Method of the "autonomous" modeling of turbulent flows under intermittency conditions. Part 1 – Problem formulation

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Аннотация

The statistical theory of turbulent flows under intermittency conditions is revised, and a new method is developed for statistical description of such flows. It is shown that the model as constructed here is more accurate than those previously proposed for intermittent flows. Some results are presented of conditioned and unconditioned averaged characteristics of several turbulent shear flows as predicted by the model. These predictions are applied via comparison with experimental date.

Nomenclature

$\langle f(x,t)\rangle$– unconditioned average value
$f'(x,t)$– fluctuation
$\langle f(x,t)\rangle_r$– conditioned averages
$P\{f\}$–event probability
$P(f)$–one-point PDF
$P(u,\zeta)$–joint PDF
$P(u|\zeta)$–conditional PDF
$I(x,t)$–intermittency function
$\gamma(x)$–intermittency factor
Ω–flow range (sample space)
Ω_t–flow range of turbulent liquid
Ω_n–flow range of non-turbulent liquid
$\tau(x)$–observation time
Subscription
r–for turbulent $(r=t)$ or non-turbulent $(r=n)$ medium; for generalized flow - r is absent

1 Introduction

At present in modeling of the turbulent flows a great attention is devoted to the DNS and LES methods together with their various modifications. However, the questions appear: did the method RANS exhaust itself? And have we the prospects of further improvement of statistical theory of the turbulent flows with the subsequent development of more effective mathematical models with heightened accuracy? The matter boils down to this.

Statistical theory of the developed turbulent flows, which based on the averaged Navier-Stokes equations, describes an energy-containing structure of such flows. Semiempirical character of the statistical theory leads to a necessity of mathematical modeling. Essence of the mathematical modeling leads to obtain a closed system of the differential equations for statistical averages of hydrodynamic values. The models, constructed under such approach, clearly have semiempirical character.

The RANS method is based on the unconditioned averaging of the Navier-Stokes equations and intended for modeling large-scale (energy-containing) structure of turbulent flow [1]. It is well-known that the mathematical (differential) models of the developed turbulent flows, which are constructed by the RANS method, have not the universality property and do not provide a high enough accuracy of calculation of the flow statistical characteristics. In particular it concerns to correlations of the various "pulsation" values. The results of numerous regular round of the collective testing of the turbulence models, organised, for example, by Stanford University ([1], [2], etc.) have been recognized as "unfavourable" and it confirms our words.

Also it is known that the classical RANS method does not take into account an intermittency of various dynamic fields of the considered flow, i.e. it does not take into account the effects, are connected with "alternation" of the areas with "turbulent" and "non-turbulent" fluid, and their dynamics has essential distinctions. At the same time this effect concerns to the inalienable properties of any turbulent flow, including the well-developed one, [3], [4], [5], [6], [7], [8]. As result the models, are constructed by the RANS method, do not allow modeling of so-called "conditional averages" of the hydrodynamic values of each of the intermittent mediums and, as consequence, they are not capable to provide a more detailed description of energy-containing structure of the considered turbulent flow.

Let to consider the essence of the mathematical modeling under conditions of intermittency.

2 Mathematical modeling under intermittency condition

The first attempts to modify the RANS method with the purpose of taking into account the intermittency effects and to obtain the equations for the conditional averages have been undertaken in the works of P.Libby, C.Dopazo, V.Sabel'nikov and so on (see, for example, [9], [10], [11], [12]). Such update ideology is to the effect that operation of unconditioned averaging of the Navier-Stokes equations is carried out after pre-multiplying of these equations by an intermittency function. However, equations for conditioned averages, obtained in such way, accept a bulky appearance, contain additional unknown terms of singular character and require attracting of advanced closure hypotheses, which resist to physical description. All this facts hamper an improvement of mathematical models of intermittency turbulent flows.

Proceeding to construct mathematical models, first of all it is necessary to determine, what kind of statistical characteristics are most suitable for modeling. In the classical RANS method such characteristics are total (unconditional) averages. In the methods which are taking into account an intermittency phenomenon, such characteristics are conditioned averages of hydrodynamic values of each of intermittent mediums.

Let's consider in details how a process of mathematical modeling is carried out in this case. The initial equations system of hydrodynamics of a viscous fluid is chosen in the form of the Navier-Stokes equations (together with the equation of continuity). In the tensor kind it looks as (see [13], etc.):

[1] Small-scale (dissipative) structure is described by Kolmogorov's theory. It is supposed that influence of the small-scale structure upon large-scale structure is taken into account via a current of turbulent energy over spectrum of scales of the turbulent eddies.

$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i u_k}{\partial x_k} = \frac{\partial}{\partial x_k}\left(\sigma_{ik} - P\delta_{ik}\right), \quad \frac{\partial \rho}{\partial t} + \frac{\partial \rho u_k}{\partial x_k} = 0, \quad i = 1,2,3 \tag{1}$$

Further, the indicator of a turbulent medium, i.e. the intermittency function of the regions with turbulent and non-turbulent fluid, is entered:

$$I(x,t) = \begin{cases} 1 & if\ \ (x,t) \in \Omega_t\ \ - is\ the\ region\ of\ a\ turbulent\ fluid \\ 0 & if\ \ (x,t) \in \Omega_n\ \ - is\ the\ region\ of\ a\ nonturbulent\ fluid \end{cases} \tag{2}$$

Now it is necessary to choose some operation of averaging of hydrodynamic values. It can be operation of the time averaging (so-called “averaging of Reynolds”, is designated further by the angular brackets), application of which to some function $f(\boldsymbol{x},t)$ gives unconditioned average value of this function:

$$\langle f(x,t)\rangle = \frac{1}{\tau_0}\int\limits_0^{\tau_0} f(x,t)dt \tag{3}$$

It is usually considered that the integral (3) exists at the achievement of certain plenty large enough value of τ_0=*const*, and the itself result by time equated to the statistical averages[2].

Applying averaging, for example, to an intermittency function we obtain so-called “factor of an intermittency”$\gamma(x) = \langle I(x,t)\rangle$, the value of which determines probability of supervision of turbulent medium in specified point of the space. Now we can obtain system of the equations for conditional average velocities according to the known, above mentioned, approach. Let's obtain this system, for example, for turbulent fluid.

Let's multiply the equations of Navier-Stokes by intermittency function $I(\boldsymbol{x},t)$, integrate these equations by time (together with dividing by τ_0), and commutate integration and differentiation operations (in the known works *the legitimacy of such commutation is accepted a priori*). Consequently we obtain equations system for conditional average velocities of the turbulent medium (hereinafter conditioned averages are designated by angular brackets with an index corresponding to given medium, t-turbulent, n – non-turbulent). Such system in case of incompressible flow according to C. Dopazo looks like (see [10]):

$$\frac{\partial\gamma\langle u_i\rangle_t}{\partial x_i} = E; \quad E = \lim_{V\to 0}\left\langle \frac{1}{V}\int\limits_S v^e dS\right\rangle \quad M_i = \lim_{V\to 0}\left\langle \frac{1}{V}\int\limits_S \rho u_i v^e dS\right\rangle$$
$$\frac{\partial\gamma\langle u_i\rangle_t\langle u_j\rangle_t}{\partial x_i} + \frac{\partial\gamma\left\langle u_i' u_j'\right\rangle_t}{\partial x_i} = M_i - \frac{1}{\rho}\frac{\partial\gamma\langle P\rangle_t}{\partial x_i} - F_i(x);$$

$$F_i(x) = \frac{1}{\rho}\lim_{V\to 0}\left\langle \frac{1}{V}\int\limits_S\left(\sigma_{ij} - p\delta_{ij}\right)n_j dS\right\rangle + \nu\frac{\partial}{\partial x_j}\lim_{V\to 0}\left\langle \frac{1}{V}\int\limits_S\left(u_i n_j + u_j n_i\right)dS\right\rangle \tag{4}$$

[2]Such approach is applicable to modeling of the statistically stationary turbulent flows when the initial conditions of formation of such flows do not change. Namely only such flows are considered here.

According to Sabel'nikov's work [11], in which the same approach was used, the specified system of the equations looks like:

$$
\begin{aligned}
&\frac{\partial\gamma\langle u_i\rangle_t}{\partial x_i} = \dot{m};\ \dot{m} = \dot{m}_0 + \dot{m}_1;\ \dot{m}_0 = \langle N\rangle\left[\frac{\partial P_t}{\partial z}\right]_{z=0},\ \dot{m}_1 = -\langle N\rangle\left[\frac{\partial P_t}{\partial z}\right]_{z=1}\\
&\frac{\partial\gamma\langle u_i\rangle_t\langle u_j\rangle_t}{\partial x_j} = -\gamma\Omega_i - \frac{1}{\rho}\frac{\partial\gamma R^t_{ij}}{\partial x_j} + \dot{m}_0\langle u_i\rangle_{t,z=0} + \dot{m}_1\langle u_i\rangle_{t,z=1};\\
&R^t_{ij} = \left\langle u'^t_i u^t_j\right\rangle_t, u'^t_i = u_i - \langle u\rangle_i;\quad \Omega_i = \int \omega_i P_t\,(u,z) d^3u dz =\\
&= -\frac{\partial\langle p\rangle}{\partial x_i} - \frac{R}{T_u}\left(\langle u_i\rangle_t - \langle u_i\rangle\right);\quad R = const,\quad T_u = \langle R_{kk}\rangle/\langle\varepsilon\rangle
\end{aligned}
\tag{5}
$$

The equation systems for conditionally averaged velocities of the flow of a non-turbulent medium are deduced similarly with only difference, that in this case the Navier-Stokes equations are multiplied by value of the $1 - I(x,t)$ and instead of γ we obtain $(1-\gamma)$, etc., see [10] and [11]. For brevity of our text these equations we did not write out here.

We see that the written out equations (4) and (5) contain the intermittency factor in expression for a partial derivative. It is notable that ones contain the *source* members, requiring additional closes hypotheses and creating thus additional difficulties in modeling. Moreover, the equations system (5) contains *unconditioned averages* of the values of dissipation, velocity and some other ones for definition of which we have to attract the corresponding experimental data (as it was made in [11]), or attract the Reynolds equations with purpose of calculation these unconditioned averages.

3 New approach to modeling of the intermittent turbulent flows

In the [7], [14] has been undertaken the attempt to develop essentially new approach in modeling of the turbulent flows which takes into account an intermittency effect and, as consequence, allows to build more effective mathematical models with higher accuracy.

According to this approach the equations system for conditioned averages of the flow, for example, of the turbulent medium is

$$
\frac{\partial\langle u_i\rangle_t\langle u_k\rangle_t}{\partial x_k} + \frac{\partial\left\langle u'_i u'_j\right\rangle_t}{\partial x_k} = \frac{\partial}{\partial x_k}\left(\langle\sigma_{ik} - p\delta_{ik}\rangle_t/\ \langle\rho\rangle_t\right),\quad \frac{\partial\langle u_k\rangle_t}{\partial x_k} = 0 \tag{6}
$$

Apparently that it system essentially differs from systems (4) and (5). Relative simplicity of the equations (6) which are formally (i.e. only by general form) coincide with Reynolds's equations and make them rather attractive to mathematical and numerical modeling.

Materially this method concerns to the RANS method, which takes into account effects of intermittency. *The fundamental difference of the method from known methods is that its development is based on conditional averaging of the Navier-Stokes equations.*

At the same time the approach, which was used in [7, 14], requires a serious substantiation both in physical and mathematical plan. *The main purpose of the given work is just such substantiation.*

4 Problem formulation

A process of the modeling contains three consecutive stages – they are: 1) integration of the Navier-Stokes equations (with purpose of their averaging); 2) commutation of integration and differentiation operations (with purpose obtaining the equations for required statistical averages); 3) the attracting of close hypotheses (with purpose closing of the obtained system of the equations). In case of the modeling intermittent turbulent flows there are same problems.

First, the unconditioned average values are defined as [15]:

$$\langle f(x,t)\rangle = \lim_{\tau_0\to\infty}\frac{1}{\tau_0}\int\limits_0^{\tau_0} f(x,t)dt \tag{7}$$

The conditioned average values are defined as

$$\langle f(x,t)\rangle_r = \lim_{\tau_r\to\infty}\frac{1}{\tau_r}\int\limits_0^{\tau_r} f(x,t)dt \tag{8}$$

Assuming that these limits exist for a plenty large enough value of τ_0=*const*. Now we need the connection between (7) and (8).

Secondly, we address the issue regard ins the legitimacy of commutation of differentiation and integration operations. In troth, it is typically assumed

$$\left\langle\frac{\partial f}{\partial x}\right\rangle = \frac{\partial\langle f\rangle}{\partial x} \tag{9}$$

which, let us say for the continuity equation of incompressible turbulent flows, implies:

$$\frac{\partial u_i}{\partial x_i} = 0 \to \quad \left\langle\frac{\partial u_i}{\partial x_i}\right\rangle = 0 \to \quad \frac{\partial\langle u_i\rangle}{\partial x_i} = 0 \tag{10}$$

There is a problem with (9). In our case of intermittent flow we have well-known expression:

$$\langle u_i\rangle = \gamma\langle u_i\rangle_t + (1-\gamma)\langle u_i\rangle_n \tag{11}$$

here $\gamma = \tau_t(x)/\tau_0$. Therefrom

$$\frac{\partial\langle u_i\rangle}{\partial x_i} = \gamma\frac{\partial\langle u_i\rangle_t}{\partial x_i} + (1-\gamma)\frac{\partial\langle u_i\rangle_n}{\partial x_i} + (\langle u_i\rangle_t - \langle u_i\rangle_n)\frac{\partial\gamma}{\partial x_i} \tag{12}$$

Since the last term in Eq. (12) is not generally zero, there is dilemma:

$$or \quad \frac{\partial\langle u_i\rangle}{\partial x_i} = 0, \quad or\ corr \quad \frac{\partial\langle u_i\rangle_r}{\partial x_i} = 0 \tag{13}$$

Lastly, according to our approach we need to formulation a guideline for closing of the obtained systems of the conditional averaging of the Navier-Stokes equations.

As it is transpiring, we need to develop a new method of construction of mathematical models of the turbulent flows. Proceeding to realization of the marked plan, first of all we will set task to give the argued answers to the following questions:

Wherefore the method RANS does not allow to build mathematical models with a high quality?

Which of statistical characteristics are most suitable for a modeling?

Wherefore the known methods which take into account intermittency effects, do not provide high accuracy of modeling of the conditioned averages?

Which of the approaches in mathematical modeling is the most effective?

5 About necessity of taking into account the intermittency phenomenon

Рис. 1: Coherent eddies on one of borders of an initial region of a free jet, [16].

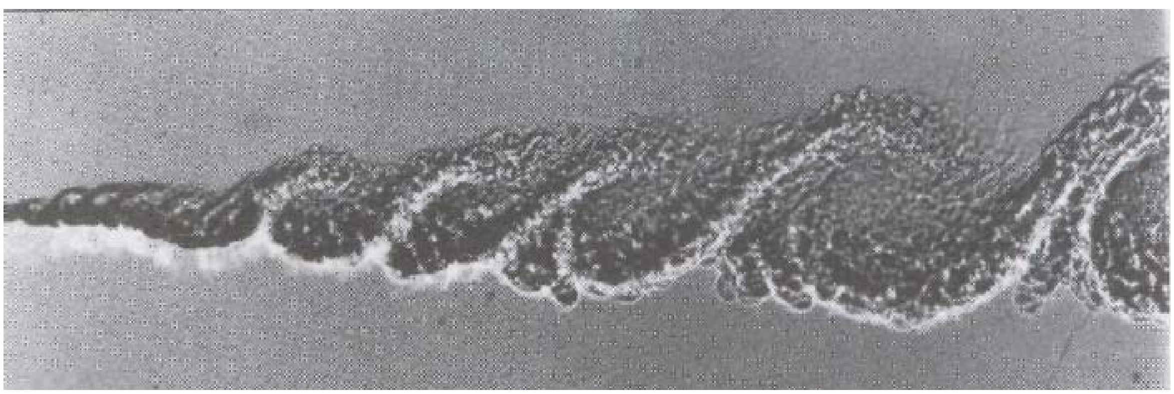

Рис. 2: A visualization of the flow of a plane mixing layer along with a co-streams with $u_2/u_1 = 0.38$ of helium and nitrogen, [17]. Reynolds's number $Re = 1.2 * 10^5$ and corresponds to developed turbulent flow.

Let's consider characteristic photos for turbulent flows, fig. 1, 2 and 3. Here first of all pays attention to itself a presence of expressive coherent large-scale structures. The presence of such structures leads to involving of a surrounding (non-turbulent) fluid inwards of turbulent stream along with subsequent turbulization of this fluid downstream. At the same time it is possible to notice the areas with unmixed fluid (carbonic gas) even in central regions of a turbulent jet, fig.3.

The presence of the enough sharp borders between so-called turbulent and non-turbulent fluid is the other characteristic property of the turbulent streams. Hence it appears a question about character of behavior of the momentary, random nonaveraged hydrodynamic characteristics over various regions of the considered turbulent flow. To answer this question let's attract the oscillograms which have been taken off by velocity sensor at the various points of turbulent jet, fig. 4.

Because of the various behaviors of momentary characteristics of turbulent and non-turbulent fluid it's necessary to reconsider the approaches in mathematical modeling of the turbulent flows. Really, let's imagine a picture of the unconditional and conditional averaging

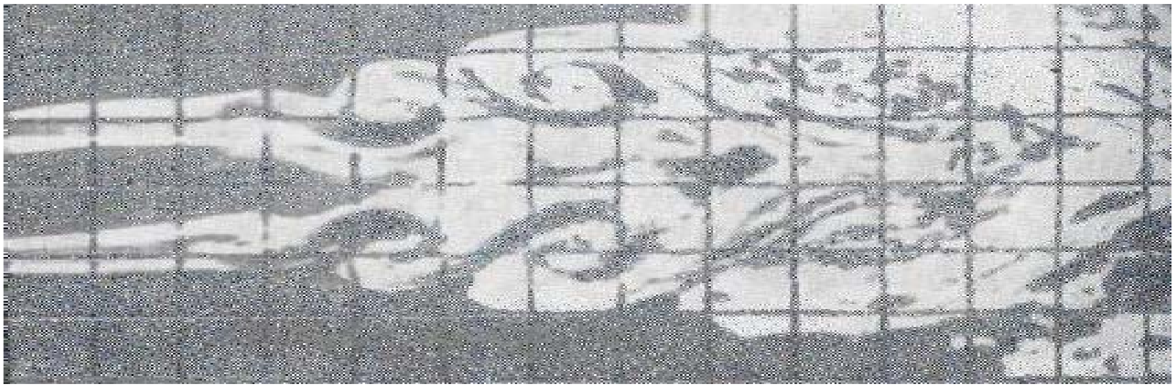

Рис. 3: Tepler's photo of carbonic gas jet incidental out of a flat nozzle into air, $Re = 5 * 10^3$, [18].

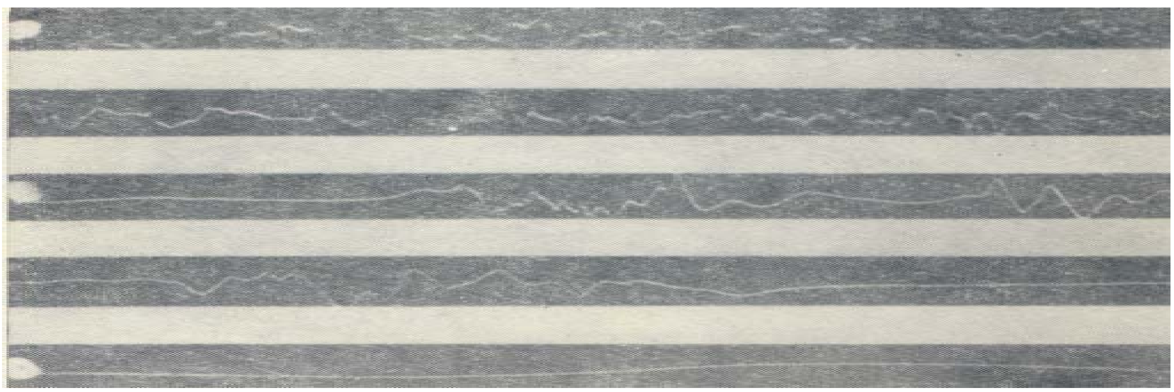

Рис. 4: Oscillograms of momentary velocity's value at the various points of turbulent jet, [19]. During the movement of the sensor from the center of jet to periphery developed turbulent flow (two pictures from above) is replaced by the intermittent (two average pictures) and potential (below) flow.

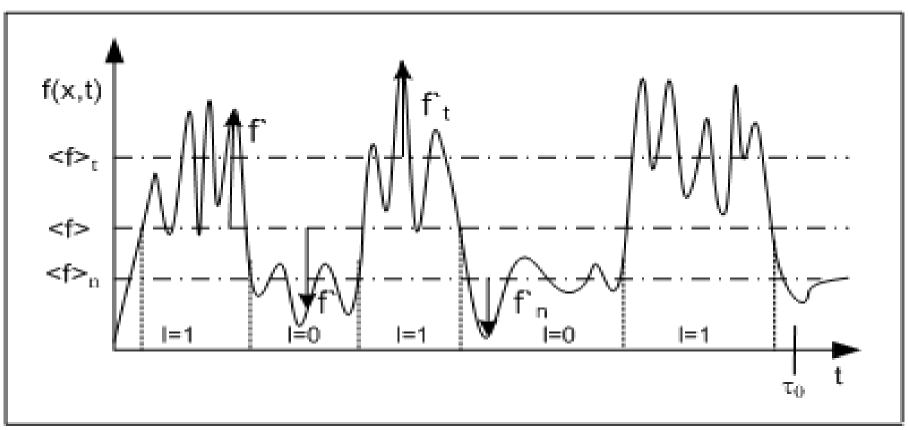

Рис. 5: An illustration of behavior of the momentary hydrodynamic variable, taken off by the sensor in the specified point of a turbulent flow $\boldsymbol{x}$ =*const* in area with a strong intermittency.

(over each of intermittent mediums) of these characteristics according to fig.4 in the physical region ($\mathbf{x}$,t) with strong intermittency, fig.5.

Evidently, the conditional averages of hydrodynamic characteristics of turbulent ($\langle f\rangle_t$, $I = 1$) and non-turbulent ($\langle f\rangle_n$, $I = 0$) medium do not coincide (that is proved by the numerous experimental data). Meanwhile the value of unconditional (total) average $\langle f\rangle$ lies between the conditional averages $\langle f\rangle_t$ and $\langle f\rangle_n$.

As seen, the total averages of hydrodynamic characteristics have some "intermediate" character. And the "fluctuations" f', which are counted out from this average value, are not always quite random as it demands the RANS method. Meanwhile the mentioned "fluctuations" can not change their own sign during all the time of observation, but only in process of alternation of the regions with turbulent and non-turbulent liquid. Hence it appears that

$$\left\langle \frac{\partial f'}{\partial x} \right\rangle \neq \frac{\partial \langle f'\rangle}{\partial x} = 0 \tag{14}$$

when the value $\tau_0$is arbitrary large, but finite. It means that the intermittency can lead to infringement of Leibnitz conditions at the generalized field of flow.

6 About commutation of differentiation and integration operations

Let's consider now a question about the averaging of the Navier-Stokes equations in order to obtain our equations in the spatial area ($\mathbf{x}$,t). With decomposition of the time interval of averaging $\tau_0 = \tau_t(\boldsymbol{x}) + \tau_n(\boldsymbol{x})$, we have

$$\langle u\rangle = \frac{1}{\tau_0}\int_0^{\tau_0} u(x,t)dt = \frac{1}{\tau_0}\left\{ \int_0^{\tau_t(x)} u(x,t)dt + \int_0^{\tau_n(x)} u(x,t)dt \right\} = \gamma\langle u\rangle_t + (1-\gamma)\langle u\rangle_n \tag{15}$$

and the conditionally averaged values are

$$\langle u\rangle_t = \frac{1}{\tau_t}\int_0^{\tau_t(x)} u(x,t)dt, \quad \langle u\rangle_n = \frac{1}{\tau_n}\int_0^{\tau_n(x)} u(x,t)dt \tag{16}$$

And define

$$J(\tau_t(x),x) = \frac{1}{\tau_0}\int_0^{\tau_t(x)} u(x,t)dt, i.e. J(\tau_t(x),x) = \gamma\langle u\rangle_t$$

Then on the basis of

$$\frac{dJ(\tau_t(x),x)}{dx} = \frac{\partial J}{\partial x}\Big/_{\tau_t = Const} + \frac{\partial J}{\partial \tau_t}\Big/_{x = Const}\frac{d\tau_t}{dx} \tag{17}$$

we get [3]

$$\frac{dJ(\tau_t(x),x)}{dx} = \frac{1}{\tau_0}\int\limits_0^{\tau_t(x)} \frac{\partial u(x,t)}{\partial x}dt + \frac{\partial}{\partial \tau_t}\left(\frac{\tau_t}{\tau_0}\frac{1}{\tau_t}\int\limits_0^{\tau_t(x)} u(x,t)dt\right)\frac{d\tau_t}{dx} =$$

$$= \gamma\left\langle\frac{\partial u}{\partial x}\right\rangle_t + \langle u\rangle_t\frac{d\gamma}{dx} + \tau_t\frac{\partial\langle u\rangle_t}{\partial\tau_t}\frac{d\gamma}{dx} =$$
$$= \gamma\frac{d\langle u\rangle_t}{dx} + \langle u\rangle_t\frac{d\gamma}{dx}$$

Since $\langle u\rangle_t$ does not depend on τ_t under condition $\tau_0 \to \infty$, we have $\partial\langle u\rangle_t/\partial\tau_t = 0$. Similarly, we have $\partial\langle u\rangle_n/\partial\tau_n = 0$. So, with $J(\tau_n(x),x) = (1-\gamma)\langle u\rangle_n$, we have

$$\frac{d\langle u\rangle_t}{dx} = \left\langle\frac{\partial u}{\partial x}\right\rangle_t; \quad \frac{d\langle u\rangle_n}{dx} = \left\langle\frac{\partial u}{\partial x}\right\rangle_n \tag{18}$$

Generalizing to 3-D yields:

$$\left\langle\frac{\partial u_i}{\partial x_i}\right\rangle \neq \frac{\partial\langle u_i\rangle}{\partial x_i}, \quad \left\langle\frac{\partial u_i}{\partial x_i}\right\rangle = 0 \tag{19}$$

And

$$\frac{\partial\langle u_i\rangle}{\partial x_i} = [\langle u_i\rangle_t - \langle u_i\rangle_n]\frac{\partial\gamma}{\partial x_i} \neq 0 \tag{20}$$

Thus, commutation of the integration and differentiation is not allowed for the unconditional averages as it leads to an infringement of the Leibnitz rule. But all of the Leibnitz conditions are satisfied within each of the two regions, allowing commutation.

7 Some comments

The mathematical rigor of the developed theory allows to prove legitimacy of the Eg. (14). Indeed, let's choose as fluctuation value, for example, fluctuation of the longitudinal velocity. According to fig.5 the one can be submitted as

$$u' = Iu' + (1-I)u' = \begin{cases} \langle u\rangle_t + u'_t - \langle u\rangle, & I(x,t) = 1 \\ \langle u\rangle_n + u'_n - \langle u\rangle, & I(x,t) = 0 \end{cases}$$

Hence it follows that $\langle u'\rangle = 0$. Consider now the $\partial u'/\partial x$. After unconditional averaging of this value we have:

$$\left\langle\frac{\partial u'}{\partial x}\right\rangle = \left\langle I\frac{\partial u'}{\partial x} + u'_{st}\frac{\partial I}{\partial x} + (1-I)\frac{\partial u'}{\partial x} - u'_{sn}\frac{\partial I}{\partial x}\right\rangle =$$
$$= \gamma\left\langle\frac{\partial u'}{\partial x}\right\rangle_t + (1-\gamma)\left\langle\frac{\partial u'}{\partial x}\right\rangle_n + \left(u'_{st} - u'_{sn}\right)\frac{\partial I}{\partial x}$$

Since in our case the function $u' = f'(x,t)$ is smooth and, hence, the values of velocities at the border between turbulent and non-turbulent liquid (i.e. at the surface S) are equal among themselves, $u'_{st} = u'_{sn}$.

[3] We consider that $\tau_0 \to \infty$, i.e. $\tau_t(\mathbf{x}) \to \infty$ and $\tau_n(\mathbf{x}) \to \infty$, however the values $\tau_t(\mathbf{x})/\tau_0 = \gamma(\mathbf{x})$ and $\tau_n(\mathbf{x})/\tau_0$=1-$\gamma(\mathbf{x})$ are limited. It does not contradict the condition $\gamma(\mathbf{x})$=0 or $\gamma(\mathbf{x})$=1, when there is no the turbulent or non-turbulent liquid.

Then we obtain

$$\left\langle \frac{\partial u'}{\partial x} \right\rangle = \gamma \left\langle \frac{\partial u'}{\partial x} \right\rangle_t + (1-\gamma) \left\langle \frac{\partial u'}{\partial x} \right\rangle_n$$

Substituting here the expressions for u' and noting that

$$\left\langle \frac{\partial u'_t}{\partial x} \right\rangle_t = 0 = \left\langle \frac{\partial u'_n}{\partial x} \right\rangle_n$$

by definition, we find:

$$\left\langle \frac{\partial u'}{\partial x} \right\rangle = -\left(\langle u \rangle_t - \langle u \rangle_n \right) \frac{\partial \gamma}{\partial x} \neq 0$$

Inasmuch as $\langle u' \rangle = 0$ we obtain the Eg. (14).

In regard to time derivative it is easy to show that in case of statistically stationary turbulent flows, i.e. when $\partial\gamma/\partial t = 0$, such commutation is possible:

$$\left\langle \frac{\partial u}{\partial t} \right\rangle = \frac{\partial \langle u \rangle}{\partial t}$$

(It is interesting to notice that the same results of the averaging of "dimensional" and "time" derivatives have been obtained in the LES method, see e.g. [8]).

Well then what is the reason of the inequality $\langle \partial u/\partial x \rangle \neq \partial \langle u \rangle/\partial x$? To answer this question finally we shall consider the expression for derivative:

$$\left\langle \frac{\partial u}{\partial x} \right\rangle = \left\langle \lim_{\Delta x \to 0} \frac{u(x+\Delta x) - u(x)}{\Delta x} \right\rangle = \lim_{\Delta x \to 0} \frac{\langle u(x+\Delta x) \rangle - \langle u(x) \rangle}{\Delta x}$$

Let's notice now, that if $x_1 = x + \Delta x \in \Omega_t,\ x_2 \in \Omega_n$, i.e. if the considered points are on the different sides from border of phase spaces with turbulent and non-turbulent liquid, then $u(x_1) = u|_{(I=1)} = u_t, \quad u(x_2) = u|_{(I=0)} = u_n$ and we find, that the limit

$$\lim_{\Delta x \to 0} \tfrac{\langle u(x+\Delta x) \rangle - \langle u(x) \rangle}{\Delta x} = \lim_{\Delta x \to 0} \tfrac{\langle u_t(x_1) \rangle - \langle u_n(x_2) \rangle}{\Delta x} =$$
$$= \lim_{\Delta x \to 0} \tfrac{\gamma \langle u \rangle_t - (1-\gamma) \langle u \rangle_n}{\Delta x} \to \infty$$

i.e. by virtue of finite value of the numerator this limit does not exist. If the both points x_1 and x_2 are located within turbulent (or non-turbulent) medium, then we have

$$\lim_{\Delta x \to 0} \tfrac{\langle u(x+\Delta x) \rangle - \langle u(x) \rangle}{\Delta x} = \lim_{\Delta x \to 0} \tfrac{\langle u_t(x_1) \rangle - \langle u_t(x_2) \rangle}{\Delta x} =$$

$$= \lim_{\Delta x \to 0} \tfrac{\gamma \langle u \rangle_t (x_1) - \gamma \langle u \rangle_t (x_2)}{\Delta x} = \lim_{\Delta x \to 0} \tfrac{0}{0} = const.$$

8 Conclusion

On basis of the carried analysis it may be concluded, that it's necessary to develop a new, more effective method for modeling the mathematical models of turbulent flows with an opportunity of more detailed description of their hydrodynamic structure, which would allow modeling the conditioned averages of turbulent and non-turbulent liquid without using the

Reynolds equations. But such method leads to a necessity of a "splitting" of the random generalized field of considering turbulent flow on two stochastic independent selective fields. Such splitting has a lot of specific features which should be taken into account by development of the statistical theory of intermittent turbulent flows. For example, after introduction of an intermittency function in order to "split" our flow on regions with "turbulent" and "non-turbulent" liquid, the partial derivatives are uniformly continuous only inside of each mentioned regions and exist only as unilateral limits on their borders.

Thus, from a perspective of statistical hydrodynamics, a turbulent flow is characterized by some generalized random field of hydrodynamic variables. This field consists from two selective fields, which is available in case of intermittency of turbulent and non-turbulent medium. The operation of unconditioned averaging of the Navier-Stokes equations leads to superposition (to imposing) of the mentioned selective fields and, consequently, to "smoothing" of a detailed flow picture, that is due intermittency.

Hence it follows a far-reaching conclusion: total averages which have been using in the methods of mathematical modeling and, particularly in the RANS method, don't quite conform to physics of turbulent flows. They don't suit to modeling and cannot provide a high quality of the mathematical models, which based on Reynolds differential equations.

The conditioned averages, i.e. statistical characteristics of each of two selective fields of a turbulent flow, are conventional method, used for modeling conditional averages (such as [10, 11]) cannot provide high precision of the description of statistical characteristics, because they are based on the operation of unconditioned averaging, because of the additional "source" terms with random character, and also in view of intermittency factor in the differential equations of hydrodynamics, whose procedure of determination is not quite clear till now.

One of the basic results received in the given work is the conclusion about impossibility of commutation of operations of integration and differentiation in process of the unconditional averaging of the hydrodynamics equations. This conclusion is very important in the sphere of modeling the turbulent flows possessing a property of intermittency.

As it is found out, the educt is the consequence of various flow structures of turbulent and non-turbulent media and it is expressed in essential distinction of the PDF functions of hydrodynamical characteristics of these media. Here it is interesting to draw an analogy between intermittent turbulent flow and turbulent flow of "biphase" media.

So then, we have the analytical results as applied to a spatial area $(\mathbf{x}, t)$. Our next intention is to develop a mathematical tools technique as applied to statistical theory of the turbulent intermittent flows. (To be continued).

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