Simulation of Supersonic Flows with the Non-Reflecting Boundary Conditions

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Abstract

This paper presents a formulation of the NSCBC (Navier- Stocks characteristic boundary conditions) for the problem of turbulent supersonic gas flow in a plane channel with a perpendicular injection jets. The non-reflection boundary conditions for direct modeling of compressible viscous gases are studied through boundaries for the subsonic inflow and subsonic non-reflection outflow situations. Verification of the constructed algorithm of boundary conditions is carried out by solving a test problem of perpendicular sound of jets injection into a supersonic gas flow in a plane channel at different three inflow Mach Numbers ($M_\infty = 2.75, M_\infty = 3.75$ and $M_\infty = 4.75$.).

Introduction

For the problem of turbulent supersonic gas flow in a plane channel with a perpendicular injection jets there are two types of boundary conditions, the first is physical which is specifies the known physical behavior of one or more of the dependent variables at the boundaries, and the second - is numerical boundary conditions which are necessary when the number of physical boundary conditions are less than the number of dependent variables. Also the numerical boundary conditions are necessary to exclude all wrong reflections of propagating incoming waves from the inside of the domain to the outside (subsonic non-reflecting outflow) as in [1], and from the outside of the domain to the inside (subsonic non-reflecting inflow). These waves require a specific treatment as quoted here from [2-3] for the Navier-Stokes equations. In this paper is numerically simulated planar turbulent supersonic air flow with a transverse hydrogen injection from the channel walls based on the method developed for solving the Navier-Stokes equations for multi-component gas mixture flow at($M_\infty = 2.75, M_\infty = 3.75$ and $M_\infty = 4.75$.). For the convenience of computation is considered that, the injecting of the jet with the bottom wall. Flow scheme is shown in Fig. 1. The computational code is developed On the basis of the fourth order weighted essentially non-oscillatory (WENO) schemes, the NSCBC (Navier- Stocks characteristic boundary conditions) for supersonic flows of compressible viscous multi-component gas at the boundariesis introduced in the following situations:

I. A subsonic inflow.

II. A subsonic non-reflection outflow.

1.Model Equations

In Cartesian coordinates two-dimensional Reynolds-averaged Navier-Stokes equations for multi- components flow in conservation form are:

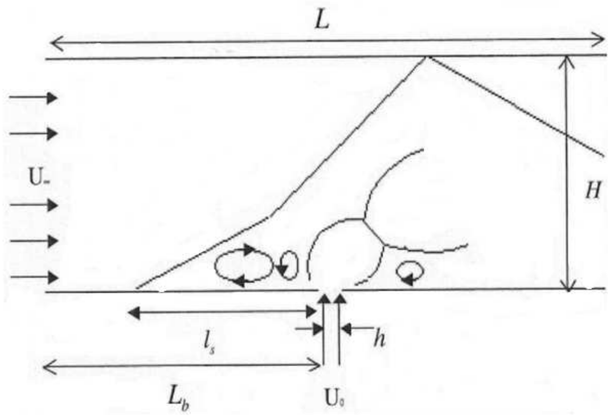


Fig.1 Flow diagram

$$\frac{\partial \vec{U}}{\partial t}+\frac{\partial\left(\vec{E}-\vec{E}_v\right)}{\partial x}+\frac{\partial\left(\vec{F}-\vec{F}_v\right)}{\partial z}=0 \qquad (1)$$

$$\vec{U}=\begin{pmatrix}\rho\\ \rho u\\ \rho w\\ E_t\\ \rho Y_k\end{pmatrix},\quad \vec{E}=\begin{pmatrix}\rho u\\ \rho u^2+P\\ \rho uw\\ (E_t+P)\,u\\ \rho uY_k\end{pmatrix},\quad \vec{F}=\begin{pmatrix}\rho w\\ \rho uw\\ \rho w^2+P\\ (E_t+P)\,w\\ \rho wY_k\end{pmatrix},$$

The components $\vec{E}_v$, $\vec{F}_v$ are written in the form:

$$\vec{E}_v=(0,\quad \tau_{xx},\quad \tau_{xz},\quad u\tau_{xx}+u\tau_{xz}-q_x,\quad J_{kx})^T,$$

$$\vec{F}_v=(0,\quad \tau_{xz},\quad \tau_{zz},\quad w\tau_{xz}+w\tau_{zz}-q_z,\quad J_{kz})^T,$$

$$P=\frac{\rho T}{\gamma_\infty M_\infty^2}(\sum_{i=1}^{N}\frac{Y_i}{W_i}),\quad \sum_{i=1}^{N}Y_i=1, \qquad (2)$$

$$E_t=\frac{\rho}{\gamma_\infty M_\infty^2}\sum_{i=1}^{N}Y_ih_i-P+\frac{1}{2}\rho(u^2+w^2), \qquad (3)$$

$$h_i=h_i^0+\int_{T_0}^{T}c_{Pi}dT,\quad c_{Pi}=C_{Pi}/W.$$

Where the molar specific heat C_{Pi} of the $i-$th components is given in terms of the fourth degree polynomial with respect to temperature which constants can be found in the JANAF Thermochemical Tables [4], Y_i is the mass fraction of the $i-$th components,$\tau_{xx}, \tau_{xz}, \tau_{zz}, \tau_{zx}$,are the viscous stress tensors , q_x, q_z, J_{xi}, J_{zi} are the heat and diffusion flux (diffusion fluxes are defined from Fick's Law).$\mu=\mu_L+\mu_t$ is the sum of the coefficients of the laminar and turbulent viscosity. The Baldwin - Lomax model is used for determiningμ_t. The system equation

(1)-(3) is written in a dimensionless form and conventional notation. The governing parameters are the entrance parameters, the pressure and total energy are normalized to $\rho_\infty u_\infty^2$ the enthalpy - to $R^0 T_\infty / W_\infty$, the molar specific heat - to R^0. The boundary conditions have the following form: At the entrance: $W_i = W_{i0}, P = nP_\infty, T = T_0, w = M_0\sqrt{\frac{\gamma_0 R^0 T_0}{W_0}}, u = 0, Y_i = Y_{i0}, z = 0, L_b \leq x \leq L_b + h$;
($n = P_0/P_\infty$is the jet pressure ratio, is the jet pressure, and is the flow pressure); on the lower wall the no-slip condition and the adiabatic wall condition are imposed; on the upper boundary the condition of symmetry is assumed; on the outflow the nonreflecting boundary condition is derived.

2. Method of the solution

The numerical solution of the system (1)-(3) is calculated by the two steps. The first is defined dynamic parameters and second mass species. The approximation of convection terms is performed on the basis of the fourth order weighted essentially non-oscillatory (WENO) scheme. The WENO scheme is constructed on the basis of ENO scheme [1]. However, in WENO scheme instead of choosing one interpolating polynomial, is used a convex combination of all corresponding polynomials. This is done by introducing weight coefficients to the convex combination. In approximation of derivatives in diffusion terms were used second-order central-difference operators.

3.The implementation of non-reflection boundary conditions

Implementation of a true non-reflecting boundary conditions based on the NSCBC theory such that variables which are not imposed by physical boundary conditions are computed on the boundaries by solving the conservation equations as in the domain. The wave propagation is assumed to be associated only with the hyperbolic part of the Navier-Stokes equations; waves associated with the diffusion process are neglected. In characteristics analysis, absence of reflection is enforced by correcting the amplitude of the incoming characteristic (wave reflected by the boundary) to zero. To formulate the NSCBC approach for the inflow and outflow boundaries first we consider a boundary located at x=0 for the inflow boundaries, and x=L for the outflow boundaries. The characteristic analysis [2] is used to modify the hyperbolic terms of Eqs. (1)-(3) corresponding to waves propagating in x=const. direction:

$$\frac{\partial \rho}{\partial t} + d_1 + \frac{\partial(\rho w)}{\partial z} = 0 \tag{4}$$

$$\frac{\partial \rho u}{\partial t} + ud_1 + \rho d_2 + \frac{\partial(\rho u w)}{\partial z} = \frac{\partial \tau_{xz}}{\partial z} \tag{5}$$

$$\frac{\partial \rho w}{\partial t} + wd_1 + \rho d_3 + \frac{\partial(\rho w^2)}{\partial z} = \frac{\partial \tau_{zz}}{\partial z} \tag{6}$$

$$\frac{\partial E_t}{\partial t} + \frac{1}{2}(u^2 + w^2)d_1 + \frac{d_4}{\gamma - 1} + \rho u d_2 + \rho w d_3 + \frac{\partial((E_t + P)w)}{\partial z} = \frac{\partial(w\tau_{xz} + w\tau_{zz} - q_z)}{\partial z} \tag{7}$$

Where

$$\begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{pmatrix} = \begin{pmatrix} \frac{1}{c^2}(Z_2 + \frac{1}{2}(Z_4 + Z_1)) \\ \frac{1}{2\rho c}(Z_4 - Z_1) \\ Z_3 \\ \frac{1}{4}(Z_4 + Z_1) \end{pmatrix} \tag{8}$$

and Z_i's are the amplitudes of the characteristic waves associated with each characteristic velocityλ_i. These characteristic velocities are given by [2]:

$$\lambda_1 = u - c, \lambda_2 = u, \lambda_3 = u, \lambda_4 = u + c \tag{9}$$

Where c is the speed of sound: $c^2 = \frac{\gamma P}{\rho}$ and Z_i's are given by:

$$Z_1 = \lambda_1(\frac{\partial P}{\partial x} - \rho c\frac{\partial u}{\partial x}), Z_2 = \lambda_2(\frac{\partial P}{\partial x} - c^2\frac{\partial \rho}{\partial x}), Z_3 = \lambda_3\frac{\partial w}{\partial x}, Z_4 = \lambda_4(\frac{\partial P}{\partial x} + \rho c\frac{\partial u}{\partial x}) \tag{10}$$

To specify the values of Z_i's of the incoming waves we use the LODI (the Local One-Dimensional Inviscid) relations [2]:

$$\frac{\partial \rho}{\partial t} + \frac{1}{c^2}[Z_2 + \frac{1}{2}(Z_4 + Z_1] = 0 \tag{11}$$

$$\frac{\partial u}{\partial t} + \frac{1}{2\rho c}(Z_4 - Z_1) = 0 \tag{12}$$

$$\frac{\partial w}{\partial t} + Z_3 = 0 \tag{13}$$

$$\frac{\partial P}{\partial t} + \frac{1}{4}(Z_4 + Z_1) = 0 \tag{14}$$

3.1 A subsonic inflow.

For a subsonic inflow the parameters u, w, and T are constant so we need only to solve two equations (4) and (7). Since u= const. and from (11) in (12) and (13) we get$Z_1 = Z_4$ and $Z_2 = (\gamma - 1)Z_1$ and from (12) we have $Z_3 = 0$ by substituting in (4) with these values of Z_i's we get,

$$\frac{\partial \rho}{\partial t} = -[\frac{\gamma}{c^2}(u - c)(\frac{\partial P}{\partial x} - \rho c\frac{\partial u}{\partial x}) + \frac{\partial \rho w}{\partial z}] \tag{15}$$

Applying an approximation for time step with second-order of accuracy then we get,

$$\rho_{ij}^{n+1} = \frac{4}{3}\rho_{ij}^{n} - \frac{1}{3}\rho_{ij}^{n-1} - \frac{2\triangle t}{3}[\frac{\gamma}{c^2}(u - c)(\frac{\partial P}{\partial x} - \rho c\frac{\partial u}{\partial x}) + \frac{\partial \rho w}{\partial z}]_{ij} \tag{16}$$

For total energy E_t we consider the following equation:

$$\frac{\partial E_t}{\partial t} = \frac{h(T)}{\gamma M_\infty^2}\frac{\partial \rho}{\partial t} + \frac{\rho}{\gamma M_\infty^2}\frac{\partial h(T)}{\partial t} - \frac{\partial P}{\partial t} + \frac{1}{2}u^2\frac{\partial \rho}{\partial t} + \rho u\frac{\partial u}{\partial t} \tag{17}$$

where $h(T)$ is the enthalpy and. Since u and T are constants and from the state equation:

$$P = \frac{\rho T}{\gamma M_\infty^2 W} \tag{18}$$

We have:

$$\frac{\partial E_t}{\partial t} = -(\frac{h(T)}{\gamma M_\infty^2} + \frac{1}{2}u^2 - \frac{T}{\gamma M_\infty^2 W})[\frac{\gamma}{c^2}(u - c)(\frac{\partial P}{\partial x} - \rho c\frac{\partial u}{\partial x}) + \frac{\partial \rho w}{\partial z}] \tag{19}$$

Again here we applying an approximation for time step with second-order of accuracy then we get:

$$(E_t)_{ij}^{n+1}=\frac{4}{3}(E_t)_{ij}^{n}-\frac{1}{3}(E_t)_{ij}^{n-1}+$$

$$+\frac{2\triangle t}{3}(\frac{h(T)}{\gamma M_\infty^2}+\frac{1}{2}u^2-\frac{T}{\gamma M_\infty^2 W})_{ij}[\frac{\gamma}{c^2}(u-c)(\frac{\partial P}{\partial x}-\rho c\frac{\partial u}{\partial x})+\frac{\partial \rho w}{\partial z}]_{ij} \quad (20)$$

Since $u=const$ and $w=const$ then

$$\frac{\partial \vec{U}}{\partial t}=(\frac{\partial \rho}{\partial t},u\frac{\partial \rho}{\partial t},w\frac{\partial \rho}{\partial t},\frac{\partial E_t}{\partial t}) \quad (21)$$

then by applying one finite difference approximation time step and from Eqs. (16) and (20) we get the following approximation at the inflow boundary

$$\vec{U}_{1j}^{n+1}=\begin{pmatrix}\rho_{1j}^{n+1}\\ u\rho_{1j}^{n+1}\\ w\rho_{1j}^{n+1}\\ (E_t)_{1j}^{n+1}\end{pmatrix} \quad (22)$$

3.2 A subsonic non-reflection outflow.

For a subsonic non-reflection outflow the static pressure at the outflow $P=P_\infty$ was imposed to define the amplitude of incoming wave as:

$$Z_1=K(P-P_\infty) \quad (23)$$

Corresponding to the negative characteristic velocity $\lambda_1=u-c$ where $K=\sigma(1-M_\infty^2)c/L$ (σ is constant and L is a characteristic size of the domain) and others physical conditions are:

$$\frac{\partial \tau_{xx}}{\partial x}=0,\frac{\partial \tau_{xz}}{\partial x}=0,\frac{\partial q_x}{\partial x}=0 \quad (24)$$

and finding the other's (Z_2, Z_3 and Z_4 by using interior points [2]. Now we substitute from Eqs. (9)-(10) and (23)-(24) in the system of equations (4)-(7). We get:

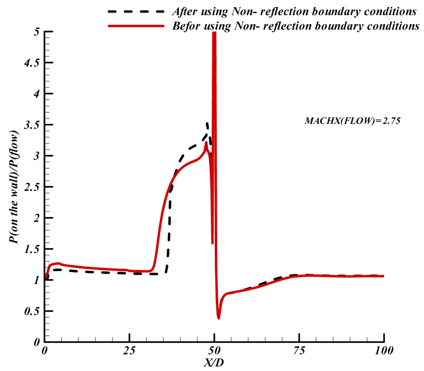
$$\frac{\partial \rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}-\frac{(u-c)}{2c^2}[\frac{\partial P}{\partial x}-\rho c\frac{\partial u}{\partial x}]+\frac{K}{2c^2}(P-P_\infty)+\frac{\partial(\rho w)}{\partial z}=0 \quad (25)$$

$$\frac{\partial \rho u}{\partial t}+\frac{\partial(\rho u^2+P)}{\partial x}-\frac{(u-c)^2u}{2c^2}[\frac{\partial P}{\partial x}-\rho c\frac{\partial u}{\partial x}]+\frac{(u-c)K}{2c^2}(P-P_\infty)+\frac{\partial(\rho uw)}{\partial z}=\frac{\partial \tau_{xz}}{\partial z} \quad (26)$$

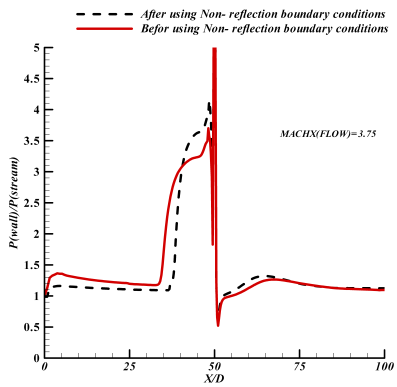
$$\frac{\partial \rho w}{\partial t}+\frac{\partial(\rho uw)}{\partial x}-\frac{(u-c)^2w}{2c^2}[\frac{\partial P}{\partial x}-\rho c\frac{\partial u}{\partial x}]+\frac{(u-c)K}{2c^2}(P-P_\infty)+\frac{\partial(\rho w^2+P)}{\partial z}=\frac{\partial \tau_{zz}}{\partial z} \quad (27)$$

$$\frac{\partial E_t}{\partial t}+\frac{\partial((E_t+P)u)}{\partial x}-[\frac{1}{2(\gamma-1)}+\frac{u^2+w^2}{4c^2}-\frac{u}{2c}][(u-c)[\frac{\partial P}{\partial x}-\rho c\frac{\partial u}{\partial x}]-K(P-P_\infty)]+$$

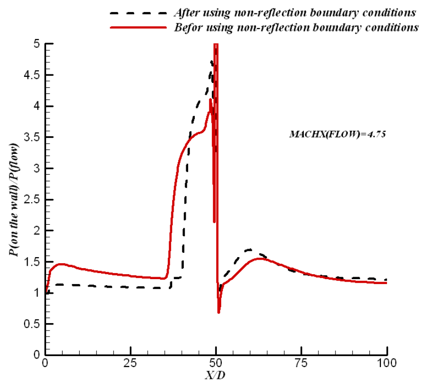
$$+\frac{\partial((E_t+P)w)}{\partial z}=\frac{\partial(w\tau_{xz}+w\tau_{zz}-q_z)}{\partial z} \quad (28)$$



(a)



(b)



(c)

Fig.2 Pressure profiles on the wall at different inflow Mach number (M_∞) $(A)M_\infty = 2.75$ $(B)M_\infty = 3.75$ $(C)M_\infty = 4.75$ $(- - -)$ *After applying NSCBC theory* $(\longrightarrow)$ *Before applying NSCBC theory*

Then we can rewrite the system of equations (25)-(28) in vector form as:

$$\frac{\partial \vec{U}}{\partial t} + \frac{\partial \vec{E}}{\partial t} - SUB + \frac{\partial \vec{F}}{\partial z} = \frac{\partial \vec{F}_v}{\partial z} \tag{29}$$

where

$$SUB = \begin{pmatrix} \frac{(u-c)}{2c^2}[\frac{\partial P}{\partial x} - \rho c\frac{\partial u}{\partial x}] + \frac{K}{2c^2}(P - P_\infty) \\ \frac{(u-c)^2 u}{2c^2}[\frac{\partial P}{\partial x} - \rho c\frac{\partial u}{\partial x}] + \frac{(u-c)K}{2c^2}(P - P_\infty) \\ \frac{(u-c)^2 w}{2c^2}[\frac{\partial P}{\partial x} - \rho c\frac{\partial u}{\partial x}] + \frac{(u-c)K}{2c^2}(P - P_\infty) \\ (\frac{1}{2(\gamma-1)} + \frac{u^2+w^2}{4c^2} - \frac{u}{2c})((u-c)[\frac{\partial P}{\partial x} - \rho c\frac{\partial u}{\partial x}] - K(P - P_\infty)) \end{pmatrix} \tag{30}$$

The finite difference approximation of (29) at the outflow boundary is given by:

$$\vec{U}^{n+1}_{Nj} = \vec{U}^n_{Nj} - \triangle t\{\frac{\partial \vec{E}^n}{\partial x} - SUB^n + \frac{\partial \vec{F}^n}{\partial z} - \frac{\partial \vec{F}^n_v}{\partial z}_{Nj}\} \tag{31}$$

4. Numerical results and discussions

The computations were done on a staggered spatial grid with range of parameters: $2.75 \leq M_\infty \leq 4.75, M_0 = 1$, $Pr = 0.7$, $2 \leq n \leq 15$, $T_0 = 642K$, $T_\infty = 800K$ $D = 0.1cm$, $H = 3.0cm$, $L_b = 5cm$ and $L = 10cm$, with Pressure ratio n = 10.26, $P_\infty = 1000Pa$, and different three inflow Mach numbers $M_\infty = 2.75$, $M_\infty = 3.75$, and $M_\infty = 4.75$, The computed results after applying the non-reflection boundaries condition at the entrance boundary are compared with that computed results before applying the non-reflection boundaries condition at the entrance boundary. We can see that from the Pressure profiles on the wall Figure 2, the implementation of the NSCBC method on the entrance boundary contributed to improve the numerical solution on the boundaries.

5. Conclusions

Numerical boundary conditions for the problem of turbulent supersonic gas flow in a plane channel with a perpendicular injection jets are constructed without any extrapolation. The NSCBC method is based on a local inviscid one-dimensional analysis of the waves crossing the boundary. The amplitude variations of the waves entering the domain are estimated from an analysis of the local one-dimensional inviscid equations. These amplitude variations are then used in a reduced set of conservation equations to determine boundary variables which were not specified by the physical boundary conditions.

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В работе предлагается граничные условия неотражения для задачи турбулентного сверхзвукового течения газа в плоском канале с перпендикулярным вдувом струй. Для этого записывается характеристическая форма системы уравнений Навье- Стокса для сверхзвуковых течений сжимаемого вязкого многокомпонентного газа на границах. После чего осуществляется постановка физических граничных условий в полученные уравнения. При его явной конечно- разностной реализации аппроксимация производных по пространственным направлениям осуществляется односторонними разностями со вторым порядком точности.

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Жұмыста жылдамдығы дыбыс жылдамдығынан жоғары тік ағыншасы бар жазық каналда турбуленттік ағындарды есептеуде шағылыспайтын шекаралық шарттар ұсынылады. Бұл үшін сығылмалы тұтқырлы көпкомпонентті газға арналған Навье-Стокс теңдеуі шекарада қорытылып, алынған теңдеулерге физикалық шекаралық шарттар қойылады. Жазықтық бойынша туындылардың ақырлы айырымдары үшін екінші ретті бір бағыттағы айырымдар сұлбасы қолданылады.