Reduction in the Research of Large-Scale Dynamics with Allowance of the Effects of Magnetic Field Diffusion

Sergey Peregudin, Svetlana Kholodova

Department of Information System, Saint Petersburg State University,
Universitetskii av. 35, Petergof, 198504 Saint Petersburg, Russia
National Research University of Information Technologies, Mechanics and Optics,
Kronverksky prospekt 49, 197101 St. Petersburg, 197101 Saint Petersburg, Russia
{peregudinsi,kholodovase}@yandex.ru

Abstract. A system of nonlinear partial differential equations is considered that models perturbations in a layer of an ideal electrically conducting rotating fluid bounded by spatially and temporally varying surfaces with allowance for inertial forces and diffusions of magnetic field. The system is reduced to a scalar equation. The solvability of initial boundary value problems arising in the theory of waves in conducting rotating fluids can be established by analyzing this equation. Solutions to the scalar equation are constructed that describe small-amplitude wave propagation in an infinite horizontal layer and a long narrow channel.

Keywords: Ideal fluid dynamic problems, magnetohydrodynamic equations, reduction of vector equations to scalar equations, analytical method, diffusions of magnetic field.

1 Introduction

The system of nonlinear partial differential equations describing the dynamics of a rotating layer of an ideal conducting incompressible fluid is difficult to investigate because of its vector character. Therefore, it is natural to try to reduce it to equivalent scalar equations for auxiliary functions.

We consider the nonlinear system of partial differential equations that model perturbations in a layer of an ideal conducting rotating fluid bounded by spatially and temporally varying surfaces with allowance for inertial forces and diffusions of magnetic field. The purpose of this study is to reduce this system to a scalar equation and to construct analytical solutions to the corresponding boundary value problems.

The accounting of diffusive members is necessary when studying dynamics of waves of more local character, i.e., when the horizontal scale of change of hydromagnetic sizes much less than a radius of a considered layer, and also at very great time scales. It would be desirable to see influence of diffusion of a magnetic field on its generation. Whether there will be able to be a magnetic field as much as long time and whether it will exist at shutdown of an inoculating field.

The motion of a conducting fluid in a magnetic field causes electric currents. These currents change the magnetic field. At the same time, the forces acting on the currents in the magnetic field can change the character of the fluid motion. Hence, hydrodynamic motion and electromagnetic phenomena are interrelated. This relation is described by the joint system of field equations and the equations of motion of a fluid. According to the works by the well-known Swedish physicist and astrophysicist G. Alfven, the interrelation between electromagnetic and hydrodynamic phenomena strengthens as the linear scale of a phenomenon increases. For large-scale phenomena, this interrelation can be rather strong. For example, this is true of star interiors and the Earth's liquid core [1] .

Large-scale motions of an electrically conducting fluid have been intensively studied. In particular, we note [2]–[6], which investigate a model constructed in the approximation of fast rotation. Within the framework of this theory, the inertial force is ignored in the equation of motion. As a result, the inertial, Alfven, and Rossby waves are filtered out. Furthermore, in the limit of fast rotation, the velocity v is not determined uniquely but rather up to a term representing the geostrophic velocity. The reason for this is that the geostrophic velocity does not satisfy the magnetostrophic equation. To overcome these difficulties, viscous forces are invoked and the viscosity is neglected when possible.

In [7], [8], large-scale motions of a conducting fluid in a layer between the planes $z = 0$ and $z = d$ were studied in the magnetostrophic approximation taking into account viscous forces.

In this study, we assume that the boundaries of the layer are not stationary but vary in space and time. Furthermore, the inertial forces are taken into account in the equation of motion.

2 Dynamics of a thin rotating layer of an ideal electrically conducting incompressible fluid

Consider a thin layer of an ideal conducting incompressible fluid rotating at an angular velocity ω. The layer is bounded from below by a moving bottom specified by $z = h_B(x; y; t)$, where $h_B(x; y; t)$ is an unknown function and $z = 0$ is the reference level. The layer is bounded from above by a known surface $Z(x; y)$. The axis of fluid rotation coincides with the z-axis.

2.1 Governing equations of a horizontal structure of an electrically conducting rotation fluid

The governing magnetohydrodynamic equations for the problem under consideration are written in projections onto the coordinate axes [1], [9]–[12]:

$$\operatorname{div} \mathbf{v} = 0, \tag{1}$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\,\mathbf{v} = -\frac{\nabla p}{\rho} - 2\,\boldsymbol{\omega} \times \mathbf{v} - g\mathbf{z} + \frac{1}{\mu\rho} \operatorname{rot} \mathbf{b} \times \mathbf{B}, \tag{2}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \operatorname{rot}\,(\mathbf{v} \times \mathbf{B}) + \frac{1}{\sigma\mu} \Delta \mathbf{b}, \tag{3}$$

$$\operatorname{div} \mathbf{b} = 0, \tag{4}$$

where v_x, v_y and v_z are the velocity components of the fluid; p is the pressure; $\mathbf{g}$ is the acceleration of gravity; ρ is the density; b_x, b_y and b_z are the magnetic induction components; μ is the magnetic permeability; σ is the electrically conduction of the medium; and ω is the angular velocity of the Earth.

Consider the following characteristic scales of the variables in (1)–(4): D for vertical motion (where D is the average depth of the fluid layer $h_B(x; y; t) - Z(x; y)$), L for horizontal motion, U for the horizontal velocity component, $\mathbf{b}$ for the horizontal field components, H for the vertical field component, T for time, and P for the pressure field.

The basic equations of magnetohydrodynamics (1)–(4) of the problem in projections on the coordinate axes are of the form

$$\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z} = -\frac{1}{\rho}\frac{\partial}{\partial x}\left(p + \frac{b^2}{2\mu}\right) + \\ +2\omega v_y + \frac{1}{\mu\rho}\left(b_x\frac{\partial b_x}{\partial x} + b_y\frac{\partial b_x}{\partial y} + b_z\frac{\partial b_x}{\partial z}\right), \tag{5}$$

$$\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z} = -\frac{1}{\rho}\frac{\partial}{\partial y}\left(p + \frac{b^2}{2\mu}\right) - \\ -2\omega v_x + \frac{1}{\mu\rho}\left(b_x\frac{\partial b_y}{\partial x} + b_y\frac{\partial b_y}{\partial y} + b_z\frac{\partial b_y}{\partial z}\right), \tag{6}$$

$$\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z} = -\frac{1}{\rho}\frac{\partial}{\partial z}\left(p + \frac{b^2}{2\mu}\right) - g + \\ +\frac{1}{\mu\rho}\left(b_x\frac{\partial b_z}{\partial x} + b_y\frac{\partial b_z}{\partial y} + b_z\frac{\partial b_z}{\partial z}\right), \tag{7}$$

$$\frac{\partial b_x}{\partial t} + v_x\frac{\partial b_x}{\partial x} + v_y\frac{\partial b_x}{\partial y} + v_z\frac{\partial b_x}{\partial z} - b_x\frac{\partial v_x}{\partial x} - b_y\frac{\partial v_x}{\partial y} - b_z\frac{\partial v_x}{\partial z} = \lambda\Delta b_x, \tag{8}$$

$$\frac{\partial b_y}{\partial t} + v_x\frac{\partial b_y}{\partial x} + v_y\frac{\partial b_y}{\partial y} + v_z\frac{\partial b_y}{\partial z} - b_x\frac{\partial v_y}{\partial x} - b_y\frac{\partial v_y}{\partial y} - b_z\frac{\partial v_y}{\partial z} = \lambda\Delta b_y, \tag{9}$$

$$\frac{\partial b_z}{\partial t} + v_x\frac{\partial b_z}{\partial x} + v_y\frac{\partial b_z}{\partial y} + v_z\frac{\partial b_z}{\partial z} - b_x\frac{\partial v_z}{\partial x} - b_y\frac{\partial v_z}{\partial y} - b_z\frac{\partial v_z}{\partial z} = \lambda\Delta b_z, \tag{10}$$

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0, \tag{11}$$

$$\frac{\partial b_x}{\partial x} + \frac{\partial b_y}{\partial y} + \frac{\partial b_z}{\partial z} = 0, \qquad b^2 = b_x^2 + b_y^2 + b_z^2. \tag{12}$$

We pass from the system (5)–(12) to the corresponding system in dimensionless variables. Previously we introduce the characteristic scales of the variables system. Let D — characteristic vertical scale equal characteristic value of the average depth of the liquid layer $-Z(X,y) + h_B(x,y,t)$ and L — the characteristic scale of the horizontal movement. We assume that

$$\delta = \frac{D}{L} \ll 1.$$

We introduce further in consideration of the characteristic scale: U is scale of horizontal speed; W is scale of vertical speed; B is value b_x, b_y; H is value of b_z; T is value of time t; P is value of the fields of pressure.

In the equation (11) first and second terms of the order $O\left(\frac{U}{L}\right)$, so the order third term $O\left(\frac{W}{D}\right)$ no more than $O\left(\frac{U}{L}\right)$. Therefore, using the equation (11), we obtain

$$W \le O(\delta U).$$

Similarly, using the equation (12), we obtain

$$H \le O(\delta B).$$

Given the extent of the connection, we have system (5)–(10) to dimensionless variables. The a result we obtain a system

$$\frac{U}{T}\frac{\partial v_x}{\partial t}+\frac{U^2}{L}\left(v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}+v_z\frac{\partial v_x}{\partial z}\right)=-\frac{1}{\rho L}\left(P+\frac{(1+\delta^2)B^2}{2\mu}\right)\cdot$$
$$\cdot\frac{\partial}{\partial x}\left(p+\frac{b^2}{2\mu}\right)+2\omega U v_y+\frac{B^2}{L\mu\rho}\left(b_x\frac{\partial b_x}{\partial x}+b_y\frac{\partial b_x}{\partial y}+b_z\frac{\partial b_x}{\partial z}\right), \tag{13}$$
$$\frac{U}{T}\frac{\partial v_y}{\partial t}+\frac{U^2}{L}\left(v_x\frac{\partial v_y}{\partial x}+v_y\frac{\partial v_y}{\partial y}+v_z\frac{\partial v_y}{\partial z}\right)=-\frac{1}{\rho L}\left(P+\frac{(1+\delta^2)B^2}{2\mu}\right)\cdot$$
$$\cdot\frac{\partial}{\partial y}\left(p+\frac{b^2}{2\mu}\right)-2\omega U v_x+\frac{B^2}{L\mu\rho}\left(b_x\frac{\partial b_y}{\partial x}+b_y\frac{\partial b_y}{\partial y}+b_z\frac{\partial b_y}{\partial z}\right), \tag{14}$$
$$\frac{\delta U}{T}\frac{\partial v_z}{\partial t}+\frac{\delta U^2}{L}\left(v_x\frac{\partial v_z}{\partial x}+v_y\frac{\partial v_z}{\partial y}+v_z\frac{\partial v_z}{\partial z}\right)=-\frac{1}{\rho D}\left(P+\frac{(1+\delta^2)B^2}{2\mu}\right)\cdot$$
$$\cdot\frac{\partial}{\partial z}\left(p+\frac{b^2}{2\mu}\right)-g+\frac{\delta B^2}{L\mu\rho}\left(b_x\frac{\partial b_z}{\partial x}+b_y\frac{\partial b_z}{\partial y}+b_z\frac{\partial b_z}{\partial z}\right), \tag{15}$$
$$\frac{B}{T}\frac{\partial b_x}{\partial t}+\frac{UB}{L}\left(v_x\frac{\partial b_x}{\partial x}+v_y\frac{\partial b_x}{\partial y}+v_z\frac{\partial b_x}{\partial z}-b_x\frac{\partial v_x}{\partial x}-b_y\frac{\partial v_x}{\partial y}-\right.$$
$$\left.-b_z\frac{\partial v_x}{\partial z}\right)=\frac{\lambda B}{L^2}\Delta b_x,$$
$$\frac{B}{T}\frac{\partial b_y}{\partial t}+\frac{UB}{L}\left(v_x\frac{\partial b_y}{\partial x}+v_y\frac{\partial b_y}{\partial y}+v_z\frac{\partial b_y}{\partial z}-b_x\frac{\partial v_y}{\partial x}-b_y\frac{\partial v_y}{\partial y}-\right.$$
$$\left.-b_z\frac{\partial v_y}{\partial z}\right)=\frac{\lambda B}{L^2}\Delta b_y,$$
$$\frac{\delta B}{T}\frac{\partial b_z}{\partial t}+\frac{\delta UB}{L}\left(v_x\frac{\partial b_z}{\partial x}+v_y\frac{\partial b_z}{\partial y}+v_z\frac{\partial b_z}{\partial z}-b_x\frac{\partial v_z}{\partial x}-b_y\frac{\partial v_z}{\partial y}-\right.$$
$$\left.-b_z\frac{\partial v_z}{\partial z}\right)=\frac{\lambda\delta B}{L^2}\Delta b_z. \tag{16}$$

Hereinafter dimensionless variables denote the same symbols.

From equations (13) and (14), it follows that scale dynamic pressure P and the magnetic pressure $\dfrac{B^2}{\mu}$ is equal to the highest value set of parameters $\dfrac{\rho UL}{T}$, ρU^2, $2\omega\rho UL$, otherwise the acceleration of the flow of traffic will be zero.

Let us turn to a simplified version of the studied system of differential equations.

Leaving equation (15) principal terms, we obtain

$$\frac{\partial}{\partial z}\left(p+\frac{b^2}{2\mu}\right)=-\rho g,$$

or, after integration in z

$$p+\frac{b^2}{2\mu}=-\rho gz+C(x,y,t).$$

Here and in what follows, we retain the same notation for dimensionless variables as for dimensional ones.

All the terms in (13) and (14) are used unchanged in the subsequent study.

The ratio of convective member in equations of induction (16)–(16) to diffusion member expressed through the characteristic velocity of liquid U and the characteristic length L is a dimensionless parameter $\frac{LU}{\lambda}$, which is called the magnetic number of Reynolds. It characterizes the relationship between a plasma flow and a magnetic field. Under laboratory conditions usually $R_m \ll 1$, and this relationship is weak, whereas in astrophysics usually $R_m \gg 1$, and this relationship is strong [1]. Equation of induction determines the behavior of the magnetic field given the velocity, and this behavior depends significantly on the value of Reynolds magnetic number R_m. In the general case, the magnetic power lines are partially transferred by the plasma flow, and partially diffuse through it.

We will consider this general case. Thus we let $R_m = 1$, and assume that the diffusion members have the same order us the convection members.

Accounting for diffusion members is required when studying the dynamics of waves of more local nature, i.e. when L is much less than the radius of the layer, and at very large time scale T. It would like to examine the influence of diffusion of a magnetic field on its generation. Can such a field exist for an arbitrarily long time, and will it exist after switching the inoculating field off.

Define the total depth function $H = h_B\check{\ }Z$. Assume that the thickness of the fluid layer at rest is $H_0(x, y)$. The function $H(x, y, t)$ is represented in the form $H(x, y, t) = H_0(x, y) + \eta(x, y, t)$, where (x, y, t) is a small perturbation such that $\eta \ll H_0$. To describe the propagation of small perturbations, we use the standard linearization method applied in continuum mechanics to systems of differential equations describing the behavior of a medium. A solution to system is sought in the form

$$\mathbf{v} = \mathbf{v}_0 + \mathbf{v}'(x, y, t), \qquad \mathbf{b} = \mathbf{b}_0 + \mathbf{b}'(x, y, t), \tag{17}$$

assuming that the small perturbations of the horizontal velocity $\mathbf{v}'$ and horizontal magnetic field $\mathbf{b}'$ propagate against a certain steady-state uniform background described by the constants $\mathbf{v}_0$ and $\mathbf{b}_0$. Consider the case $\mathbf{v}_0 = 0$. We obtain the new system of equations

$$\frac{\partial v_x}{\partial t} - \alpha v_y = g\frac{\partial \eta}{\partial x} + \frac{1}{\mu\rho}\left(b_{0x}\frac{\partial b_x}{\partial x} + b_{0y}\frac{\partial b_x}{\partial y}\right), \tag{18}$$

$$\frac{\partial v_y}{\partial t} + \alpha v_x = g\frac{\partial \eta}{\partial y} + \frac{1}{\mu\rho}\left(b_{0x}\frac{\partial b_y}{\partial x} + b_{0y}\frac{\partial b_y}{\partial y}\right), \tag{19}$$

$$\frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x}(H_0 v_x) + \frac{\partial}{\partial y}(H_0 v_y) = 0, \tag{20}$$

$$H_0\left(\frac{\partial b_x}{\partial x} + \frac{\partial b_y}{\partial y}\right) + b_{z0}^{(e)}(x, y, t) - b_{z0}(x, y, t) = 0, \tag{21}$$

$$\frac{\partial b_x}{\partial t} - b_{0x}\frac{\partial v_x}{\partial x} - b_{0y}\frac{\partial v_x}{\partial y} = \frac{1}{R_m}\Delta b_x, \tag{22}$$

$$\frac{\partial b_y}{\partial t} - b_{0x}\frac{\partial v_y}{\partial x} - b_{0y}\frac{\partial v_y}{\partial y} = \frac{1}{R_m}\Delta b_y. \tag{23}$$

where $\alpha = 2\omega$.

Substituting, we obtain the following equation for $\xi(x, y, t)$:

$$\mathcal{D}\left(\mathcal{D}_t^2 + \alpha^2\right)^2\left(\left(\mathcal{D}_t - \frac{\Delta}{R_m}\right)\mathcal{D}_t - \frac{\mathcal{D}^2}{\mu\rho}\right)\Delta_2\xi = \frac{b_{z0} - b_{z0}^{(e)}}{(\mu\rho)^2 H_0}. \tag{24}$$

The above reasoning implies the following result.

Theorem *Any solution $\mathbf{v}(x, y, t)$, $\mathbf{b}(x, y, t)$, and $\eta(x, y, t)$ to the small perturbation problem in a layer of an ideal incompressible homogeneous conducting rotating fluid with allowance effects of diffusions of magnetic field satisfying the necessary smoothness conditions can be represented* *in the form*

$$\mathbf{b}(x,y,t) = \mu\rho\mathcal{D}_t\left(\mathcal{D}_t^2+\alpha^2\right)\widetilde{\mathbf{b}},\ \eta = \frac{1}{g}\mathcal{D}_t\left(\mathcal{D}_t^2+\alpha^2\right)\widetilde{\eta},\ \widetilde{\widetilde{\eta}} = \mathcal{D}_t\widetilde{\eta},\ \widetilde{\widetilde{\mathbf{b}}} = \mathcal{D}_t\widetilde{\mathbf{b}}, \tag{25}$$

$$\begin{pmatrix} v_x \\ v_y \end{pmatrix} = \begin{pmatrix} \mathcal{D}_t & \alpha \\ -\alpha & \mathcal{D}_t \end{pmatrix}\begin{pmatrix} \widetilde{\widetilde{\eta_{\mathbf{x}}}} + \mathcal{D}\widetilde{\widetilde{b}}_x \\ \widetilde{\widetilde{\eta_{\mathbf{y}}}} + \mathcal{D}\widetilde{\widetilde{b}}_y \end{pmatrix}, \tag{26}$$

$$\begin{pmatrix} \widetilde{\widetilde{b}}_x \\ \widetilde{\widetilde{b}}_y \end{pmatrix} = \mathcal{D}\left(\mathcal{D}_t^2+\alpha^2\right)\begin{pmatrix} \mu\rho\mathcal{D}_t\mathcal{R}-\mathcal{D}^2 & \alpha\mu\rho\mathcal{R} \\ -\alpha\mu\rho\mathcal{R} & \mu\rho\mathcal{D}_t\mathcal{R}-\mathcal{D}^2 \end{pmatrix}\begin{pmatrix} \xi_x \\ \xi_y \end{pmatrix}, \tag{27}$$

$$\widetilde{\widetilde{\eta}}(x,y,t) = \left(\mathcal{F}^2 + (\alpha\mathcal{D}^2)^2\right)\xi(x,y,t),\ \mathcal{F} = \mu\rho\left(\mathcal{D}_t^2+\alpha^2\right) - \mathcal{D}^2\mathcal{D}_t, \tag{28}$$

$$\mathcal{D} = b_{0x}\frac{\partial}{\partial x} + b_{0y}\frac{\partial}{\partial y},\ \mathcal{R} = \left(\mathcal{D}_t - \frac{\Delta}{R_m}\right), \tag{29}$$

where $\xi(x,y,t)$ is a solution to (24).

The converse is also valid: any solution to (24) generates a solution to system (18)–(22), which governs small perturbations in a thin layer of ideal incompressible homogeneous conducting rotating fluid, if the functions v, b, and з defined by formulas (25)–(28) satisfy the smoothness conditions in the domain under consideration.

Consider free linear oscillations of a conducting rotating fluid layer; that is, investigate small-amplitude wave propagation in an infinite horizontal layer and a long narrow channel.

Assume that $H_0 = \text{const}$ and $b_{z0} - b_{z0}^{(e)} = \text{Re}\ Be^{i(kx+ly-\sigma t)}$. Then, (24) has the solution

$$\zeta = \text{Re}\ Ae^{i(kx+ly-\sigma t)},$$

if the dispersion relation

$$\left(\sigma^2-\alpha^2\right)^2\left(\sigma^2 - \frac{(b_{0x}k+b_{0y}l)^2}{\mu\rho} + i\frac{(k^2+l^2)}{R_m}\sigma\right)\left(k^2+l^2\right)(b_{0x}k+b_{0y}l) = \frac{B}{Ai(\mu\rho)^2H_0} \tag{30}$$

is fulfilled. In particular, for $b_{z0} = b_{z0}^{(e)}$, (30) implies

$$\left(\sigma^2-\alpha^2\right)^2\left(\sigma^2 - \frac{(b_{0x}k+b_{0y}l)^2}{\mu\rho} + i\frac{(k^2+l^2)}{R_m}\sigma\right) = 0,$$

wherefrom

$$\sigma = \pm\alpha, \qquad \sigma = \pm\sqrt{\frac{(b_{0x}k+b_{0y}l)^2}{\mu\rho} - \frac{(k^2+l^2)^2}{4R_m^2}} - i\frac{(k^2+l^2)}{2R_m}.$$

We have two strong different branches for frequencies σ. The first type of oscillations is inertial waves. Inertia and Coriolis forces play main role here. Inertial waves have real frequencies and they are stable. The second type of oscillations is magnetic waves with complex frequencies. These magnetic waves are not stable, because they have negative imaginary part of frequencies σ.

In case $\mathbf{b}_0 = 0$, we obtain

$$\sigma = \pm i\sqrt{\frac{(k^2+l^2)^2}{4R_m^2}} - i\frac{(k^2+l^2)}{2R_m} = \pm i\frac{(k^2+l^2)}{2R_m} - i\frac{(k^2+l^2)}{2R_m} = -i\frac{(k^2+l^2)}{R_m},$$
$$\zeta = \Re e A\exp i(kx+ly+i\tfrac{(k^2+l^2)}{R_m}t) = A\exp -\tfrac{(k^2+l^2)}{R_m}t\cos(kx+ly).$$

The diffusion of magnetic field causes damping of the field. We have the stationary process for infinite value of Reynolds magnetic number. It means that the induced magnetic field can exist for an arbitrarily long time.

We obtain the well-known dispersion relation for Alfvens wave with $R_m \to \infty$.

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