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Analytical method of definition of internal forces taking into account the distributed dynamical loads in links of robotic systems and mechanisms with statically indeterminate structures

In this paper the technique of analytical determination of internal forces in links of planar mechanisms and manipulators with statically indeterminate structures taking into account the distributed dynamical loads, a dead weight and the operating external loads is designed. The dynamic equilibrium equations for the discrete model of the element under the action of cross and axial inertial trapezoidal loads are derived. Also, the dynamic equilibrium equations for elements and joints that expressed in terms of the unknown parameters of the internal forces of elements under the action of the distributed trapezoidal loads are obtained. The compliance matrix of an element is received from the expressions of energy for rods, so as the replacement of construction by a set of discrete elements is based on the equality of the energies of the real structures and its discrete model. The programs in the MAPLE18 system are made on the given algorithm and animations of the motion of mechanisms with construction on links the intensity of cross and axial distributed inertial loads, the bending moments, cross and axial forces, depending on kinematic characteristics of links are obtained.
Key words: Mechanisms, Manipulators, Distributed inertial forces, Internal forces, Dynamic equilibrium, Linkage compliance, Kinematic parameters, Statically indeterminate mechanisms, CAD, Animation.

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Құрылымы статикалық анықталмаған механизмдер мен роботты-техникалық жүйелердің буындарындағы тарқалған динамикалық жүктемелерді ескеріп ішкі күштерді анықтаудың аналитикалық әдісі

Құрылымы статикалық анықталмған жазық стерженді механизмдер мен манипуляторлардың буындарындағы ішкі күштерді тарқалған динамикалық жүктемелерді, салмақты және сыртқы күштерді ескеріп анықтаудың аналитикалық әдісі әзірленді. Қарқындылықтары трапеция түрінде таралған көлденең және бойлық инерция күштерінің әсерінде тұрған, сәйкесінше, қимасы тұрақты және сызықты өзгеретін элементтердің дискретті модельдерінің динамикалық тепе-теңдік теңдеулері алынды. Сонымен бірге, элементтерге трапециялық таралған жүктемелер әсер еткендегі ішкі күштердің ізделінетін параметрлері арқылы өрнектелген, шарнирлі және қатаң түйіндердің тепе-теңдік теңдеулері алынды. Құрылымы статикалық анықталмаған механизмдердің буындарындағы ішкі күштерді анықтағанда, буындардың жаншылғыштық матрицасының қажеттілігі туады. Конструкцияны дискретті элементтердің жиынтығымен ауыстыру нақты конструкцияның және оның дискретті моделінің энергияларының теңдігіне негізделгендіктен, элементтердің жаншылғыштық матрицалары стержендердің энергиясын анықтау үшін жазылған өрнектерден табылды.

Келтірілген алгоритм бойынша MAPLE18 жүйесінде программалар құрылып, механизмдердің буындарына қойылған көлденең және бойлық таралған инерция күштерінің, иілу моменттерінің, бойлық және көлденең күштердің қарқындылықтары көрсетілген қозғалыс анимациялары алынды.
Түйін сөздер: механизмдер, манипуляторлар, тарқалған инерция күштері, ішкі күштер, динамикалық тепе-теңдік, буын жаншылғыштығы, кинематикалық параметрлер, статикалық анықалмаған механизмдер, программалау, анимация құру.

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Аналитический метод определения внутренних усилий с учетом распределенных динамических нагрузок в звеньях робототехнических систем и механизмов со статически неопределимыми структурами

Разработана методика аналитического определения внутренних усилий в звеньях плоских стержневых механизмов и манипуляторов со статически неопределимыми структурами с учетом распределенных динамических нагрузок, собственного веса и от действующих внешних нагрузок. Выведены динамические уравнения равновесия для дискретной модели элемента под действием поперечных и продольных инерционных нагрузок трапецеидального вида. Также выявлены уравнения равновесия шарнирных и жестких узлов, выраженные через искомые параметры внутренних усилий элементов под влиянием распределенных нагрузок трапецеидального вида. Матрицы податливости элемента получены из энергетических выражений для стержней, так как замена конструкции совокупностью дискретных элементов основывается на равенстве энергии реальной конструкции и ее дискретной модели. По приведенному алгоритму составлены программы в системе MAPLE18 и получены анимации движения механизмов с построением на звеньях интенсивности поперечных и продольных распределенных инерционных нагрузок, изгибающих моментов, поперечных и продольных сил, зависящие от кинематических характеристик звеньев.
Ключевые слова: механизмы, манипуляторы, распределенные инерционные силы, внутренние усилия, динамическое равновесие, податливость звеньев, кинематические параметры, статически неопределимые механизмы, программирование, анимация.

Introduction

There are a variety of graphic-analytical and numerical calculation methods of rod mechanisms and robotic systems on strength and rigidity, which don't consider the distributed inertia forces of complex nature [1-9]. Assur groups, that form the designed scheme of mechanism, can be statically determinate, and also statically indeterminate in concept of determination of internal forces. Statically indeterminate group will be those included in the calculated scheme of the mechanism, which consists of three or more basic joints, and the links are connected rigidly. In this paper we propose a new analytical approach of solution of problems of dynamic calculation on strength and rigidity taking into account the distributed dynamic loads in links of robotic systems and mechanisms with statically definable structures. The distributed inertia forces of complex nature appear in links of rod mechanisms within the motion process. The intensity of distribution of inertia forces along the link depends on the mass distribution along the link and the kinematic characteristics of the mechanism changing rapidly. Therefore, relations between the intensity of distributed inertia forces and link weight with geometrical, physical and kinematic characteristics are determined in our work. The distribution laws of inertia forces and dead weight, make it possible at each position of links deduce the laws of distribution of internal forces along the axis of the link, in which loads are found at any point of link. Their maximum values allow to optimize the design parameters of the link, providing the strength and rigidity of links and, entirely, of robotic systems and mechanisms. As internal loads of each continued link are all defined by a set of internal loads in its separate cross-sections, and by the matrixes of approximations, so the task was to calculate the internal loads in finite number of cross-sections of elements. As a result, we refer to discrete model of elastic calculation of links of rod mechanisms. In the work of elastic calculation of planar rod mechanisms for each instantaneous position of the mechanisms they are brought to link structures, which degree of freedom is equal to zero based on Dalamber's principle. For definition of internal loads in links of designed scheme of mechanism, the structure is divided into elements, and both pin and rigid joints. This is the first time the elements are divided into three types of beam. Discrete models of these beams having constant cross-sections which are under the action of cross and axial the distributed loads of a trapezoidal view are constructed. These constructed discrete models allow to determine the quantity of the independent dynamic equilibrium equations, components of a vector of forces in calculated cross-sections and to construct discrete model of all structure. The dynamic equilibrium equations for discrete model of an element of the link with constant cross-sections under the action of cross and axial inertial loads of a trapezoidal view are also received in this work as well as the equilibrium equations of pin and rigid joints expressed through required parameters of internal forces. If we integrate the equations of dynamic equilibrium of elements and joints in a single system, we will receive the equations of dynamic equilibrium of all discrete model of system. A sort of systems of equations is sufficient for definition of internal forces in links of mechanisms, which structure is statically definable. The vector of forces and vector of loads in calculated cross-sections of discrete models of mechanisms are formed from vectors of forces and vectors of loads in calculated cross-sections of their separate elements, respectively. For mechanisms, with the Assur groups in its structure and having statically indeterminate links, the number of equilibrium equations is less than the number of unknowns by the number of degree of statically indetermination of construction. Hence, for definition of internal forces of statically indeterminate mechanisms the flexibility matrices were constructed optional for entire discrete model of rod mechanism. On the given algorithm the programs in the MAPLE system were made and animations of the motion of mechanisms with construction on links the intensity of cross and axial distributed inertial loads, the bending moments, cross and axial forces, depending on kinematic characteristics of links were obtained.

1 Inertia forces and the approximation matrix

Considering the plane-parallel motion of an **k**th link of mechanism with constant cross-sections comparatively fixed system of coordinates OXY, the following laws of distribution of the cross and axial inertia forces along a link, that arise from self mass of a link are defined [10]:

$$\begin{aligned} q_k(x'_k) &= a_{kq} + b_{kq}x'_k, \\ n_k(x'_k) &= a_{kn} + b_{kn}x'_k, \end{aligned} \tag{1}$$

where $a_{kq} = \gamma_k A_k \cos\theta_k - \frac{\gamma_k A_k}{g}\omega_{kp}^{y'_k}$, $b_{kq} = -\frac{\gamma_k A_k}{g}\varepsilon_k$, $a_{kn} = -\gamma_k A_k \sin\theta_k - \frac{\gamma_k A_k}{g}\omega_{kp}^{x'_k}$, $b_{kn} = \frac{\gamma_k A_k}{g}\varepsilon_k^2$, θ_k - an angle, which determines the position of the **k**th link comparatively fixed system of coordinates OXY, respectively, ω_k, ε_k - angular velocity and angular acceleration of the **k**th link, respectively, $\omega_{kp}^{y'_k}, \omega_{kp}^{x'_k}$ - components of P_k (pole) point acceleration of the **k**th link put on the axis of link and perpendicular to it, respectively, γ_k - specific weight of material of the **k**th link, A_k - square of cross-section of the **k**th link, g - acceleration of gravity. The obtained expressions show that the distribution of cross and axial inertia forces along the axis of link with constant cross-sections is characterized by trapezoidal law. For the **k**th link, which is under the influence of axial trapezoidal distributed load (see fig. 1), the bending moments along the length of element are distributed by the law of polynomial of third-degree.

$$M_k(x'_k) = a_0 + a_1 x'_k + a_2 (x'_k)^2 + a_3 (x'_k)^3, \tag{2}$$

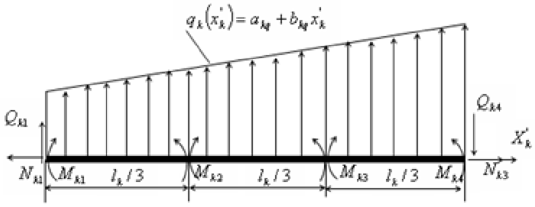

Figure 1 – Axial trapezoidal distributed load acting on the element

Now, let express the bending moments in x'_k cross-section through the sought bending moments $M_{k1}, M_{k2}, M_{k3}, M_{k4}$ in the cross-sections demonstrated in Fig. 1, respectively. For this purpose it is enough to express coefficients a_0, a_1, a_2, a_3 through $M_{k1}, M_{k2}, M_{k3}, M_{k4}$. As a result we have [10]:

$$\begin{aligned} M_k(x'_k) = [1 - \frac{11}{2l_k} x'_k + \frac{9}{l_k^2}(x'_k)^2 - \frac{9}{2l_k^3}]M_{k1} + [\frac{9}{l_k} x'_k - \frac{45}{2l_k^2}(x'_k)^2 + \frac{27}{2l_k^3}(x'_k)^3]M + \\ [-\frac{9}{2l_k} x'_k + \frac{18}{l_k^2}(x'_k)^2 - \frac{27}{2l_k^3}(x'_k)^3]M_{k3} + [\frac{1}{l_k} x'_k - \frac{9}{2l_k^2}(x'_k)^2 + \frac{9}{2l_k^3}(x'_k)^3]M_{k4k2}. \end{aligned} \tag{3}$$

Differentiating $M_k(x'_k)$ to x'_k gives the equation of shear force:

$$\begin{aligned} Q_k(x'_k) = [-\frac{11}{2l_k} + \frac{18}{l_k^2} x'_k - \frac{27}{2l_k^3}(x'_k)^2]M_{k1} + [\frac{9}{l_k} - \frac{45}{l_k^2} x'_k - \frac{81}{2l_k^3}(x'_k)^2]M_{k2} + \\ [-\frac{9}{2l_k} + \frac{36}{l_k^2} x'_k - \frac{81}{2l_k^3}(x'_k)^2]M_{k3} + [\frac{1}{l_k} - \frac{9}{l_k^2} x'_k + \frac{27}{2l_k^3}(x'_k)^2]M_{k4}. \end{aligned} \tag{4}$$

Let the element be affected by the axial trapezoidal distributed load, except the distributed shear force. In that case, the axial force in arbitrary cross-section of an element can be anologously expressed to previous by means of axial forces in calculated cross-sections as follows:

$$N_k(x'_k) = [1 - \frac{3}{l_k} x'_k + \frac{2}{l_k^2}(x'_k)^2]N_{k1} + [\frac{4}{l_k} x'_k - \frac{4}{l_k^2}(x'_k)^2]N_{k2} + [-\frac{1}{l_k} x'_k + \frac{2}{l_k^2}(x'_k)^2]N_{k3}, \tag{5}$$

Thus, for the elemnt which is acted by cross and the axial trapezoidal distributed loads, the approximation matrix connecting interal loads in arbitrary cross-section of the element with values of internal loads in cross-sections has an appearance:

$$[H_k(x'_k)] = \begin{bmatrix} h_{11}(x'_k) & h_{12}(x'_k) & h_{13}(x'_k) & h_{14}(x'_k) & 0 & 0 & 0 \\ h_{21}(x'_k) & h_{22}(x'_k) & h_{23}(x'_k) & h_{24}(x'_k) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & h_{35}(x'_k) & h_{36}(x'_k) & h_{37}(x'_k). \end{bmatrix} \quad (6)$$

Elements of the first line of this matrix can be seen from the equation (3), elements of the second line can be seen from the equation (4), and elements of the third line can be seen from the equation (5), respectively. The given expression of approximation matrix of loads defines dependence between a vector of forces $S_k(x'_k)$ in arbitrary section of an element and a vector of forces S_k in the appointed cross-sections. For an element of rod system the approximation matrix is accurately obtained as it is solved on the basis of known laws of distribution of sought forces. Note, the equations of the bending moment, the cross and axial forces (3,4,5) respectively, which are expressed by the same values in calculated cross-sections, show that for definition of internal loads of each element of the mechanism it is enough to know values of these loads in final number of cross-sections of each of these elements. Number of sections in which it is necessary to know values of internal loads, are defined by polynomial degrees of external actions. Thus, internal loads of each continual link are unambiguously determined by a set of internal loads in its separate cross-sections and by the approximation matrixes, therefore, the task is reduced to calculation of internal forces in final number of cross-sections of elements. Hence, we come to a discrete model of elastic calculation of links of rod mechanisms.

2 Discrete models of elastic calculation of elements and mechanisms in general

For elastic calculation of rod mechanisms based on Dalamber's principle, all inertial, external forces, gravity of links are loaded and the unknown driving moments (forces) are applied, providing the predetermined laws of motion. If the pin that connects drive link with the frame will be replaced by rigid fixing, then the structures with zero degree of freedom are received. For definition of internal loads in links (in elements) of calculated scheme of mechanism, the structure is divided into elements and joints. The link or its part can act as the elements, whereas the joints are the pins connecting links and cross-sections in the middle part, where concentrated external stress is occurred. For definition of internal loads in links (in elements) of designed scheme of mechanism, the structure is divided into elements and joints. As an element can be the link or half link, as the joints – the pin that joins adjacent links and sections loaded by intensive external forces. The process of structure sectioning is made up from giving function and signs for elements' calculated sections. While dividing the elements of calculated scheme of structure into calculated cross-sections and joints, it is necessary to set what internal relations between elements are remained or removed. If we reject any internal relations or their combinations in the element, so the element breaks up to two elements which can turn, move or be removed relatively each other. With the purpose to prevent it, internal forces-loads have to be applied at the joint rejecting places. Thereafter, these loads are regarded as primary unknowns. Let's decompose an element of planar rod mechanisms on three types of beams, for the convenience of composing of resolving equations to determine the internal loads in the appointed cross-sections of elements of the mechanism [10]. The first type is a beam, which both ends are fixed rigidly. Such beams can be the links of complex link, which are connected rigidly among them.

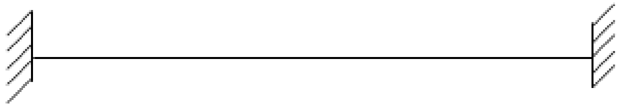

Figure 2 – Beam's both ends are fixed rigidly (first type of a beam)

For determination of coefficients of expressions of the bending moment, it is necessary to know values of the bending moments in four cross-sections, and for determination of coefficients of expressions of axial force, it is necessary to know values in three sections of an element. Therefore, we will choose four sections with unknown bending moments and three sections with unknown axial forces in this beam. Then, by means of conditional schemes with the corresponding unknown, we will construct discrete model of the considered beam as in figure 3.

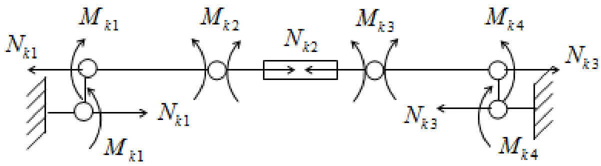

Figure 3 – Discrete model of the first type beam under the action of the distributed trapezoidal load

Then the vector of forces in calculated cross-sections of the beam's discrete model is expressed by the following vector:

$$\{S_k\} = \{M_{k1}, M_{k2}, M_{k3}, M_{k4}, N_{k1}, N_{k2}, N_{k3}\}^T. \tag{7}$$

There is dependence between degree of freedom of discrete model m, number of the attached external loads n and degree of redundancy of calculated scheme k [10]:

$$m = n - k. \tag{8}$$

The matter is that total number of loads of calculated cross-sections is counted easily, and degree of redundancy of calculated scheme is obtained by formula *k=3K-Ш*, where K - number of the closed contours, *Ш* - number of simple (single) joints, k - degree of redundancy of calculated scheme of mechanism. Degree of freedom of discrete model m determines the quantity of necessary independent equations of statics.

For instance, the fourth class mechanism that is shown in fig. 4 can be considered as geometric stable system (see fig. 5), if the rotational kinematic pair of drive link and frame is replaced by rigid fixing.

To define the internal loads in links (in elements) of designed scheme of mechanism, we divide the structure into elements and joints (see fig. 6).

For determination whether the system statically indeterminate we use the formula [9]:

$$k = 3K - , \tag{9}$$

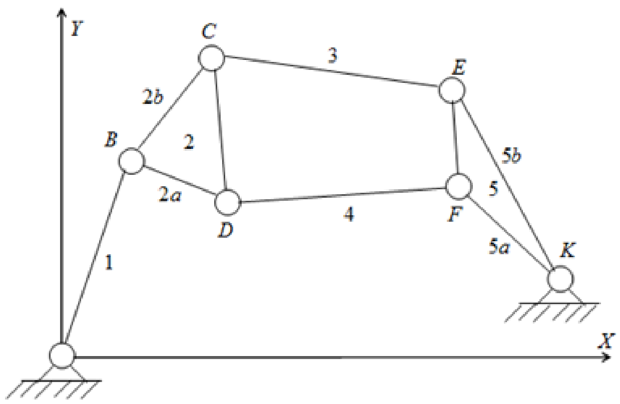

Figure 4 – The fourth class planar mechanism

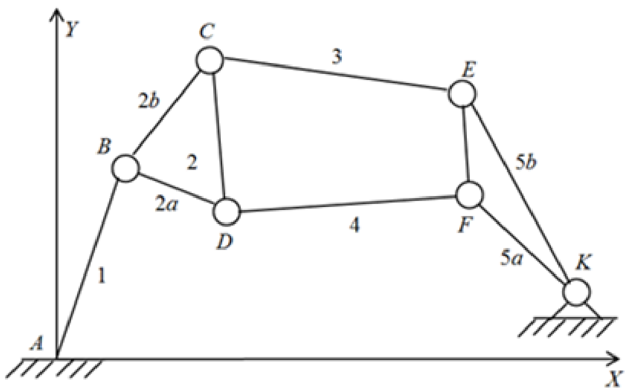

Figure 5 – A designed scheme of the fourth class planar mechanism

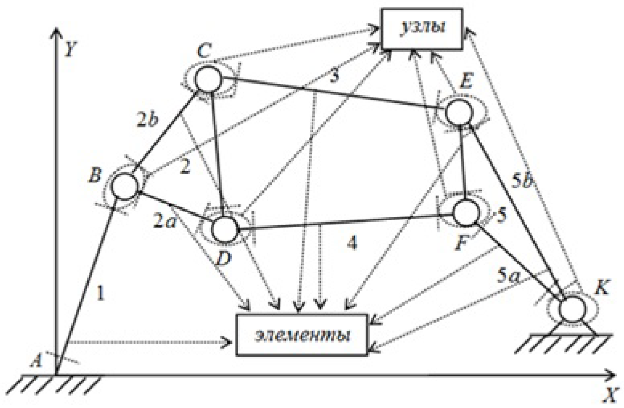

Figure 6 – Decomposing of structure into elements and joints

where *K*- the number of closed contours; *Ш*- the number of simple (single) pins; *k*- the degree of redundancy of designed scheme of mechanism. If the rods of basic links 2 and 5 are rigidly interconnected, the number of single pins will be equal to Ш=6 and k=6, and this system will be six times statically indeterminate. Now, we construct discrete model for calculation on elasticity of the fourth class mechanism (see fig. 4). Let all the rods of mechanism have stable cross-sections, as well as the rods of basic links 2 and 5 are rigidly interconnected, as shown in figure 7.

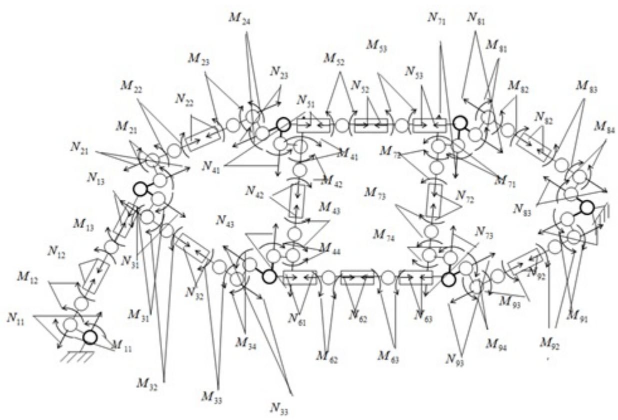

Figure 7 – The discrete model of the fourth class mechanism with stable cross-sections of links and statically indeterminate structure

Thus, we construct the vectors of forces in calculated cross-sections of links for investigated mechanism. So we have:

$\{S_1\} = \{M_1 1, M_1 2, M_1 3, N_1 1, N_1 2, N_1 3\}^T; \{S_2\} = \{M_2 1, M_2 2, M_2 3, M_2 4, N_2 1, N_2 2, N_2 3\}^T;$
$\{S_3\} = \{M_3 1, M_3 2, M_3 3, M_3 4, N_3 1, N_3 2, N_3 3\}^T; \{S_4\} = \{M_4 1, M_4 2, M_4 3, M_4 4, N_4 1, N_4 2, N_4 3\}^T;$
$\{S_5\} = \{M_5 2, M_5 3, N_5 1, N_5 2, N_5 3\}^T; \{S_6\} = \{M_6 2, M_6 3, N_6 1, N_6 2, N_6 3\}^T;$
$\{S_7\} = \{M_7 1, M_7 2, M_7 3, M_7 4, N_7 1, N_7 2, N_7 3\}^T; \{S_8\} = \{M_8 1, M_8 2, M_8 3, M_8 4, N_8 1, N_8 2, N_8 3\}^T;$
$\{S_9\} = \{M_9 1, M_9 2, M_9 3, M_9 4, N_9 1, N_9 2, N_9 3\}^T.$

The vector of forces in calculated cross-sections for discrete model of investigated the fourth class mechanism has the form:

$$\{S\} = \{M_1 1, M_1 2, M_1 3, N_1 1, N_1 2, N_1 3, M_2 1, M_2 2, M_2 3, M_2 4, N_2 1, N_2 2, N_2 3,$$
$$M_3 1, M_3 2, M_3 3, M_3 4, N_3 1, N_3 2, N_3 3, M_4 1, M_4 2, M_4 3, M_4 4, N_4 1, N_4 2, N_4 3,$$
$$M_5 2, M_5 3, N_5 1, N_5 2, N_5 3, M_6 2, M_6 3, N_6 1, N_6 2, N_6 3, M_7 1, M_7 2, M_7 3, M_7 4, N_7 1, N_7 2, N_7 3,$$
$$M_8 1, M_8 2, M_8 3, M_8 4, N_8 1, N_8 2, N_8 3, M_9 1, M_9 2, M_9 3, M_9 4, N_9 1, N_9 2, N_9 3\}^T.$$

3 Dynamic equilibrium equations of discrete models of elements and joints

Let's remove the equations of dynamic equilibrium of an element. From the attached concentrated external loads (Q_{k1}, M_{k1}) and from the cross trapezoidal distributed loads on the axis of element, in arbitrary cross-section of x'_k element, which is expressed through the sought moments in calculated cross-sections, is solved by the equation (3). If the equations (2) and (3) will be differentiated three times on x'_k, then they will be equated and substituted to value b'_k, respectively, then the primary equation of dynamic equilibrium of element will be:

$$-\frac{27}{l_k^3}]M_{k1} + \frac{81}{l_k^3}M_{k2} - \frac{81}{l_k^3}M_{k3} + \frac{27}{l_k^3}M_{k4} = -\frac{y_k A_k}{g}\varepsilon. \quad (10)$$

Relation between the values of the unknown quantities of bending moments in the calculated cross-sections and geometric, physical and kinematic characteristics of **k**th element of mechanism is found. Thus, the second equation is expressed through relation of the sum of moments of all the acting forces on k - element to center of gravity of *k4* cross-section, (relatively to fig. 1):

$$Q_{k1}l_k + a_{kq}fracl_k^2 2 + b_{kq}\frac{l_k^3}{6} + M_{k1} - M_{k4} = 0, \quad (11)$$

where $Q_{k1} = -\frac{11}{2l_k}M_{k1} + \frac{9}{l_k}M_k 2 - \frac{9}{2l_k}M_{k3} + \frac{1}{l_k}M_{k4}$, this equations is easy to get, if the value $x'_k = 0$ is substituted into the equation (4).

Substituting the values Q_{k1} and a_{kq}, b_{kq} into equation (1), and summing the coefficients of same name unknowns, and also substituting the known quantities into the right end of the equation, the second equilibrium equation can be written as:

$$-\frac{9}{2}M_{k1} + 9M_{k2} - \frac{9}{2}M_{k3} = (\gamma_k A_k \cos\theta_k + \frac{\gamma_k A_k}{g}\omega_{k1}^{y'_k})\frac{l_k^2}{2} + \frac{\gamma_k A_k}{g}\varepsilon_k\frac{l_k^3}{6}. \quad (12)$$

From the axial trapezoidal distributed loads acting on the element, as well as from the force N_{k1} of *k1* cross-section, in the x'_k cross-section of element the axial force is occurred, which can be solved by equation:

$$N_k(x'_k) = N_{k1} - a_{kn}x'_k - b_{kn}\frac{(x'_k)^2}{2}. \quad (13)$$

On the other hand, the axial force in the x'_k cross-section of the element, expressed by means of axial forces in the calculated cross-section, has the form (5). Differentiating twice on x'_k the equation (10 and 5), respectively, equating them and substituting the value b_{kn}, the third equation of equilibrium can be expressed as:

$$\frac{4}{l_k^2}N_{k1} - \frac{8}{l_k^2}N_{k2} + \frac{4}{l_k^2}N_{k3} = -\frac{\gamma_k A_k}{g}\omega_k^2. \quad (14)$$

Projecting all forces acting on the **k**th element on the x'_k axis and substituting the values a_{kn}, b_{kn}, the third equation of equilibrium is found. Thus

$$-N_{k1} + N_{k3} = (\gamma_k A_k \sin\theta_k + \frac{\gamma_k A_k}{g}\omega_{k1}^{x'_k})l_k - \frac{\gamma_k A_k}{g}\omega_k^2\frac{l_k^2}{2}. \quad (15)$$

Obtained system of equation, which consists of equations (9), (11), (13) and (21) are assembled in a matrix form as:

$$[A_k]\{S_k\} = \{F_k\}, \tag{16}$$

where

$$[A_k] \begin{bmatrix} -\frac{27}{l_k^3} & \frac{81}{l_k^3} & -\frac{81}{l_k^3} & \frac{27}{l_k^3} & 0 & 0 & 0 \\ -\frac{9}{2} & 9 & -\frac{9}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{4}{l_k^2} & -\frac{8}{l_k^2} & \frac{4}{l_k^2} \\ 0 & 0 & 0 & 0 & -1 & 0 & 1, \end{bmatrix}$$

$$\{S_k\} = \{M_{k1}, M_{k2}, M_{k3}, M_{k4}, N_{k1}, N_{k2}, N_{k3}\}^T, \tag{17}$$

$$\{F_k\} = \{b_{kq}, -a_{kq}\frac{l_k^2}{2} - b_{kq}\frac{l_k^3}{6}, -b_{kn}, -a_{kn}l_k - b_{kn}\frac{l_k^2}{2}\}^T. \tag{18}$$

Let the two elements j and k of mechanism form a rotational kinematic pair, i.e. permit rotational motion relative to each other. Also let the elements have constant cross-sections along its length. We cut out a kinematic pair from mechanism with elements' surrounding cross-sections, constituting this pair. Then, in the cross-sections of the elements adjacent to the joint (to the kinematic pair) there are internal loads, as shown in figure 8. There are two equilibrium conditions for these joints:

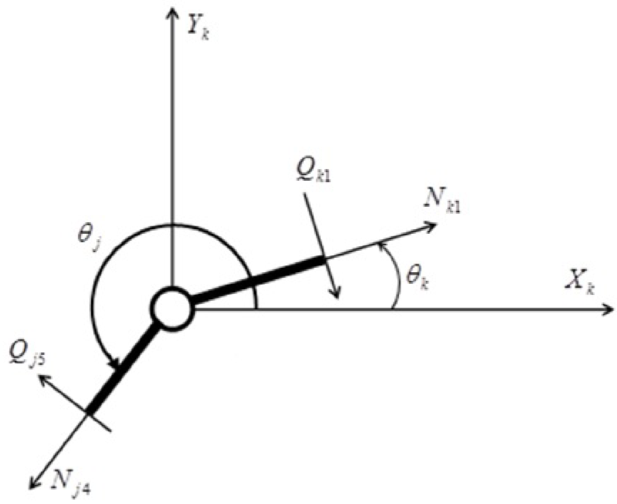

Figure 8 – The pin joint of mechanism with constant cross-sections of elements

The equilibrium equation for considered joint can be described as:

$$\begin{cases} N_{k_1}\cos\theta_k + Q_{k1}\sin\theta_k + N_{j3}\cos\theta_j + Q_{j4}\sin\theta_j = 0; \\ N_{k_1}\sin\theta_k - Q_{k1}\cos\theta_k + N_{j3}\sin\theta_j - Q_{j4}\cos\theta_j = 0. \end{cases}$$

Further, the values will be expressed by means of sought moments in the calculated cross-sections of discrete model of the element, for this purpose we use the equation (4), substituting here the values $x'_k = 0$ and $x'_j = l'_j$, respectively, hence:

$$\begin{aligned} Q_{k1} &= -\frac{11}{2l_k}M_{k1} + \frac{9}{l_k}M_{k2} - \frac{9}{2l_k}M_{k3} + \frac{1}{l_k}M_{k4};\\ Q_{j4} &= -\frac{1}{l_j}M_{j1} + \frac{9}{2l_j}M_{j2} - \frac{9}{l_j}M_{j3} + \frac{11}{2l_j}M_{j4}. \end{aligned} \qquad (19)$$

Now, substituting the values Q_k1 and Q_j4 in the equation (17), the following equilibrium equations for the joint have an appearance:

$$\begin{cases} -\frac{11\sin\theta_k}{2l_k}M_{k1} + \frac{9\sin\theta_k}{l_k}M_{k2} - \frac{9\sin\theta_k}{2l_k}M_{k3} + \frac{\sin\theta_k}{l_k}M_{k4} + \cos\theta_k N_{k1} - \\ -\frac{\sin\theta_j}{l_j}M_{j1} + \frac{9\sin\theta_j}{2l_j}M_{j2} - \frac{9\sin\theta_j}{l_j}M_{j3} + \frac{11\sin\theta_j}{2l_j}M_{j4} + \cos\theta_j N_{j3} = 0; \\ \frac{11\cos\theta_k}{2l_k}M_{k1} + \frac{9\cos\theta_k}{l_k}M_{k2} + \frac{9\cos\theta_k}{2l_k}M_{k3} - \frac{\cos\theta_k}{l_k}M_{k4} + \sin\theta_k N_{k1} + \\ +\frac{\cos\theta_j}{l_j}M_{j1} - \frac{9\cos\theta_j}{2l_j}M_{j2} + \frac{9\cos\theta_j}{l_j}M_{j3} - \frac{11\cos\theta_j}{2l_j}M_{j4} + \sin\theta_j N_{j3} = 0. \end{cases}$$

The cross-sections of links can also be rigid joints, the concentrated external loads are attached here. For example, the concentrated force $P_k x'_k, P_k y'_k$ and the concentrated moment M_k are occurred in the G cross-section of **k**th link (see fig. 9). Then the **k**-link is divided into two elements, for **k**th and **i**th. If the cross-sections of elements are constant along the length of the link, then by means of cutting the **G**-joint out of mechanism, the scheme of **G**-joint with adjacent internal loads in cross-sections is displayed below. For this joint it is possible to write three equilibrium equations that are expressed through sought parameters of elements.

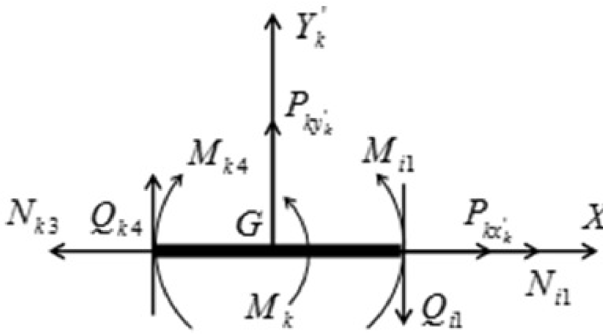

Figure 9 – Rigid joints of a link with a constant cross-sections of elements, where the external concentrated forces are applied

4 The matrix of compliance of the element with constant cross-sections under the distributed cross and axial trapezoidal loads

The approximation matrices allow defining the physical characteristics of the element, i.g. the matrix of compliance of discrete element $[D_k]$ [9]. The physical characteristics of the element can be obtained from the expressions of energy for rods, so as the replacement of construction by a set of discrete elements based on the equality of the energies of the real structures and its discrete model. The matrix of compliance of the element is defined by equation [9]:

$$[D_k] = \int_{l_k} [H_k(x'_k)]^T [D_{kx'_k}][H_k(x'_k)]dx'_k. \qquad (20)$$

where $[D_k]$ is a matrix of compliance of link cross-section. For the rods working on bending and stretching-shrunking the equation (20) is used:

$$[D_{kx'_k}] = \begin{bmatrix} \frac{1}{E_kI_k} & 0 & 0 \\ 0 & \frac{\mu}{G_kA_k} & 0 \\ 0 & 0 & \frac{1}{E_kA_k}. \end{bmatrix}$$

where $\frac{1}{E_kI_k}$ - bending compliance of the cross-section; $\frac{\mu}{G_kA_k}$ - displacement compliance of the cross-section; $\frac{1}{E_kA_k}$ - stretching-shrunking compliance of the cross-section, $[H_k(x'_k)]$ is defined by equation (6). Therefore, the overall view of matix of compliance taking into account the bending, stretching-shrunking and the actions of the distributed cross and axial trapezoidal loads has an appearance:

$$[D_k] = \begin{bmatrix} \frac{8l_k}{105E_kI_k} & \frac{33l_k}{560E_kI_k} & -\frac{3l_k}{140E_kI_k} & \frac{19l_k}{1680E_kI_k} & 0 & 0 & 0 \\ \frac{33l_k}{560E_kI_k} & \frac{27l_k}{70E_kI_k} & -\frac{27l_k}{560E_kI_k} & -\frac{3l_k}{140E_kI_k} & 0 & 0 & 0 \\ -\frac{3l_k}{140E_kI_k} & -\frac{27l_k}{560E_kI_k} & \frac{27l_k}{70E_kI_k} & \frac{33l_k}{560E_kI_k} & 0 & 0 & 0 \\ \frac{19l_k}{1680E_kI_k} & -\frac{3l_k}{140E_kI_k} & \frac{33l_k}{560E_kI_k} & \frac{8l_k}{105E_kI_k} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2l_k}{15E_kI_k} & \frac{l_k}{15E_kI_k} & -\frac{l_k}{30E_kI_k} \\ 0 & 0 & 0 & 0 & \frac{2l_k}{15E_kI_k} & \frac{l_k}{15E_kI_k} & -\frac{l_k}{30E_kI_k} \\ 0 & 0 & 0 & 0 & -\frac{l_k}{30E_kI_k} & \frac{l_k}{15E_kI_k} & \frac{2l_k}{15E_kI_k}. \end{bmatrix}$$

5 Resolving equations of determination of internal forces

By combining the equilibrium equations of elements and joints into a single system, the equilibrium equations of the discrete model of entire mechanism is obtained. They can be written in general form:

$$[A]\{S\} = \{F\}. \qquad (21)$$

Such systems of equations are sufficient to determine the internal forces in the links of the mechanism, which frame includes a statically definable group of Assur. The matrix of equilibrium equations for the discrete model of mechanisms consists of matrices of equilibrium equations of their individual elements, as well as the equilibrium equations of their joints. The matrix of dynamic equilibrium equations of discrete models of mechanisms is as follows:

$$[A] = \begin{bmatrix} [A_1] & 0 \cdots & 0 & \\ 0 & [A_2] & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & [A_n] \\ Uravn.ravnovesiyauzlov. & & & \end{bmatrix}$$

For discrete models of mechanisms the vector of force and the vector of loads in calculated cross-sections are formed by vector of forces and loads in calculated cross-sections of their separate elements. These vectors have the following vector form, respectively:

$$\{F\} = \{\{F_1\}, \{F_2\}, \cdots, \{F_n\}\}^T; \{S\} = \{\{S_1\}, \{S_2\}, \cdots, \{S_n\}\}^T. \quad (22)$$

For determination of internal forces of statically indeterminate mechanisms (in the way of determination of internal forces), it is necessary to build a compliance matrix for the entire discrete model of the rod mechanism. The matrix of compliance $[D]$ for the entire discrete model of the rod mechanism consists of the matrixes of individual elements $[D_k]$ [10]. It is expressed in block-diagonal form:

$$\begin{bmatrix} [D_1] & 0 \cdots & 0 & \\ 0 & [D_2] & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & [D_n]. \end{bmatrix}$$

where n is a number of the elements of discrete model of mechanism. A formula below is used to determine the components of the vector of loads {S}[10]:

$$\{S\} = [K][A]^T([A][K][A]^T)^{-1}\{F\}, \quad (23)$$

$[K] = [D]^{-1}$ is taken into consideration.

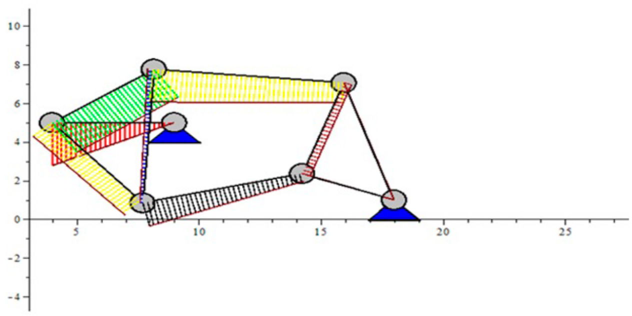

Figure 10 – The diagram of axial distributed inertial forces, originated from link weight of investigated mechanism

Now, for determination of internal loads in links, we give an example of four-bar second class statically indeterminate mechanism with single drive link as shown in figure 4. The computer programs for determination and construction of the inertia forces and internal

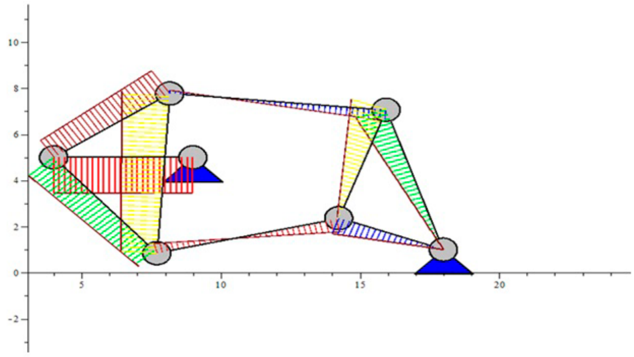

Figure 11 – The diagram of cross distributed inertial forces, originated from link weight of investigated mechanism

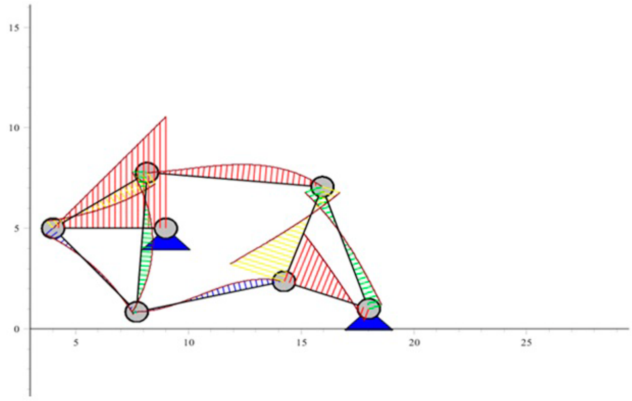

Figure 13 – The diagram of axial forces, originated from distributed inertial forces acting on links of mechanism with statically indeterminate structure

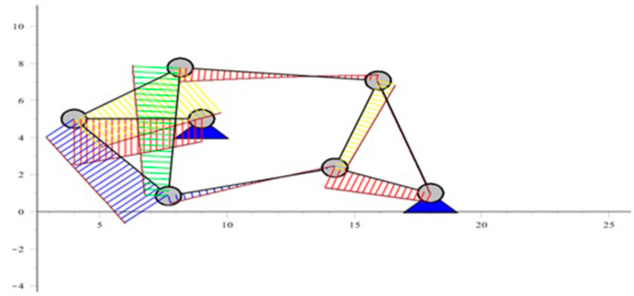

Figure 14 – The diagram of cross forces, originated from distributed inertial forces acting on links of mechanism with statically indeterminate structure

loads on the links by means of using the MAPLE18 system are made. Therefore, the results of obtained inertia forces and internal loads for some positions of the mechanism are shown in figures 10-13.

Conclusion

The technique of analytical determination of internal forces in links of planar mechanisms and manipulators with statically indeterminate structures taking into account the distributed dynamical loads, a dead weight and the operating external loads is designed. The programs in the MAPLE18 system are made on the given algorithm and animations of the motion of mechanisms with construction on links the intensity of the distributed cross and axial inertial loads, the bending moments, the cross and axial forces, depending on kinematic characteristics of links are obtained. The developed technique can be applied in the study of stress-strain state of the projected and existing mobile and fixed beam systems with statically definable structures (planar link mechanisms, manipulators, frames, etc.).

Acknowledgements

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