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Zhilisbayeva K.S.[1*], Saspayeva A.D.[2**]

[1] Kazakh National University after al-Farabi,
Republic of Kazakhstan, Almaty
[2] Joint stock company «National center of space researshes and technologies»,
Republic of Kazakhstan, Almaty
E-mail: *zhilisbaeva@mail.ru, **asem.saspaeva@mail.ru

Programed motion of the magnetized spacecraft

This paper focuses on the problem of creation of control of the rotational motion of the magnetized dynamically symmetric Earth's satellite in a polar circular orbit plane in the geomagnetic field. It is assumed that the permanent magnetic moment of the satellite is directed along the axis of its dynamic symmetry. The rotational motion of the satellite is caused by interaction of the magnetic moment of the satellite and the Earth's magnetic field, which is modeled by the direct dipole. Influence of gravitational moment is not considered. The objective of the control system is to implement the required programed motion. The satellite's rotation around its own axis is selected as the programed motion (in this case the angle deviation of the axis should be constant). Equations of programed motion of the magnetized satellite in the semi-combined coordinate system are derived. The control moments that ensure the specified programed motion are obtained. The solutions of the motion equations of the control system are found by Runge-Kutta method using the mathematical package Maple. According to the results the graphs of change of the kinematic parameters of the motion are obtained with and without control moments which demonstrate uncontrolled motion of the satellite. It is shown that the programed motion can be implemented by selection of the values of the deviation angle and the angular velocity, even when there is an asymptotic instability of Lyapunov's programmed motion.
Key words: magnetized satellite, geomagnetic field, control moment, program motion.

Жилисбаева К.С., Саспаева А.Д.
Программное движение намагниченного космического аппарата

В данной работе рассматривается задача построения управления вращательным движением намагниченного динамически симметричного спутника Земли по полярной плоской круговой орбите в геомагнитном поле. Предполагается, что постоянный магнитный момент спутника направлен по оси его динамической симметрии. Вращательное движение спутника обусловлено взаимодействием магнитного момента спутника и магнитного поля Земли, которое моделируется прямым диполем. Влияние гравитационного момента не учитывается. Задачей системы управления является реализация требуемого программного движения. В качестве программного движения выбрано вращение спутника вокруг собственной оси с постоянной угловой скоростью, при этом угол отклонения оси должен быть постоянным. Построены уравнения программного движения намагниченного спутника в полусвязанной системе координат. Получены управляющие моменты, обеспечивающие заданное программное движение. С помощью математического пакета Maple методом Рунге-Кутта найдены решения уравнений движения управляемой системы. По результатам решений получены графики изменения кинематических параметров движения с управляющими моментами и в случае отсутствия управляющих моментов, которые демонстрируют неконтролируемые движения спутника. Показано, что подбором значений угла отклонения оси и угловой скорости можно реализовать программные движения даже при асимптотической неустойчивости программного движения по Ляпунову.
Ключевые слова: намагниченный спутник, геомагнитное поле, управляющий момент, программное движение.

Жылысбаева Қ.С., Саспаева Ә.Д.

Магниттелген ғарыштық аппараттың бағдарламалық қозғалысы

Мақалада геомагниттік өрістегі динамикалық симмериялы, магниттелген ғарыштық аппараттың полярлы жазық орбитадағы айналмалы қозғалысын басқаруды құру есебі қарастырылған. Серіктің тұрақты магниттік моменті динамикалық симметрия өсімен бағытталған. Серіктің айналмалы қозғалысы серіктің магниттік моменті мен тура дипольмен моделденетін Жердің магниттік өрісінің өзара әрекетімен беріледі. Гравитациялық моменттің әсері ескерілмеген. Басқару жүйесінің есебі - қажетті бағдарламалық қозғалыс болып табылады. Бағдарламалық қозғалыс ретінде серіктің өз өсі бойында тұрақты бұрыштық жылдамдықпен айналатын қозғалысы алынған. Аталған жағдайда өстен ауытқу бұрышы тұрақты болып қалады. Магниттелген серіктің қозғалыс теңдеуі жартылай бекітілген санақ жүйесінде құрылды. Берілген бағдарламалық қозғалысты қанағаттандыратын басқару моменттері табылды. Басқарылатын жүйенің қозғалыс теңдеуінің шешімі Maple математикалық пакетінде Рунге-Кутта әдісімен шығарылды. Шешімнің нэтижесі бойынша қозғалыстың кинематикалық параметрлерлерінің графиктері алынып, келесідей қорытынды жасалынды: басқару моменттері ескерілмеген кезде серіктің қозғалысы басқарылмайтындығы анықталды. Асимтотикалық орнықсыздық кезінде өстің ауытқу бұрышы мен бұрыштық жылдамдықтың мәнін таңдап алу барысында бағдарламалық қозғалысты жүзеге асыруға болатындығы көрсетілді.
Түйін сөздер: магниттелген серік, геомагниттік өріс, басқарушы момент, бағдарламалық қозғалыс.

1 Introduction

Today, the interest in problem of rotational motion control of magnetized satellites has risen due to the practical needs of progressing technology of space flights and systems of magnetic stabilizing [1-2]. Especially, it concerns targets of orientation and stabilization of the magnetized artificial satellites of Earth in the geomagnetic field. Non-uniform rotation of the vector of geomagnetic field strength in inertial space and variation in its magnitude during the motion of the satellite's center of mass along an orbit does not allow providing exact orientation of a longitudinal axis of the satellite along this vector. In addition, the presence of forcing oscillations of the satellite relative to the local vector of geomagnetic field strength causes danger of onset of resonances between a basic frequency of the satellite and a frequency of the forcing moment. Therefore, there is a question to reduce the amplitude of forcing oscillations of the satellite by selection of control input. The physical meaning of control input can be of different nature: it can be directly the control forces and moments or control signals that are supplied to these mechanisms when their functioning is needed to take into account. This paper deals with the problem of forming the control moments through rotational motion of dynamically symmetric Earth satellite that has permanent magnetic moment directed along the axis of its dynamic symmetry. The magnetized satellite moves on a flat circular polar orbit in the geomagnetic field simulated by the dipole, which is antiparallel to the Earth spin axis. The orbits will be assumed to be independent of the satellite motion regarding their own center of mass.

2 Equations of motion

Hereafter, we consider the rotational motion of magnetized dynamically symmetric Earth satellite moving on a flat circular polar orbit in the geomagnetic field. It is expected that

Table 1. The table of direction cosines

-	x_2	y_2	z_2
x	$\cos\psi$	$\sin\psi$	0
y	$-\sin\psi\cos\theta$	$\cos\psi\cos\theta$	$\sin\theta$
z	$\sin\psi\sin\theta$	$-\cos\psi\sin\theta$	$\cos\theta$

the rotational motion of satellite is caused by interaction between the satellite's magnetic moment and the Earth magnetic field, which is simulated by the direct dipole. The effect of the gravitational torque is not taken into account. Derivation of equations of satellite motion will be carried out relative to the system of $x_2y_2z_2$ axes with the origin at its center of mass G, parallel regarding the geocentric reference frame (Fig.1).The permanent magnetic moment of satellite is directed along the axis of its dynamic symmetry. We will choose the semi-related right system of Rezal axes in the satellite body:$z$axis is directed on a symmetry axis; then x and y axes will lie in the equatorial plane, and the x axis will be directed perpendicularly to the zGz_2 plane so that looking through its end it will be seen the turn from the z_2 axis to z anticlockwise on the θangle. An angle between the x_2 and axes is defined via ψ [3]. Mutual position of introduced reference frames can be determined by the table of direction cosines (Table 1). By means of projection of the vector of magnetic strength H onto the x and y axes this yields:

$$H_x = H_x^/ \cos\psi + H_y^/ \sin\psi \qquad H_y = -H_x^/ \sin\psi\cos\theta + H_y^/ \cos\psi\cos\theta + H_z^/ \sin\theta,$$

Then the magnetic torque is defined as follows:

$$\vec{M} = \vec{I} \times \vec{H}$$

and for the projections of the magnetic torque onto the satellite axes we have:

$$\begin{cases} M_x = -\frac{I\mu_e}{R^3}(1.5\sin i\sin 2u\sin\psi\cos\theta + [sin\nu - 3\sin i\cos(\nu-1)\sin^2 u]cos\theta + \\ +[\cos\nu + 3\sin i\sin(\nu-1)sin^2 u]\sin\theta) \\ M_y = \frac{I\mu_e}{R^3}(-1.5\sin i\sin 2u\cos\psi + (\sin\nu - 3\sin i\cos(\nu-1)\sin^2 u)\sin\psi) \\ M_z = 0 \end{cases} \quad (1)$$

where I is the magnetic moment of the satellite.

Projection of the satellite's resulting angular velocity $\vec{\omega}$ onto the Rezal axes has the form:

$$\omega_x = \dot{\theta}, \omega_y = \dot{\psi}\sin\theta, \omega_z = \dot{\psi}\cos\theta + \dot{\varphi}$$

where $\dot{\varphi}$ is the angular velocity of satellite rotation with respect to xyz reference frames.

Hence, the equations of satellite motion regarding the center of mass will be written as follows [4]:

$$\begin{cases} A\ddot{\theta} - A\dot{\psi}^2\sin\theta\cos\theta + Cr\dot{\psi}\sin\theta = M_x \\ A\ddot{\psi}\sin\theta + 2A\dot{\psi}\dot{\theta}\cos\theta - Cr\dot{\theta} = M_y \\ r = r_0 \end{cases} \quad (2)$$

These equations do not depend on φ, i.e., the angle at which the satellite turned around its axis. Then the projection of angular velocity onto the axis of symmetry r is permanent, and the θ and ψ angles are defined from the first two equations (Eq. 2).

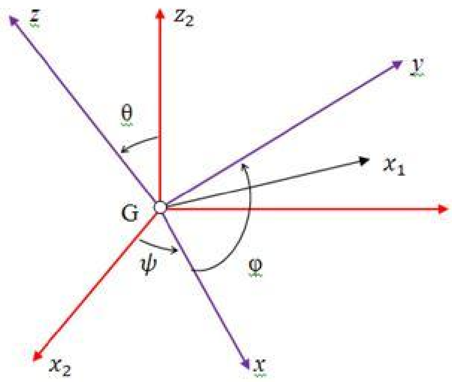

Figure 1 - The coordinate system

3 Building the programm motion

In this work as the program motion we will understand a task of determination of the active forces and moments applied to the spacecraft at which the motion with the given properties is one of its possible motions. The task of definition of the control vector, which provides the program of motion, leads to the solution of the linear equation relative to the control vector. The obtained common solution allows defining the sought differential equations for which using numerical methods of solution is possible to derive the methods of solution of non-linear equations system. Now, let us pass to consideration of building the program control for the prescribed motions of the satellite described in this paper. As a program motion we will choose the satellite's rotation around its axis with permanent angular velocity $\dot{\psi}_p = k_2$, by the same time saving the constant magnitude of angle of the axis deviation $\theta_p = k_1$, then we have:

$$\begin{cases} \theta_p = k_1 \\ \psi_p = k_2 t \end{cases} \tag{3}$$

In order to realize this prescribed motion (Eq. 3) we will add the control moments M_1 and M_2 to the right side of equations of the motion of satellite (Eq. 2), and, when the polarity of the satellite $i = \frac{\pi}{2}, \nu = \frac{\pi}{2}$ is taken into account, we will get:

$$\begin{cases} A\ddot{\theta} - A\dot{\psi}^2 \sin\theta\cos\theta + Cr\dot{\psi}\sin\theta = \\ = -\frac{I\mu_e}{R^3}(1.5\sin 2u\sin\psi\cos\theta + (1 - 3\sin^2 u)\cos\psi\cos\theta) + M_1 \\ A\ddot{\psi}\sin\theta + 2A\dot{\psi}\dot{\theta}\cos\theta - Cr\dot{\theta} = \\ = \frac{I\mu_e}{R^3}(-1.5\sin 2u\cos\psi + (1 - 3\sin^2 u)\sin\psi) + M_2 \end{cases} \tag{4}$$

To find these control moments, which are program control, we will substitute the values $\theta_p = k_1$ and $\psi_p = k_2 t$ relative to prescribed motion. Then, for defining of control moments we obtain the following expressions:

$$\begin{cases} M_1 = Ak_2^2 \sin k_1 \cos k_1 + Crk_2 \sin k_1 + \\ + \frac{I\mu_e}{R^3}(1.5\sin 2u \sin k_2 t\cos k_1 + (1 - 3sin^2 u)\cos k_2 t\cos k_1) \\ M_2 = -\frac{I\mu_e}{R^3}(-1.5\sin 2u\cos k_2 t + (1 - 3\sin^2 u)\sin k_2 t) \end{cases} \tag{5}$$

In consideration of Eq. 5 the equations of motion of control system will assume the form:

$$\begin{cases} A\ddot{\theta} - A\dot{\psi}^2 \sin\theta\cos\theta + Cr\dot{\psi}\sin\theta = \\ = -\frac{I\mu_e}{R^3}(1.5\sin 2u \sin\psi\cos\theta + (1-3\sin^2 u)\cos\psi\cos\theta) + Ak_2^2 \sin k_1 \cos k_1 + Crk_2 \sin k_1 + \\ +\frac{I\mu_e}{R^3}(1.5\sin 2u \sin k_2 t \cos k_1 + (1-3sin^2 u)\cos k_2 t \cos k_1) \\ A\dot{\psi}\sin\theta + 2A\dot{\psi}\dot{\theta}\cos\theta - Cr\dot{\theta} = \\ = \frac{I\mu_e}{R^3}(-1.5\sin 2u\cos\psi + (1-3\sin^2 u)\sin\psi) - -\frac{I\mu_e}{R^3}(-1.5\sin 2u \cos k_2 t + (1-3\sin^2 u)\sin k_2 t) \end{cases} \tag{6}$$

Note that investigated program motion (Eq. 3) is a solution of this system of equations by the choice of control moments M_1 and M_2. Let's resolve these equations regarding the second derivatives:

$$\begin{cases} \ddot{\theta} = \dot{\psi}^2 \sin\theta\cos\theta + k_2^2 \sin k_1 \cos k_1 + \frac{C}{A} r(k_2 \sin k_1 - \dot{\psi}\sin\theta + \\ +\frac{I\mu_e}{R^3}(1.5\sin 2u(\sin k_2 t \cos k_1 - \sin\psi\cos\theta) + (1-3\sin^2 u)(\cos k_2 t\cos k_1) - \cos\psi\cos\theta) \\ \ddot{\psi} = 2\dot{\psi}\dot{\theta}\coth\theta + \frac{Cr\dot{\theta}}{A\sin\theta} + \frac{I\mu_e}{AR^3\sin\theta}(-1.5\sin 2u(\cos\psi - \cos k_2 t) + (1-3\sin^2 u)(\sin\psi) - \sin k_2 t) \end{cases} \tag{7}$$

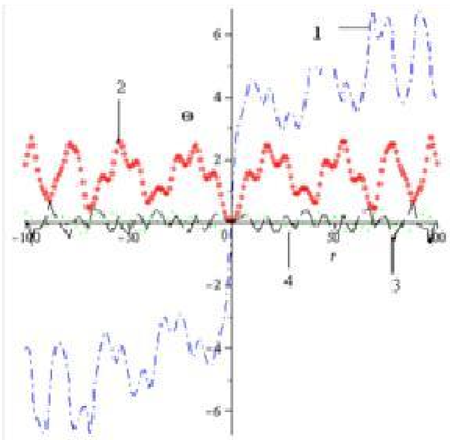

Figure 2 -Solution of the equations of motion without control

4 The solution of the motion equations

The motion equations of control system are obtained by the Runge - Kutta method using a mathematical package Maple. Solutions of equations of non-control motion show uncontrolled motion of the satellite. The primary program motion is a solution of the received system, however, it will be one of possible for physical realization only in case of its asymptotic stability by Lyapunov. If the decision is asymptotically unstable by Lyapunov, then the possibility of unlimited growth of deviations of magnitudes from their preset values actually means that the system makes the uncontrollable motions (see Fig. 2). Similar results were obtained in [7-9].Here are following descriptions in the Figures 2, 3: 1 - $\psi(t)$ (precession angle), 2 - $\theta(t)$ (nutation angle), 3 - $q(t)$, 4- $p(t)$ (the projections of satellite's angular velocity onto the equatorial principal axes of inertia of the satellite).

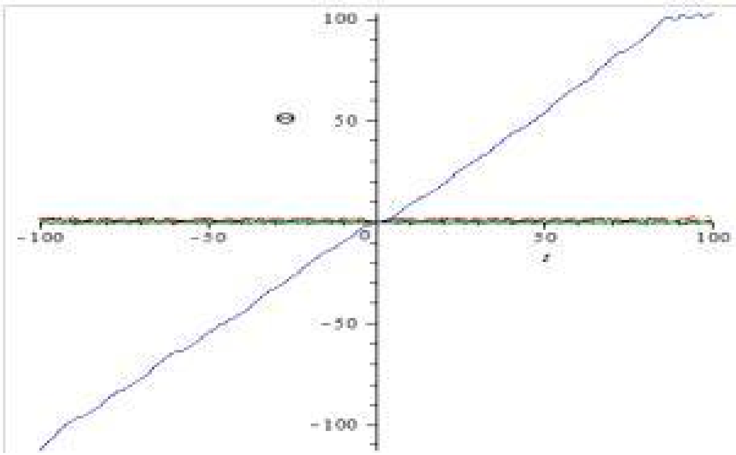

Figure 3 -Solution of the equations of motion with control

The figure 2 represents the graphs of the variation of precession angle, nutation angle and the projections of satellite's angular velocity onto the equatorial principal axes of inertia of the satellite in case of absence of control moments that demonstrate the non-control motion of satellite. Similarly, the figure 3 shows the graphs with presence of control moments, which provide prescribed motion, also are possible for realization. The results point out that through selection of values $k_1 = \frac{2\pi}{3}$ and $k_2 = 0.5$ it is possible to realize the program motion even at an asymptotic instability by Lyapunov.

5 Conclusion

The equations of control motion of magnetized satellite that moves on a flat circular polar orbit in the geomagnetic field simulated by the dipole, are constructed in this paper. The control moments that provide prescribed motion are obtained for chosen program motion. The solution of motion equations of control system is obtained by the Runge - Kutta method using a mathematical package Maple. Also we received the graphs of the variation of precession angle, nutation angle and the projections of satellite's angular velocity onto the equatorial principal axes of inertia of the satellite in case of absence of control moments that demonstrate the non-control motion of satellite. It is shown that through selection of values k_1 and k_2 it is possible to realize the program motion even at an asymptotic instability by Lyapunov. This work was partially supported by the grant of the Committee of Science of the Ministry of Education and Science of the Republic of Kazakhstan (project 0091/GF4 MON RK)

References

[1] *M.J.Sidi.* Spacecraft dynamics and control. Cambridge: Univercity Press, 2002.- 409p.

[2] *Ivanov D.S., Ovchinnikov M.Y.,Tkachev S.S. C.C.* Upravlenie orientasiei tverdogo tela, podveshannogo na strune s ispolsovaniem ventiliatornych dvigatelei //Isvestia RAN. Theory i systemy upravlenia. 2011.–№1. C.127-139 (in Russian)

[3] *Hentov A.A.* Passivnaya stabilisasiya iskusstvennych sputnikov po magnitnomu polu Zemli // Kosmisheskie issledovannie. - 1967. - Tom. 5, № 4. - S. 540-553.(in Russian)

[4] *Appel P.* Teoretisheskaya mechanika. - M.: Fizmatkis, 1960. - 487 s.(in Russian)

[5] *Mukhametzyanov I.A., Mucharlyamov R.G.* Uravnenie programmnych dvijeni. M.: Izd. RUDN, 1986 (in Russian)

[6] *Bezglasnyi S.P.* Stabilisathsya i upravleniya dvijeniem dynamisheskich system . Electronnoe ushebnoe posobie. - Samara, 2010. - 102 s.(in Russian)

[7] *Zhilisbayeva K.S,Tureeva Zh.U.* O beznutatsionnom dvijenii namagnishennogo dynamisheski simmetrishnogo sputnika // Vestnik KazGU, ser. matem., mechanika, informatika. - 2005. - №3(46), - C. 85-90.

[8] *Zhilisbayeva K.S.* O kolebanyach namagnishennogo sputnika v okrestnosti statsionarnogo dvijenia . // Sb. Methody experimentalnoi phyzyki. - 2010. - S. 49-52.(in Russian)

[9] *Zhilisbayeva K.S., Ismailova A.* Passive Magnetic Stabilization of the Rotational Motion of the Satellite in its Inclined Orbit // Applied Mathematical Sciences. - 2015. - Vol. 9, no. 16. - P. 791-802.

[10] *Saspayeva A.D* Postoenie uravnenya dvijenia iskustvennogo sputnika Zemli v gravitatsionnych polyach Zemli i Luny // Mezhdunarodaya konferensia "Actualnye problemy matematiki i matematisheskogo modelirovanya". -Almaty, Kazakhstan. - 2015. - S. 352-353. (in Russian)