Controllability Criterion of Nonlinear Dynamical Systems

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Abstract. This paper devotes to controllability of nonlinear dynamical systems. Main result of this work is controllability criteria, received on the basic of application of the interval analysis.

Keywords: control, criterion, interval

1 Introduction

Currently, increased requirements for the design, operation of complex technical objects and technological processes, as well as management. In connection with this circumstance developed new mathematical models of dynamic processes described by nonlinear differential equations. Thus there is a need for further development of the theory of nonlinear dynamical systems, improve understanding of the goals of management. Many technical problems of the structure of controlled dynamic systems and its parameters are known with some error. Application of interval analysis will take into account such errors.

2 Formulation of the Problem

Consider a system of control described by nonlinear ordinary differential equations

$$\dot{x} = f(x, u, t) \tag{1}$$

where - n-n-vector, elements of which are the continuously differentiable functions by their arguments, x - n-dimensional system state vector, u -u - scalar control.

There given the limitations for control

$$u(t) \in U = u(t) : -L \leq u(t) \leq L, t \in [t_0, t_1]. \tag{2}$$

Investigating the problem of the existence of control, which satisfies to constraints (2) and transfers the system from the initial state

$$x(t_0) = x_0 \tag{3}$$

to the specified final state

$$x(t_1) = x_1 \tag{4}$$

in a fixed time $t_1 - t_0$ [1].

By the properties imposed on the right side of the system of equations of the Cauchy problem (1) - (3) for a fixed control $u(t) \in U$ followed the conditions of theorems of existence and uniqueness of solution $x(t), t \in [t_0, t_1]$ [2].

Rewrite the Cauchy problem (1) - (3) in the form of integral recurrence

$$x_{k+z}(t) = x_0 + \int_{t_0}^{t} f(x_k(\tau), u(\tau), \tau) d\tau \tag{5}$$

By the properties imposed on the right-hand side of equation (1), and limitations on the function $u(t)$ in [1] it is proved that the method of successive approximations (5) converges to the solution absolutely and uniformly for any fixed control.

Then the problem of control is reduced to the investigation of the following problem: whether there is at least one control $u(t) \in U$, in which the solution of the integral equation (5) at the time t_1 satisfies the condition (4).

To solve this problem we apply the results of the interval analysis [4]. We denote by $[f]_i = (f_i, 0)$ - the interval with center f_i and with radius 0, by $[\nu] = (0, L)$ - the interval from - L to L.

Substituting in equation (5) interval $[\nu] = (0, L)$ instead of function $u(t)$ we obtain the interval integral equation

$$[x]_{k+1}(t) = x_0 + \int_{t_0}^{t} [f]([x]_k(\tau), [\nu], \tau) d\tau \tag{6}$$

Theorem. In order for the investigating system was managed, there is necessary and sufficient that the given vector x_1 of the right-hand side of (4) belonged to interval vector $[x]_{k+1}(t)$.

As an example, consider the problem of Zermelo [5]. Managed process is described by the following system of differential equations

$$\dot{x_1} = \cos(x_3), \quad \dot{x_2} = \sin(x_3), \quad \dot{x_3} = u. \tag{7}$$

There given the limitations for control

$$u(t) \in U = u(t) : -0.5 \leq u(t) \leq 0.5, t \in [t_0, t_1]. \tag{8}$$

As an initial state given the coordinates

$$x(0) = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

As a final state given the coordinates

$$x(10) = \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}.$$

In [6] there are the calculations that the optimal speed time for the sample is in the range 3.5-5 seconds.

3 Program Code

For a numerical calculation with using of library of the interval mathematics [6] there developed a program in Delphi, and the main text of which is given below:

```
Const n=3;m=100;h=0.05;izik=10;kt=10;
Type Interval=record
   med,rad:real;
   end;
   MatIntr=array[1..n,1..n] of Interval;
   VecIntr=array[0..n] of Interval;
```

```
    TForm1 = class(TForm);

    var
  Form1: TForm1;
  x,xk,xn:array[1..m] of VecIntr;
  rr,rm,u:Interval;
  i,j,k,ij:integer;
  r1,r2,r3,r4,r5,r6,x1t,x2t,x3t,h1:real;
  ch:string;
  fotl:TextFile;
odin_p_int,odin_m_int,null_int:Interval;

begin
  odin_p_int.med:=1.0;  odin_p_int.rad:=0.0;
  odin_m_int.med:=-1.0; odin_m_int.rad:=0.0;
  null_int.med:=0.0; null_int.rad:=0.0;
AssignFile(fotl,'d:\yprav.txt');   rewrite(fotl);
  x1t:=3.0; x2T:=4.0; x3t:=0.0;
  u.med:=0.0; u.rad:=0.5;
   for i:=1 to m do begin
    for j:=1 to 3 do xk[i,j].rad:=0.0;
    xk[i,1].med:=(i-1)*x1t/m;
    xk[i,2].med:=(i-1)*x2t/m;
    xk[i,3].med:=(i-1)*x3t/m;
   end;
  for k:=1 to izik do begin
   for j:=1 to 3 do begin xn[1,j].med:=1.0;  xn[1,j].rad:=0.0;  end;
  { xn[1,3].med:=0.0;}
   for i:=1 to m-1 do begin
    rm:=xn[i,1];
    r5:=xk[i,3].med-xk[i,3].rad;  r3:=cos(r5);
    r6:=xk[i,3].med+xk[i,3].rad;  r4:=cos(r6);
    r1:=r3;
    r2:=r4;
    if (r1>r4) then r1:=r4;
    if (r2<r3) then r2:=r3;
    h1:=(r6-r5)/kt;
    for ij:=1 to kt-1 do begin
     r5:=r5+h1; r3:=cos(r5);
     if (r1>r3) then r1:=r3;
     if (r2<r3) then r2:=r3;
     end;
    r3:=0.5*(r1+r2); r4:=0.5*abs(r2-r1);
    rr.med:=h*r3;    rr.rad:=h*r4;
    AddInN(rm,rr,xn[i+1,1]);

    ch:='r1='+FormatFloat('######0.00',r1)+' ';
    ch:=ch+'r2='+FormatFloat('######0.00',r2)+' ';
```

```
    ch:=ch+'rr=('+FormatFloat('######0.00',rr.med)+' ';
    ch:=ch+FormatFloat('######0.00',rr.rad)+') ';
    ch:=ch+'rm=('+FormatFloat('######0.00',rm.med)+' ';
    ch:=ch+FormatFloat('######0.00',rm.rad)+') ';
    ch:=ch+'xn=('+FormatFloat('######0.00',xn[i+1,1].med)+' ';
    ch:=ch+FormatFloat('######0.00',xn[i+1,1].rad)+') ';

{    Writeln(fotl,ch);}
        rm:=xn[i,2];
    r5:=xk[i,3].med-xk[i,3].rad;  r3:=sin(r5);
    r6:=xk[i,3].med+xk[i,3].rad;  r4:=sin(r6);
    r1:=r3;
    r2:=r4;
    if (r1>r4) then r1:=r4;
    if (r2<r3) then r2:=r3;
    h1:=(r6-r5)/kt;
    for ij:=1 to kt-1 do begin
     r5:=r5+h1; r3:=sin(r5);
     if (r1>r3) then r1:=r3;
     if (r2<r3) then r2:=r3;
     end;
    r3:=0.5*(r1+r2); r4:=0.5*abs(r2-r1);
    rr.med:=h*r3;    rr.rad:=h*r4;
    AddInN(rm,rr,xn[i+1,2]);
       rm:=xn[i,3];
    rr.med:=h*u.med;    rr.rad:=h*u.rad;
    AddInN(rm,rr,xn[i+1,3]);

    ch:='k='+IntToStr(k)+' i='+IntToStr(i)+' ';
    ch:=ch+'t='+FormatFloat('######0.00',i*h)+' ';
    r1:=xn[i+1,1].med;r2:=xn[i+1,1].rad;
    ch:=ch+'('+FormatFloat('######0.00',r1)+' ';
    ch:=ch+FormatFloat('######0.00',r2)+') ';
        r1:=xn[i+1,2].med;r2:=xn[i+1,2].rad;
    ch:=ch+'('+FormatFloat('######0.00',r1)+' ';
    ch:=ch+FormatFloat('######0.00',r2)+') ';
        r1:=xn[i+1,3].med;r2:=xn[i+1,3].rad;
    ch:=ch+'('+FormatFloat('######0.00',r1)+' ';
    ch:=ch+FormatFloat('######0.00',r2)+') ';
    Writeln(fotl,ch);
  end;
  for i:=1 to m do for j:=1 to 3 do xk[i,j]:=xn[i,j];

    ShowMessage(ch);
  end;
  CloseFile(fotl);
end.
```

4 Results

The software has an iterative linearization to compute the values of interval nonlinear equations. Integrals were considered in step 0.05. Results are presented in Table

1	0.5	(1,50 0,00)	(1,00 0,00)	(1,00 0,08)
1	1.0	(2,00 0,00)	(1,00 0,00)	(1,00 0,11)
1	1.5	(2,50 0,00)	(1,00 0,00)	(1,00 0,14)
1	2.0	(3,00 0,00)	(1,00 0,00)	(1,00 0,16)
1	2.5	(3,50 0,00)	(1,00 0,00)	(1,00 0,18)
1	3.0	(4,00 0,00)	(1,00 0,00)	(1,00 0,19)
1	3.5	(4,50 0,00)	(1,00 0,00)	(1,00 0,21)
1	4.0	(5,00 0,00)	(1,00 0,00)	(1,00 0,22)
1	4.5	(5,40 0,00)	(1,00 0,00)	(1,00 0,23)
1	4.95	(5,95 0,00)	(1,00 0,00)	(1,00 0,25)
2	0.5	(1,27 0,01)	(1,42 0,00)	(1,00 0,08)
2	1.0	(1,54 0,01)	(1,84 0,01)	(1,00 0,11)
2	1.5	(1,81 0,02)	(2,26 0,01)	(1,00 0,14)
2	2.0	(2,07 0,03)	(2,67 0,02)	(1,00 0,16)
2	2.5	(2,34 0,04)	(3,09 0,02)	(1,00 0,18)
2	3.0	(2,61 0,04)	(3,50 0,03)	(1,00 0,19)
2	3.5	(2,87 0,05)	(3,91 0,03)	(1,00 0,21)
2	4.0	(3,13 0,06)	(4,32 0,04)	(1,00 0,22)
2	4.5	(3,40 0,07)	(4,73 0,04)	(1,00 0,24)
2	4.95	(3,63 0,07)	(5,10 0,05)	(1,00 0,25)
3	0.5	(1,27 0,01)	(1,42 0,00)	(1,00 0,08)
3	1.0	(1,54 0,01)	(1,84 0,01)	(1,00 0,11)
3	1.5	(1,81 0,02)	(2,26 0,01)	(1,00 0,14)
3	2.0	(2,07 0,03)	(2,67 0,02)	(1,00 0,16)
3	2.5	(2,34 0,04)	(3,09 0,02)	(1,00 0,18)
3	3.0	(2,61 0,04)	(3,50 0,03)	(1,00 0,19)
3	3.5	(2,87 0,05)	(3,91 0,03)	(1,00 0,21)
3	4.0	(3,13 0,06)	(4,32 0,04)	(1,00 0,22)
3	4.5	(3,40 0,07)	(4,73 0,04)	(1,00 0,24)
3	4.95	(3,63 0,07)	(5,10 0,05)	(1,00 0,25)

The numerical results presented in the table, have shown the effectiveness of the proposed criterion of control and the possibility of their use in practical applications.

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