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Determination of displacements in cross-sections of four-bar mechanism links from distributed dynamic loads and their animation using MAPLE

Utenov M.U., al-Farabi Kazakh National University, Almaty, Kazakhstan, +77478976989, E-mail: umu53@mail.ru
Zhilkibayeva S.K., al-Farabi Kazakh National University, Almaty, Kazakhstan, +77755828228, E-mail: saltanatzh.kz@gmail.com
Baygunchekov Zh.Zh., K.I. Satpaev Kazakh National Research Technical University, Almaty, Kazakhstan, +77772255647, E-mail: bzh47@mail.ru
Myrzaliyev D.S., M. Auezov South Kazakhstan State University, Shymkent, Kazakhstan, +7 725 230 05 70

The links of high-speed mechanisms and manipulators are deformed under the action of inertia forces and external loads. These deformations have significantly influence on the accuracy of execution of the required law of motion by the operating point of the mechanism and the positioning of the manipulator grip. Accordingly, longitudinal and transverse displacements, angles of rotation of cross-sections of links under the action of distributed dynamic and external loads are investigated in this paper. The developed technique allows defining deformations of links of mechanisms and manipulators and can be applied at their designing. To determine the transverse displacements, the angles of rotation of the cross-sections of the links – the basic differential equation of the elastic line of the beam, to determine the longitudinal displacements of the points of the links – Hooke's law and the boundary conditions of the computed scheme of the investigated linkages for elastic computation are used. The bending moment in the basic differential equation of the elastic line of the beam and the longitudinal force in Hooke's law were determined by the theory developed by the authors of the analytical definition of internal forces in the links of planar linkages with statically determinate structures, taking into account the distributed dynamic loads from the masses of links, dead weight and from the acting external loads. According to the developed technique, programs are created in the MAPLE system and animations of the movement of mechanisms are received, with the construction on the links the diagrams of transverse, longitudinal displacements and angles of rotation of the link cross-sections. The developed analytical technique for determining deformations in the cross-sections of links is used to calculate the strength and stiffness of elements of movable linkages.
Key words: Mechanisms, movable linkages, displacements, distributed dynamic loads.

Тарқалған динамикалық жүктемелер әсеріндегі төрт буынды механизмнің буын қималарында пайда болатын орын ауыстыруларды анықтау және MAPLE жүйесінде анимациясын құру

Утенов М.У., әл-Фараби атындағы Қазақ ұлттық университеті, Алматы қ., Қазақстан Республикасы, +77478976989, E-mail: umu53@mail.ru
Жылқыбаева С.К., әл-Фараби атындағы Қазақ ұлттық университеті, Алматы қ., Қазақстан Республикасы, +77755828228, E-mail: saltanatzh.kz@gmail.com
Байгөншеков Ж.Ж., Қ.И. Сәтбаев атындағы Қазақ 5лттық техникалық зерттеу университеті, Алматы қ., Қазақстан Республикасы, +77772255647, E-mail: bzh47@mail.ru
Мырзалиев Д.С., М. Әуезов атындағы Оңтүстік Қазақстан мемлекеттік университеті, Шымкент қ., Қазақстан Республикасы, +7 725 230 05 70

Жоғарғы жылдамдықты механизмдер мен манипуляторлардың буындары инерция күштері мен сыртқы жүктемелер әсерінен деформацияланады. Бұл деформациялар механизмнің жұмыс нүктесінің қажетті қозғалу заңын атқару дәлдігіне және манипулятордың қармауышын тұрғыландыруға айтарлықтай әсер етеді. Осыған орай, бұл жұмыста тарқалған динамикалық және сыртқы жүктемелер әсеріндегі буындар қимасында пайда болатын бойлық пен көлденең орын ауыстырулар, бұралу бұрыштары зерттелінген. Әзірленген әдіс механизмдер мен манипуляторлардың буындарындағы деформацияны анықтауға мүмкіндік береді және оларды жобалау кезінде қолдануға болады. Буын қималарындағы көлденең орын ауыстыруларды, бұралу бұрыштарын анықтау үшін арқалықтың серпімді сызығының негізгі дифференциалдық теңдеуі, буын нүктелерінің бойлық орын ауыстыруларын анықтау үшін Гук заңы, және де серпімді есептеуге қажет зерттелетін серпімді жүйелердің есептеу схемаларының шекаралық шарттары қолданылады. Арқалықтың серпімді сызығының негізгі дифференциалдық теңдеуіне кіретін июші момент және Гук заңына кіретін бойлық күш авторлардың жасаған құрылымы статикалық анықталған жазық серпімді механизмдер мен манипуляторлардың буындарында пайда болатын ішкі күштерді, буындар салмағынан, сыртқы күштерден туындайтын тарқалған динамикалық жүктемелер әсерін ескеретін талдамалы жолмен анықтау теориясымен табылады. Жасалған методика бойынша MAPLE жүйесінде программалар құрылып, табылған көлденең, бойлық орын ауыстырулар мен бұралу бұрыштарының эпюрлері буындарға тұрғызылып, механизмдер қозғалысының анимациясы алынды. Буындар қимасындағы деформацияларды анықтайтын осы талдамалы әдіс қозғалмалы серпімді жүйелердің элементтерін беріктік пен қатаңдыққа есептеуге ыңғайлы.

Түйін сөздер: Механизмдер, жылжымалы сырықты жүйелер, орын ауыстырулар, тарқалған динамикалық жүктемелер.

Определение перемещений сечений звеньев четырехзвенника от распределенных динамических нагрузок и их анимация в среде MAPLE

Утенов М.У., Казахский национальный университет имени аль-Фараби, г. Алматы, Республика Казахстан, +77478976989, E-mail: umu53@mail.ru
Жилкибаева С.К., Казахский национальный университет имени аль-Фараби, г. Алматы, Республика Казахстан, +77755828228, E-mail: saltanatzh.kz@gmail.com
Байгунчеков Ж.Ж., Казахский национальный технический университет имени К.И. Сатпаева, г. Алматы, Республика Казахстан, +77772255647, E-mail: bzh47@mail.ru
Мырзалиев Д.С., Южно-Казахстанский государственный университет имени М. Ауезова, г. Шымкент, Казахстан, +7 725 230 05 70

В высокоскоростных механизмах и манипуляторах звенья деформируются под воздействием сил инерции и внешних нагрузок. Эти деформации существенно влияют на точность исполнения требуемого закона движения рабочей точкой механизма и на позиционирование схвата манипулятора. В связи с этим, в данной работе исследуются продольные, поперечные перемещения, углы поворота сечений звеньев находящихся под воздействием распределенных динамических и внешних нагрузок. Разработанный метод позволяет определять деформаций звеньев механизмов и манипуляторов и может применяться при их проектировании. Для определения поперечных перемещений, углов поворота сечений звеньев использованы основное дифференциальное уравнение упругой линии балки, для определения продольных перемещений точек звеньев – закон Гука и граничные условия расчетной схемы исследуемых стержневых систем для упругого расчета. Изгибающий момент, входящий в основное дифференциальное уравнение упругой линии балки и продольная сила, входящая в закон Гука были определены по разработанной авторами теории аналитического определения внутренних усилий в звеньях плоских стержневых механизмов и манипуляторов со статически определимыми структурами с учетом распределенных динамических нагрузок от масс звеньев, собственного веса и от действующих внешних нагрузок.

По разработанной методике составлены программы в системе MAPLE и получены анимации движения механизмов с построением на звеньях поперечных, продольных перемещений и углов поворота сечений звеньев. Разработанный аналитический метод определения деформаций в сечениях звеньев применяется для расчета прочности и жесткости элементов подвижных стержневых систем.
Ключевые слова: Механизмы, подвижные стержневые системы, перемещения, распределенные динамические нагрузки.

1 Introduction

Dynamic analysis of high-speed mechanisms and manipulators received considerable attention in the last two decades. Every frame structure is deformed under the action of large static and dynamic loads. Whenever such a load occurs, several problems persist, for instance: the problems of failure, caused by large forces of inertia; elastic deformations of the mechanism can be significant, as a consequence, the mechanism become unusable; the mechanism cannot satisfy the kinematic requirements because of the large deformations of links. When designing high-speed mechanisms, the designer must either reduce the elastic deformations of the mechanism, or take them into account in computation. To test on stiffness and stability of the structure, it is necessary to be able to determine the displacements caused by the deformation of its elements. The techniques for determining these displacements are very diverse. They mainly differ from each other by the degree of complexity and scope of application.

2 Literature review

The method of direct integration of differential equation of elastic beam line is an earlier one for determination of displacements. However, in the case of beams with a large number of cross-sections, the implementation of this method involves considerable difficulties, which are not in the integration of differential equations, but in the technique of determining the arbitrary integration constants – drafting and solving of systems of linear algebraic equations (Jindal, 2012 : 294), (Timoshenko, 1948 : 134 – 135), (Darkov, 1975 : 289).

When computing by the displacement method, the main sought values are the displacements of the nodal points caused by the deformation of the system. Knowledge of these displacements is necessary and sufficient to determine all internal forces that arise in the cross-sections of the elements of a given system (Kaveti, 2014 : 412), (Tschiras, 1989 : 111), (Pisarenko, 1979 : 85). In the works of Sadler and Sandor (Sandor, 1973 : 497 – 516), the lateral bending vibrations of the elements of mechanisms, which can be considered as pin-ended beams making planar motions, are investigated. The normal dynamic stresses caused by the concerted actions of bending and axial loads are studied. A scheme is given for minimizing the maximum stresses in the flexible linkages of a given length without increasing the total mass. This is done using an iterative method of finding a full-strength form seeking method. The study is limited to the case of a rectangular cross-section, where the only variable is the width. Longitudinal deformations are considered negligible and, therefore, are not considered here. Abe proposed an accurate mathematical model of the flexible link in two-link rigid-flexible manipulators by taking the axial displacement and nonlinear curvature arising from large bending deformation into consideration for suppressing residual vibrations in optimal trajectory planning (Akira Abe, 2009 : 1627 – 1639). Mingxiang et al. presented a kinetostatic modeling method for flexure-hinge-based compliant mechanisms with hybrid serial-parallel substructures to provide accurate and concise solutions by combining the matrix displacement method with the transfer matrix method. This work established a general kinetostatic model of the whole compliant mechanisms based on the equilibrium equation of the nodal force (Mingxiang, 2018).

Finite element method (FEM) is used to structure the system into single finite elements and the stiffness matrix of element and of the whole system provides connection between displacement of nodes of element and system, as well as forces therein (Hutton, 2007 : 387), (Gokhale, 2008 : 416). Du and Ling have developed a general non-linear finite element model for dynamic analysis of three dimensional beam-like mechanisms undergoing both large rigid body motion and large elastic deflections. They adopted the non-linear strain-displacement relationship taking into account the axial strain and the shear strains due to the pre-twist in the beams (Hejun Du, 1995 : 56). Absy and Shabana show that the consideration of longitudinal displacement caused by bending would eliminate the third and higher order terms from the strain-energy expression, if the strain energy is written in terms of axial deformation. This leads to nonlinear inertia terms and a constant stiffness matrix (El-Absy, 1997 : 207). Zhaocai studied the dynamic stress of the flexible beam element of planar flexible manipulators. Considering the effects of bending-shearing strain and tensile compression strain, the dynamic stress of the links and its position are derived by using the Kineto-Elastodynamics theory and the Timoshenko beam theory (Ding Zhaocai, 2006 : 17-20). Yue computed the maximum payload of kinematically redundant manipulators using a finite element method for describing the dynamics of a system (Shigang Yue, 2001 : 36). Korayem et al. considered a complete dynamic model to characterize the motion of a compliant link capable of large deflection (Moharam H. Korayem, 2010 : 17).

In this paper the longitudinal and transverse displacements, the angle of rotation of link cross-sections under the action of distributed dynamic loads and external forces are studied. The developed analytical technique makes it possible to accurately and quickly determine the deformations of links of mechanisms and manipulators and can be used in their design. Earlier the authors have developed a new analytical technique for determination of internal forces in the links of planar mechanisms and manipulators under the action of distributed dynamical loads and it was described in the work (Utenov, 2016 : 5-10).

3 Materials and methods

The main differential equation of elastic beam line (for the element k) has the form (Darkov, 1975 : 289), (Kaveti, 2014 : 412), (Tschiras, 1989 : 111), (Pisarenko, 1979 : 85) :

$$\frac{d^2 y_k}{d(x_k')^2} = \frac{M_k(x_k')}{E_k I_k} \tag{1}$$

When a transverse distributed trapezoidal load acts on the element, the bending moment in the cross-sections of the element is determined by (3) from the previous work (Utenov, 2016 : 5-10). Substituting the values of $M_k(x_k')$ from (3) into (1) and integrating one time, we will have expression to the rotation angles of cross-sections of the element k :

$$\widehat{O}_k(x'_k) = \frac{dy'_k}{dx'_k} = \frac{1}{E_k I_k}\int\Big\{\Big[1-\frac{11}{2l_k}x'_k+\frac{9}{l_k^2}(x'_k)^2-\frac{9}{2l_k^3}(x'_k)^3\Big]M_{k1}+\Big[\frac{9}{l_k}x'_k-\frac{45}{2l_k^2}(x'_k)^2+$$

$$+\frac{27}{2l_k^3}(x'_k)^3\Big]M_{k2}+\Big[-\frac{9}{2l_k}x'_k+\frac{18}{l_k^2}(x'_k)^2-\frac{27}{2l_k^3}(x'_k)\Big]M_{k3}+\Big[\frac{1}{l_k}x'_k-\frac{9}{2l_k^2}(x'_k)^2+\frac{9}{2l_k^3}(x'_k)^3\Big]M_{k4}\Big\}dx'_k=$$

$$\frac{1}{E_k I_k}\Big\{\Big[x'_k-\frac{11}{4l_k}(x'_k)^2+\frac{9}{3l_k^2}(x'_k)^3-\frac{9}{8l_k^3}(x'_k)^4\Big]M_{k1}+\Big[\frac{9}{2l_k}(x'_k)^2-\frac{45}{6l_k^2}(x'_k)^3+\frac{27}{8l_k^3}(x'_k)^4\Big]M_{k2}+$$

$$+\Big[-\frac{9}{4l_k}(x'_k)^2+\frac{18}{3l_k^2}(x'_k)^3-\frac{27}{8l_k^3}(x'_k)^4\Big]M_{k3}+\Big[\frac{1}{2l_k}(x'_k)^2-\frac{9}{6l_k^2}(x'_k)^3+\frac{9}{8l_k^3}(x'_k)^4\Big]M_{k4}\Big\}+C_{k1} \quad (2)$$

which contains one arbitrary constant C_{k1}. By integrating second time, we find the expression for beam deflection $y'_k(x'_k)$

$$w_k(x'_k) = y'_k(x'_k) = \frac{1}{E_k I_k}\Big\{\Big[\frac{1}{2}(x'_k)^2-\frac{11}{12l_k}(x'_k)^3+\frac{9}{12l_k^2}(x'_k)^4-\frac{9}{40l_k^3}(x'_k)^5\Big]M_{k1}+$$

$$+\Big[\frac{9}{6l_k}(x'_k)^3-\frac{45}{24l_k^2}(x'_k)^4+\frac{27}{40l_k^3}(x'_k)^5\Big]M_{k2}+\Big[-\frac{9}{12l_k}(x'_k)^3+\frac{18}{12l_k^2}(x'_k)^4-\frac{27}{40l_k^3}(x'_k)^5\Big]M_{k3}+$$

$$+\Big[\frac{1}{6l_k}(x'_k)^3-\frac{9}{24l_k^2}(x'_k)^4+\frac{9}{40l_k^3}(x'_k)^5\Big]M_{k4}\Big\}+C_{k1}x'_k+C_{k2} \quad (3)$$

which contains two arbitrary constants C_{k1} and C_{k2}. The values of these arbitrary constants C_{k1} and C_{k2} are defined from consideration of two boundary conditions, i.e. from the conditions of end restraint.

The element aspect ratio dx'_k from the longitudinal force $N_k(x'_k)$ by Hooke's law would be:

$$\Delta dx'_k = \frac{N_k(x'_k)dx'_k}{E_k I_k} \quad (4)$$

When a distributed trapezoidal load is applied to the element, the longitudinal force is given by (5) in the work (Moharam H. Korayem, 2010 : 17). Substituting it into (4) and intergrating one time, we find the following expression for longitudinal displacements of the element points:

$$u_k(x'_k) = \frac{1}{E_k A_k}\int N_k(x'_k)dx'_k = \frac{1}{E_k A_k}\int\Big\{\Big[1-\frac{3}{l_k}x'_k+\frac{2}{l_k^2}(x'_k)^2\Big]N_{k1}+$$

$$+\Big[\frac{4}{l_k}x'_k-\frac{4}{l_k^2}(x'_k)^2\Big]N_{k2}+\Big[-\frac{1}{l_k}x'_k+\frac{2}{l_k^2}(x'_k)^2\Big]N_{k3}\Big\}dx'_k = \frac{1}{E_k A_k}\Big\{\Big[x'_k-\frac{3}{2l_k}(x'_k)^2+\frac{2}{3l_k^2}(x'_k)^3\Big]N_{k1}+$$

$$+\Big[\frac{4}{2l_k}(x_k')^2-\frac{4}{3l_k^2}(x_k')^3\Big]N_{k2}+\Big[-\frac{1}{2l_k}(x_k')^2+\frac{2}{3l_k^2}(x_k')^3\Big]N_{k3}\Big\}+C_{kn} \tag{5}$$

The arbitrary constant C_{kn} is determined from the conditions of end restraint.

Let us consider the determination of displacements in the links of four-bar mechanism (Figure 1). As the displacements of the link 1 cross-section O are known ($Phi_1(0)=0$ – the angle of rotation of cross-section O, $uy10=w_1(0)=0$ – the displacement that is perpendicular to the axis of the rod of the same cross-section, $ux10=w_{1n}(0)=0$ – the displacement along the axis of the rod of the same cross-section, that it is possible to define constants C_{11}, C_{12} and C_{1n}. Substituting into (2), (3) and (5) the value of $x_k'=,0$ and taking into account above said three boundary conditions, we will receive that C_{11}, C_{12} and C_{1n} are equal to zero. It allows defining transverse and longitudinal displacements in any cross-section of the link 1. Let us introduce three Cartesian coordinate systems $BX_1'Y_1'$, BX_2', Y_2' and BX_2Y_2 at the point B (Figure 1), where X_1' is directed along the axis of the first link, X_2' is directed along the axis of the second link, X_2 is directed parallel to the axis X. Let us determine the coordinates of the point B of the first link B' (a new position of the point B after deformation) $ux1l_1$, $uy1l_1$ with respect to the coordinate system $BX_1'Y_1'$. For this we substitute into (5) and (3) the value of $X_1'=l_1$, we will get:

$$ux1l=\frac{l_1}{E_1A_1}\left(\frac{1}{6}N_{11}+\frac{2}{3}N_{12}+\frac{1}{6}N_{13}\right) \tag{6}$$

$$uy1l=\frac{l_1^2}{E_1I_1}\left(\frac{13}{120}M_{11}+\frac{3}{10}M_{12}+\frac{3}{40}M_{13}\right) \tag{7}$$

We will denote the coordinates of the point B' in the coordinate system $BX_2'Y_2'$ through $ux20$, $uy20$. Then position of the point B' with respect to the coordinate system BX_2Y_2, using the coordinates of the point B' in the coordinate system $BX_1'Y_1'$, will be equal to:

$$\left\{\begin{array}{c} x_{2b'} \\ y_{2b'} \end{array}\right\}=\left[\begin{array}{cc} \cos\theta_1 & -\sin\theta_1 \\ \sin\theta_1 & \cos\theta_1 \end{array}\right]\left\{\begin{array}{c} ux1l_1 \\ uy1l_1 \end{array}\right\} \tag{8}$$

and using the coordinates of the point B' in the coordinate system $BX_2'Y_2'$, we get:

$$\left\{\begin{array}{c} x_{2b'} \\ y_{2b'} \end{array}\right\}=\left[\begin{array}{cc} \cos\theta_2 & -\sin\theta_2 \\ \sin\theta_2 & \cos\theta_2 \end{array}\right]\left\{\begin{array}{c} ux20 \\ uy20 \end{array}\right\} \tag{9}$$

Equating the equalities (8) and (9) we receive two equations with two unknowns $ux20$ и $uy20$:

$$ux20\cos\theta_2-uy20\sin\theta_2=ux1l_1\cos\theta_1-uy1l_1\sin\theta_1$$

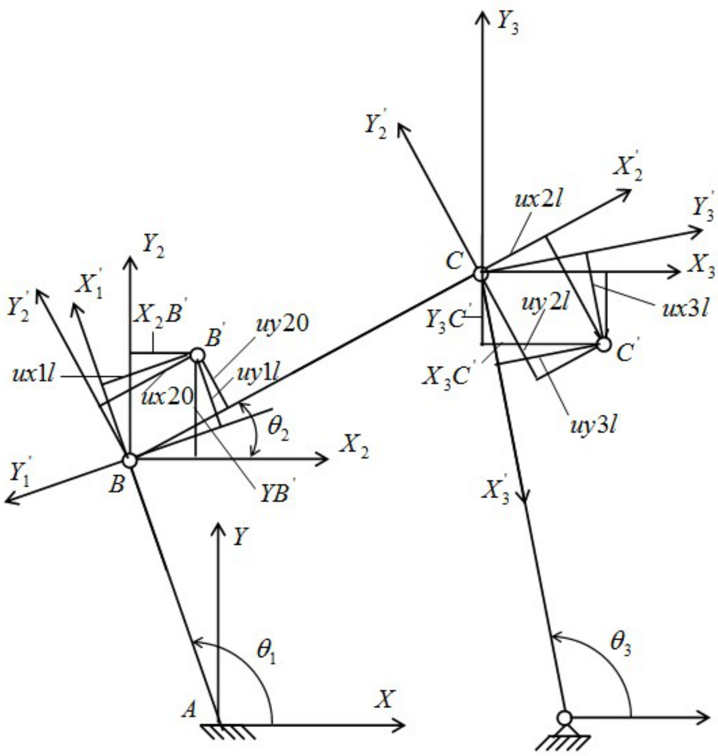

Figure 1 – The displacements in the links of four-bar mechanism

$$ux20\sin\theta_2 + uy20\cos\theta_2 = ux1l_1\sin\theta_1 + uy1l_1\cos\theta_2$$

Solving the resulting system of equations, we have:

$$ux20 = (ux1l_1\cos\theta_1 - uy1l_1\sin\theta_1)\cos\theta_2 + (ux1l_1\sin\theta_1 + uy1l_1\cos\theta_1)\sin\theta_2 \tag{10}$$

$$uy20 = (ux1l_1\sin\theta_1 + uy1l_1\cos\theta_1)\cos\theta_2 - (ux1l_1\sin\theta_1 + uy1l_1\cos\theta_1)\sin\theta_2 \tag{11}$$

Now, let us introduce three Cartesian coordinate systems $CX_2'Y_2'$, $CX_3'Y_3'$ and CX_3Y_3 in the point C where the axis X_2' is directed along the axis of the second link, the axis X_3' is directed along the axis of the third link, the axis X_3 is directed parallel to the axis X. Then position of the point C' with respect to the coordinate system CX_3Y_3, using the coordinates of the point C' in the coordinate system $CX_2'Y_2'$ will be equal to:

$$\left\{ \begin{array}{c} x_{3b'} \\ y_{3b'} \end{array} \right\} = \left[\begin{array}{cc} \cos\theta_2 & -\sin\theta_2 \\ \sin\theta_2 & \cos\theta_2 \end{array} \right] \left\{ \begin{array}{c} ux2l \\ uy2l \end{array} \right\}, \tag{12}$$

using the coordinates of the point C' in the coordinate system $CX_3'Y_3'$ will be equal to:

$$\left\{ \begin{array}{c} x_{3b'} \\ y_{3b'} \end{array} \right\} = \left[\begin{array}{cc} \cos\theta_3' & -\sin\theta_3' \\ \sin\theta_3' & \cos\theta_3' \end{array} \right] \left\{ \begin{array}{c} ux3l \\ uy3l \end{array} \right\}, \tag{13}$$

where $\theta_3' = arctg\dfrac{y_c - y_b}{x_c - x_b}$.

Equating the equalities (12) and (13) we receive:

$$-uy2l\sin\theta_2 + uy3l\sin\theta_3' = -ux2l\cos\theta_2 + ux3l\cos\theta_3' \tag{14}$$

$$uy2l\cos\theta_2 - uy3l\cos\theta_3' = -ux2l\sin\theta_2 + ux3l\sin\theta_3' \tag{15}$$

As the point D is hingedly fixed, that we have following boundary conditions:

$$\begin{cases} lux30 = 0 \\ uy30 = 0 \end{cases} \tag{16}$$

Substituting the value of $X_2' = l_2$ into (5) we get:

$$ux2l = ux20 + \frac{l_2}{E_2A_2}\left(\frac{1}{6}N_{21} + \frac{2}{3}N_{22} + \frac{1}{6}N_{23}\right)$$

Using the first boundary condition (15), and substituting $x_3^{'} = l_3$ into (5) with respect to the coordinate system $CX_3^{'}Y_3^{'}$ we have:

$$ux3l = -\frac{l_3}{E_3l_3}\left(\frac{1}{6}N_{31} + \frac{2}{3}N_{32} + \frac{1}{6}N_{33}\right)$$

Substituting the found values $ux2l$ and $ux3l$ into (14) and (15), solving in common, we get:

$$uy2l = \frac{-(-ux2l\cos\theta_2 + ux3l\cos\theta_3')\cos\theta_3' - (-ux2l\sin\theta_2 + ux3l\sin\theta_3)\sin\theta_3}{\sin\theta_2\cos\theta_3' - \sin\theta_3'\cos\theta_2}$$

$$uy3l = \frac{-(-ux2l\sin\theta_2 + ux3l\sin\theta_3')\sin\theta_2 - (-ux2l\cos\theta_2 + ux3l\cos\theta_3')\cos\theta_2}{\sin\theta_2\cos\theta_3' - \sin\theta_3'\cos\theta_2}$$

Substituting the found values $uy20$ and $uy2l$ into (2), conducting some simple transformations, we find:

$$\Phi 20 = \frac{uy2l - uy20 - \dfrac{l_2^2}{E_2 I_2}\left(\dfrac{3}{10}M_{21} + \dfrac{3}{40}M_{23}\right)}{l_2}$$

Using the second boundary condition (16), we get:

$$\Phi 30 = \frac{-uy3l - \dfrac{l_3^2}{E_3 I_3}\left(\dfrac{3}{10}M_{32} + \dfrac{3}{40}M_{33}\right)}{l_3}$$

4 Results and discussion

For the first time the authors have developed the technique for analytical determination of longitudinal and transverse displacements and the angles of rotation of cross-sections of links of the four-bar mechanism under the action of distributed dynamical loads. According to the given algorithm the programs in the Maple system were created and animations of the movement of mechanisms with the construction on the links the diagrams of transverse and longitudinal displacements and angles of rotation of the link cross-sections were received (Figure 2 – 6).

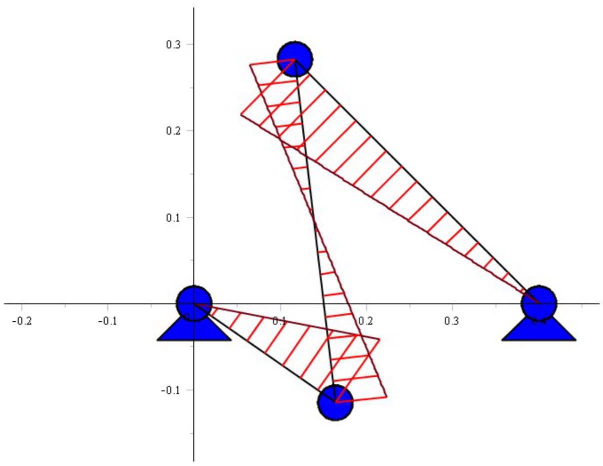

Figure 2 – The investigated mechanism with the construction on the links the diagrams of longitudinal inertial loads

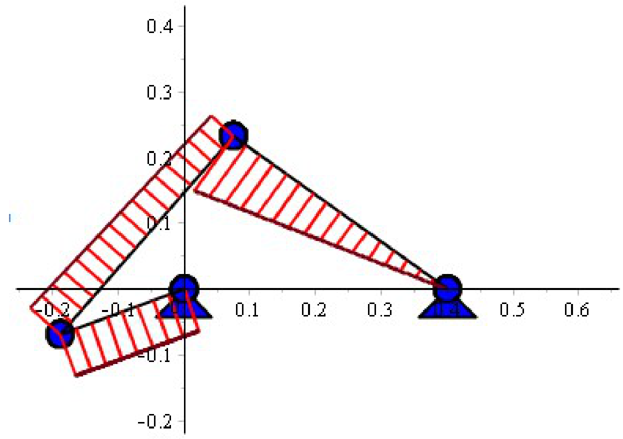

Figure 3 – The investigated mechanism with the construction on the links the diagrams of transverse inertial loads

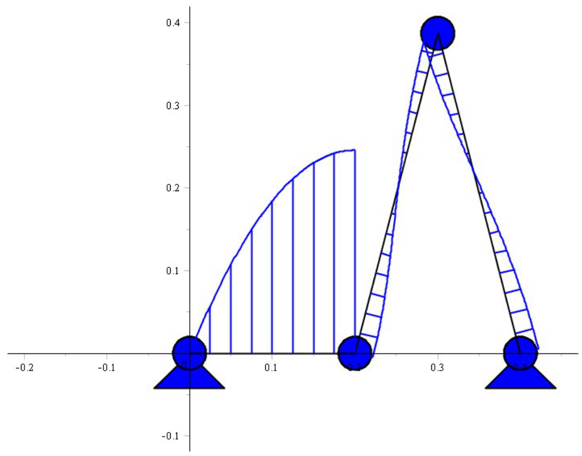

Figure 4 – The investigated mechanism with the construction on the links the diagrams of the angles of rotation of the link cross-sections

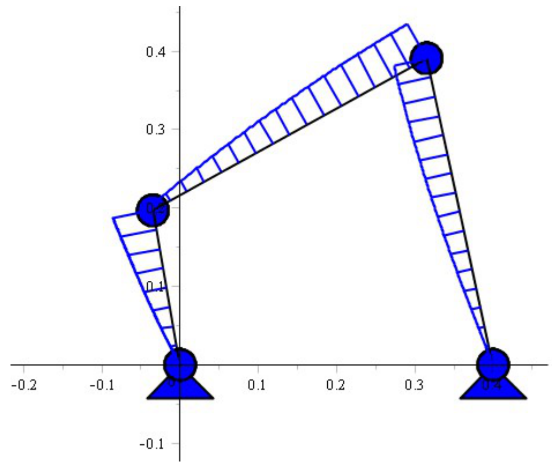

Figure 5 – The investigated mechanism with the construction on the links the diagrams of the longitudinal displacements of the link cross-sections

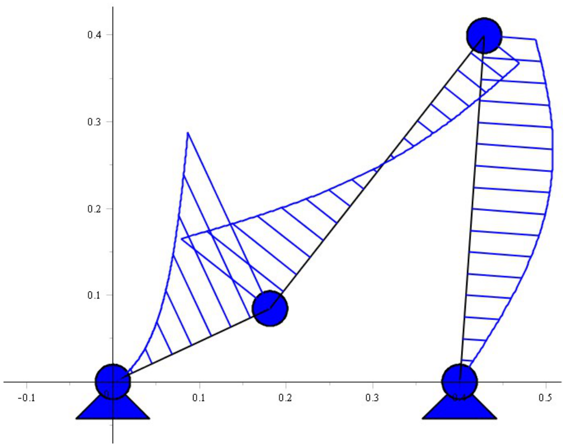

Figure 6 – The investigated mechanism with the construction on the links the diagrams of the transverse displacements (deflections) of the link cross-sections

5 Conclusion

The developed technique allows determining the deformations in the links of mechanisms and manipulators under the action of distributed dynamical loads and can be used in the study of stress-strain state of the projected and existing movable and fixed linkages (planar linkages, manipulators, frames, etc.).

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