of the dependent variable and spatial lags of the explanatory variables can then be written as

$$y_t = \rho L_s(y_t) + \sum_{j=1}^3 \beta_j X_{jt} + \sum_{j=1}^3 \theta_j L_s(X_{jt}) + \gamma y_0 + \delta L_s(y_0) + \mu + \varepsilon_t, \quad (1)$$

where

$\rho \in \mathbb{R}$ is the spatial autoregressive coefficient of the dependent variable;

β_j are local (within-region) effects of the factors;

θ_j are indirect (between-region) effects operating through spatial lags;

γ and δ capture the effects of the initial GRP per capita level and its spatial lag;

$\mu \in \mathbb{R}^N$ is the vector of region-specific effects (treated as fixed in the

fixed-effects specification or as random in the random-effects specification);

$\varepsilon_t \sim \mathcal{N}(0, \sigma^2 I_N)$ is the vector of idiosyncratic errors.

Substituting the definition of L_s into model (1), we can rewrite it as

$$y_t = \rho \widetilde{W} y_t + \sum_{j=1}^3 \beta_j X_{jt} + \sum_{j=1}^3 \theta_j \widetilde{W} X_{jt} + \gamma y_0 + \delta \widetilde{W} y_0 + \mu + \varepsilon_t. \quad (2)$$

Moving the spatial lag of the dependent variable to the left-hand side gives

$$(I_N - \rho \widetilde{W}) y_t = \sum_{j=1}^3 \beta_j X_{jt} + \sum_{j=1}^3 \theta_j \widetilde{W} X_{jt} + \gamma y_0 + \delta \widetilde{W} y_0 + \mu + \varepsilon_t. \quad (3)$$

The matrix $I_N - \rho \widetilde{W}$ is nonsingular for all values of ρ such that

$$\rho \lambda_k \neq 1 \quad \text{for all eigenvalues } \lambda_k \text{ of } \widetilde{W}.$$

In this case the solution to (3) can be written as

$$y_t = S(\rho) \left[\sum_{j=1}^3 \beta_j X_{jt} + \sum_{j=1}^3 \theta_j \widetilde{W} X_{jt} + \gamma y_0 + \delta \widetilde{W} y_0 + \mu \right] + S(\rho) \varepsilon_t, \quad (4)$$

where

$$S(\rho) = (I_N - \rho \widetilde{W})^{-1} \quad (5)$$

is the spatial multiplier. Under the condition

$$\max_k |\rho \lambda_k| < 1,$$

it admits the series representation

$$S(\rho) = I_N + \rho \widetilde{W} + \rho^2 \widetilde{W}^2 + \rho^3 \widetilde{W}^3 + \dots, \quad (6)$$

which can be interpreted as capturing neighbour effects of all orders.

In the SDM, the coefficients β_j and θ_j are not themselves marginal effects. The impact of each explanatory variable X_j on y_t is described by the matrix of partial derivatives

$$\frac{\partial y_t}{\partial X_{jt}} = S(\rho) (\beta_j I_N + \theta_j \widetilde{W}). \quad (7)$$

The diagonal elements of (7) correspond to direct (within-region) effects, while the off-diagonal elements represent indirect (spatial spillover) effects. The average impacts are summarised as follows.

Direct effect:

$$DE_j = \frac{1}{N} \text{tr}(S(\rho) (\beta_j I_N + \theta_j \widetilde{W})), \quad (8)$$

Indirect effect:

$$IE_j = \frac{1}{N} \left[\mathbf{1}^\top S(\rho) (\beta_j I_N + \theta_j \widetilde{W}) \mathbf{1} - \text{tr}(S(\rho) (\beta_j I_N + \theta_j \widetilde{W})) \right], \quad (9)$$

Total effect:

$$TE_j = DE_j + IE_j, \quad (10)$$

where $\mathbf{1} = (1, \dots, 1)^\top \in \mathbb{R}^N$. These expressions are used below to interpret the direct, indirect and total impacts of government expenditure, unemployment, investment and initial GRP per capita on regional GRP per capita.

Assuming normality of the errors, the log-likelihood function for model (2) takes the form

$$L(\beta, \theta, \gamma, \delta, \rho, \sigma^2) = -\frac{NT}{2} \ln(2\pi\sigma^2) + T \ln \det(I_N - \rho \widetilde{W}) - \frac{1}{2\sigma^2} \sum_{t=1}^T \varepsilon_t^\top \varepsilon_t, \quad (11)$$

here, $\det(I_N - \rho \widetilde{W})$ is the Jacobian term of the spatial transformation and

$$\varepsilon_t = y_t - \rho \widetilde{W} y_t - \sum_{j=1}^3 \beta_j X_{jt} - \sum_{j=1}^3 \theta_j \widetilde{W} X_{jt} - \gamma y_0 - \delta \widetilde{W} y_0 - \mu.$$

The parameters are estimated by maximum likelihood. In the empirical analysis we consider two specifications: the SDM with regional random effects and the SDM with regional fixed effects. Their comparison is based on information criteria (AIC/BIC) and the Hausman test, which indicates that the fixed-effects SDM is preferred for our data.

4 Results and discussions

This section presents the estimation results for the Spatial Durbin Model (SDM) under regional random-effects and fixed-effects specifications. In both specifications, the dependent variable is $\ln Y_{it}$ (logarithm of GRP per capita), while the explanatory variables include the logarithm of initial GRP per capita $\ln Y_{i0}$, government expenditure per capita $\ln X_{1,it}$, unemployment $\ln X_{2,it}$, investment in fixed capital per capita $\ln X_{3,it}$, and their spatial lags $\widetilde{W}(\cdot)$.

As discussed in Section 3.3, the structural coefficients β_j and θ_j in the SDM do not coincide with marginal effects. The overall impact of each regressor is characterised by the matrix of partial derivatives (7) and by the average direct, indirect and total effects defined in (8–10). In what follows, we interpret the signs and statistical significance of the estimated coefficients in a way that is consistent with these SDM impact measures.

Table 1 reports the estimation results for the SDM with regional random effects. In this specification, the coefficient on initial GRP per capita, $\ln Y_0$, is small and statistically insignificant, whereas the coefficient on its spatial lag $\widetilde{W}(\ln Y_0)$ is positive and weakly significant. This suggests that, under the random-effects specification, initial conditions in neighbouring regions may be more relevant for the current level of GRP per capita than a region's own initial level. In SDM terms, this corresponds to a positive average indirect effect of initial conditions, while the direct effect is imprecisely estimated.

Table 1 – Spatial Durbin model with random effects

Variables	Coefficients	Coefficients with spatial lag (W)	Spatial variable
$\ln Y_0$	0.0160	0.0593*	
	(0.0201)	(0.0319)	
$\ln X_1$	0.00221	-0.0514*	
	(0.0218)	(0.0281)	
$\ln X_2$	0.247**	-0.139	
	(0.0981)	(0.118)	
$\ln X_3$	0.00131	-0.0336	
	(0.0194)	(0.0219)	
ρ			0.575***
			(0.0855)
Constant	0.300**		
	(0.138)		
Number of observations	272	272	272
R-squared	0.180	0.180	0.180
Number of ID	16	16	16
Robust standard errors in parentheses.			
*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.			
Note – compiled by the authors.			

For government expenditure per capita, $\ln X_1$, the own-region coefficient is close to zero and statistically insignificant, indicating no robust direct relationship between GRP per capita and internal public spending. By contrast, the coefficient on the spatial lag $\widetilde{W}(\ln X_1)$ is negative and statistically significant: higher government expenditure in neighbouring regions is associated with lower GRP per capita in the region under consideration. This result can be interpreted as evidence of interregional competition for investment, labour and budgetary resources, i.e. a negative spillover (indirect) effect of public spending.

The unemployment variable, $\ln X_2$, enters with a positive and statistically significant coefficient, while its spatial lag is statistically insignificant. Controlling for other factors, regions with higher registered unemployment tend to exhibit higher GRP per capita. A plausible explanation is that this reflects structural unemployment and the reallocation of labour from low-productivity activities to more productive sectors. Within the SDM framework, the average direct effect of unemployment is positive, whereas spatial spillovers via neighbouring unemployment levels are weak and not precisely estimated.

Investment in fixed capital per capita, $\ln X_3$, has insignificant coefficients both for the own-region term and for its spatial lag. This suggests that, over the period and within the specification considered, the volume of investment does not exert a clear short-run impact on GRP per capita. Possible explanations include implementation lags of investment projects and the predominance of quantitative rather than qualitative improvements in the capital stock.

The spatial lag of the dependent variable is captured by the parameter ρ . Its estimate, $\rho = 0.575$, is positive and highly significant at the 1% level, which indicates strong spatial

dependence: GRP per capita in a given region is strongly influenced by GRP per capita in neighbouring regions. In terms of the partial-derivative matrix (7), this means that the spatial multiplier

$$S(\rho) = (I_N - \rho\widetilde{W})^{-1}$$

substantially amplifies both direct and indirect effects, making spatial spillovers an essential part of the system's response to changes in the explanatory variables.

Table 2 presents the estimation results for the SDM with regional fixed effects. Comparing the AIC and BIC values across alternative specifications (not reported here for brevity), together with the Hausman test, we conclude that the fixed-effects SDM provides a preferable description of the data. Fixed regional effects allow us to control for time-invariant unobserved heterogeneity, separating it from genuine spatial interactions.

Table 2 – Spatial Durbin model with spatial fixed effects

Variables	Coefficients	Coefficients with spatial lag (W)	Spatial variable
ln Y_0	0.370*** (0.105)	-0.165 (0.129)	
ln X_1	-0.0271 (0.0270)	-0.118** (0.0545)	
ln X_2	0.451*** (0.137)	-0.162 (0.155)	
ln X_3	0.0222 (0.0268)	-0.0401 (0.0338)	
ρ			0.570*** (0.0782)
Number of observations	272	272	272
R-squared	0.034	0.034	0.034
Number of ID	16	16	16
Robust standard errors in parentheses.			
*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.			
Note – compiled by the authors.			

The relatively low R -squared reported for the fixed-effects specification should be interpreted with caution. In spatial panel models with fixed effects, this statistic reflects only a limited part of model fit and is not directly comparable to the R -squared of non-spatial or pooled regressions. For this reason, model selection in the present study relies primarily on the Hausman test, information criteria, and the statistical significance of the spatial autoregressive parameter. The coefficient on initial GRP per capita, ln Y_0 , is positive and statistically significant. This confirms that regions with a higher starting level of GRP per capita tend to maintain a higher current level, even after controlling for fixed effects and spatial dependence. In terms of the SDM impact measures, this result is consistent with a positive average direct effect of initial conditions and, given the positive spatial dependence, a positive total effect, reflecting path dependence and the role of accumulated advantages.

For government expenditure, $\ln X_1$, the own-region coefficient is negative but statistically insignificant, indicating no robust direct effect of internal public spending on GRP per capita. However, the coefficient on the spatial lag $\widetilde{W}(\ln X_1)$ is negative and statistically significant at the 5% level. Thus, an increase in government expenditure in neighbouring regions is associated with a reduction in GRP per capita in the given region. This again points to interregional competition for mobile resources and public funds. In terms of the decomposition (8-10), the average indirect effect of government expenditure is negative, while the direct effect remains statistically weak. Comparing Table 1 and Table 2, we see that the negative spillover effect of government expenditure is robust to the choice between random and fixed regional effects.

The unemployment variable, $\ln X_2$, retains a positive and statistically significant association with GRP per capita in the fixed-effects specification, while the coefficient on its spatial lag remains negative and statistically insignificant. This supports the interpretation that higher measured unemployment is linked to structural adjustment processes and sectoral shifts rather than to simple underutilisation of labour. At the same time, there is no clear evidence of systematic spillover effects of unemployment from neighbouring regions: the corresponding indirect effect is statistically unstable and should be interpreted with caution.

Investment in fixed capital, $\ln X_3$, again exhibits statistically insignificant coefficients for both the own-region and spatial-lag terms. Thus, the fixed-effects SDM confirms that, in the short run and at the level of aggregate GRP per capita, investment volumes do not have a stable and precisely estimated impact. This is consistent with the presence of time lags between investment and realised output effects and with the importance of the qualitative composition of investment.

The estimated spatial autoregressive parameter of the dependent variable remains large and highly significant in the fixed-effects model, $\rho = 0.570$. This reinforces the conclusion that GRP per capita in Kazakhstan's regions is strongly interconnected across space and that spatial spillovers are a persistent feature of the regional economy.

To provide a more substantive interpretation of the SDM estimates, Table 3 reports the reconstructed average direct, indirect, and total effects implied by the fixed-effects specification. Since the coefficients of the Spatial Durbin Model do not directly coincide with marginal effects, this decomposition helps to distinguish between own-region impacts and cross-regional spillovers.

Variables	Direct effect	Indirect effect	Total effect
$\ln Y_0$	0.380	0.097	0.477
$\ln X_1$	-0.057	-0.281	-0.337
$\ln X_2$	0.472	0.200	0.672
$\ln X_3$	0.016	-0.058	-0.042
Notes – The table reports reconstructed point estimates of the average direct, indirect, and total effects implied by the fixed-effects SDM. The values are obtained from the reported FE-SDM coefficients and the baseline binary contiguity matrix under row-standardization.			
Because the original post-estimation environment is no longer fully recoverable, bootstrapped standard errors are not reported; therefore, these estimates should be interpreted as approximate.			
Note – compiled by the authors.			

The reconstructed effects in Table 3 indicate that the initial level of GRP per capita, $\ln Y_0$, has a positive direct effect and a positive total effect. This supports the conclusion that regional development in Kazakhstan is path dependent: regions with stronger initial economic positions tend to preserve their advantages over time. Government expenditure per capita, $\ln X_1$, displays the strongest negative indirect effect. This suggests that higher public spending in one region may be associated with adverse spillover effects in neighbouring regions, possibly through competition for mobile resources, fiscal advantages, or investment flows. The unemployment rate, $\ln X_2$, shows positive direct and indirect effects. In the institutional context of Kazakhstan, this result may reflect structural adjustment, labour-market formalization, and capital-intensive industrial growth rather than a purely cyclical weakening of labour demand. By contrast, investment in fixed capital per capita, $\ln X_3$, demonstrates only weak reconstructed effects overall, with a small positive direct effect and a slightly negative indirect effect.

The Hausman test,

$$\chi^2(9) = 83.69, \quad p = 0.000,$$

clearly favours the fixed-effects specification over the random-effects one. This indicates that regional heterogeneity is systematic rather than purely random and should be modelled through fixed regional effects. Consequently, the interpretation of the direct and indirect effects DE_j and IE_j in the context of regional growth should primarily rely on the fixed-effects SDM.

4.1 Moran's scatterplots

The Moran's scatterplots are used as descriptive exploratory tools and are constructed for several related regional indicators, including unemployment, investment, GRP growth, and government expenditure. Figure 1 shows Moran's scatterplot for unemployment by region, with each point representing one region of Kazakhstan. The horizontal axis of the graph shows unemployment in the region itself, while the vertical axis shows the average unemployment rate in the neighboring regions. The upward slope of the line and the positive value ($I = 0.179$)

indicate a weak positive spatial correlation, suggesting that regions with high unemployment tend to be adjacent to other regions with similarly high unemployment, and vice versa.



Figure 1: Moran's scatterplot for unemployment by region

Figure 2 shows Moran's scatterplot for investment by region. The horizontal axis of the graph shows the amount of investment in the region itself, while the vertical axis shows the average level of investment in neighboring regions. The diagram also shows a weak spatial effect ($I = 0.184$), i.e., regions with low investment are adjacent to similar regions, while regions with high investment are located next to similar regions.

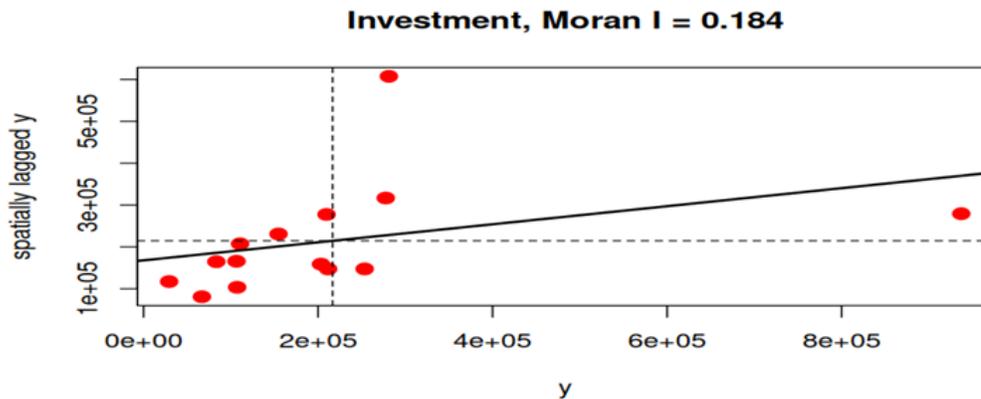


Figure 2: Moran's scatterplot for investment by region

Figure 3 shows Moran's scatterplot for GRP growth rates by region. This diagram shows a reverse spatial effect ($I = -0.247$), which means that regions with high growth are usually adjacent to regions with low growth and vice versa.

Figure 4 shows Moran's diagram for government spending by region. The diagram shows a noticeable positive spatial dependence ($I = 0.299$). Regions with high spending are adjacent

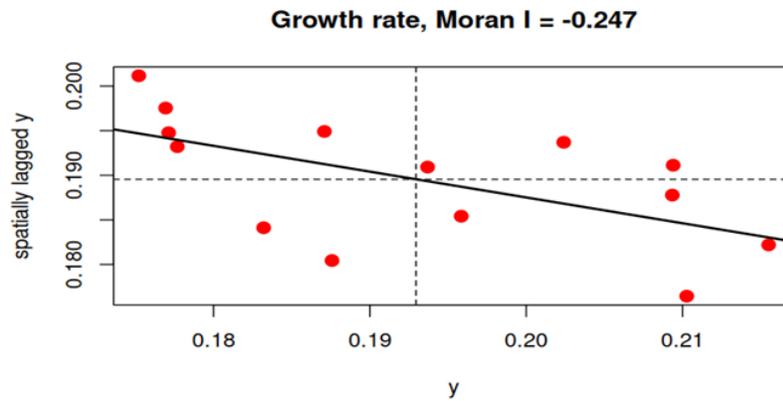


Figure 3: Moran's scatterplot for GRP growth rates by region

to similar regions, and, conversely, regions with low government spending are adjacent to similar regions.

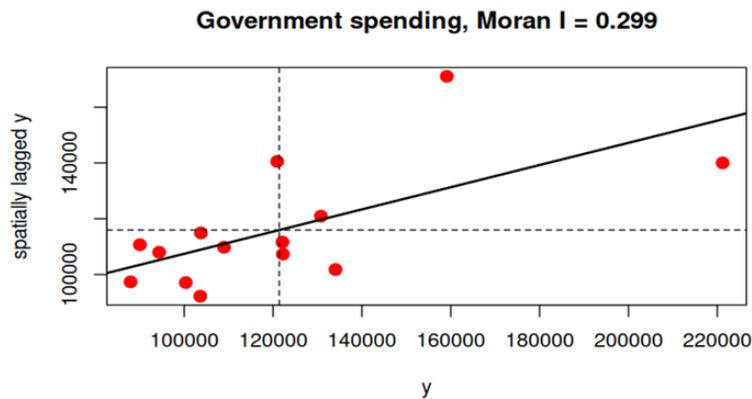


Figure 4: Moran's scatterplot for government spending by region

It is important to note that the Moran's I scatterplots presented in Figures 1–4 are constructed using the temporal averages (cross-sectional means) of the respective variables over the entire 2001-2017 study period. This methodological approach was specifically utilized to capture and illustrate stable, long-term spatial clustering patterns across the regions, effectively smoothing out any short-term annual fluctuations.

5 Conclusions

The empirical results confirm that the Fixed Effects Spatial Durbin Model (SDM) provides a better description of regional dynamics than alternative specifications, as supported by the

Akaike (AIC) and Bayesian (BIC) information criteria. The estimated spatial autoregressive parameter ($\rho \approx 0.57$) indicates significant spatial dependence in regional GRP per capita.

The reconstructed effects show that the initial level of GRP per capita has a positive direct and total effect, confirming path dependence in regional development. The unemployment rate displays positive direct and indirect effects, which in Kazakhstan are more plausibly associated with structural adjustment and capital-intensive growth than with a standard cyclical labour-market mechanism. Government expenditure exhibits the strongest negative indirect effect, suggesting adverse spillover effects across neighbouring regions.

These findings imply that reducing interregional inequality in Kazakhstan requires coordinated regional policy, with particular attention to the spatial consequences of fiscal decisions, investment allocation, and labour-market adjustment.

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