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Modelling of drill string nonlinear dynamics with a drilling fluid flow

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In this work the development of mathematical models of drill string dynamics under the influence of internal drilling fluid flow, which is of great importance for ensuring fast, cost-effective and safe process of oil and gas well drilling, is conducted. It is a reason for in-depth research of the drilling fluid influence on the motion of drill strings. The modelling is complicated by consideration of geometric nonlinearity, rotation of the drill string and the effect of external force factors. Relations of Novozhilov's nonlinear theory of elasticity form the model framework. Ostrogradsky-Hamilton's variation principle is applied for derivation of the drill string governing equations. It is assumed that the drilling fluid flow moves at constant speed. The obtained nonlinear mathematical models, describing lateral vibrations of the drill string as a compressed rod, generalize well-known linear models taking into account the effect of the drilling fluid. These models will enable to solve a wide range of problems concerning the drilling equipment nonlinear dynamics on a qualitatively new and mathematically valid level.
Key words: drill string, mathematical model, geometric nonlinearity, drilling fluid, lateral vibrations.

Жуатын сұйықтық ағыны есепке алумен бұрғылау бағанасы сызықты емес динамикасын моделдеу

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Осы жұмыста жуатын сұйықтық ішкі ағынының әсерінен бұрғылау бағанасы динамикасының математикалық моделін әзірлеу жүргізіледі. Жуатын сұйықтық тиімді, үнемді және қауіпсіз мұнай және газ ұңғымаларын бұрғылау үдерісін қамтамасыз ету үшін маңызды рөл атқарады. Бұл бұрғылау бағаналарының қозғалысына бұрғылау ерітіндісінің ықпалына байланысты терең зерттеулер жүргізуді қажет етеді. Моделдеу геометриялық сызықты еместікті ескере отыруы, бұрғылау бағанасының айналуы және сыртқы күш факторларының әсерінен қиындатылады. В.В. Новожиловтың сызықты емес серпімділік теориясы математикалық моделдің негізі құрастырады. Бұрғылау бағанасының қозғалыс теңдеулерін шығару үшін Остроградский-Гамильтон вариациялық принципі қолданылады. Жуатын сұйықтық ағыны тұрақты жылдамдықпен қозғалады деп ұйғарылады. Сығылған сырықтың ретінде бұрғылау бағанасының көлденең тербелістер сипаттайтын алынған сызықты емес теңдеулер бұрғылау ерітіндісі әрекетін ескере отырып, бағанасы көлденең тербелістердің белгілі сызықтық моделдерін жинақтап қорытады. Олар сапалы жаңа және математикалық негізделген деңгейде бұрғылау жабдықтары сызықты емес динамикасы есептердің кең классы шешуге мүмкіндік береді.
Түйін сөздер: бұрғылау бағанасы, математикалық модель, геометриялық сызықты еместік, көлденең тербелістер.

Моделирование нелинейной динамики бурильной колонны с учетом потока промывочной жидкости

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В данной работе проводится разработка математической модели динамики бурильной колонны под влиянием внутреннего потока промывочной жидкости, который играет большую роль в обеспечении эффективного, экономичного и безопасного процесса бурения нефтяных и газовых скважин. Это обуславливает необходимость проведения глубоких исследований по изучению влияния бурового раствора на движение бурильных колонн. Моделирование осложняется учетом геометрической нелинейности, вращением бурильной колонны и действием внешних силовых факторов. Основу математической модели составляют соотношения нелинейной теории упругости В.В. Новожилова. Для вывода уравнений движения бурильной колонны применяется вариационный принцип Остроградского-Гамильтона. Предполагается, что поток промывочной жидкости движется с постоянной скоростью. Полученные нелинейные уравнения, описывающие поперечные колебания бурильной колонны как сжатого стержня, обобщают известные в литературе линейные модели поперечных колебаний бурильных колонн с учетом действия бурового раствора. Они позволят решать широкий класс задач нелинейной динамики бурового оборудования на качественно новом и математически обоснованном уровне.

Ключевые слова: бурильная колонна, математическая модель, геометрическая нелинейность, промывочная жидкость, поперечные колебания.

1 Introduction

In the process of drilling of oil and gas wells by the use of rotary technique, a fluid flow, which is reffered to as drilling mud or drilling fluid, is constantly circulating in the borehole. At present, it is considered not only as a means for removing the products of destruction (sludge), but also as one of the main factors ensuring the effectiveness of the entire drilling process.

When drilling, the balance of the rocks that form the walls of the wells is disturbed. The stability of the walls in turn depends on the strength characteristics of the rocks, their changes over time under the influence of various factors. In conditions of violation of the integrity of the rock, the rock pressure plays a substantial role. In the near-well part of the well, it occurs in two directions: in the vertical and horizontal one. The lateral pressure is a consequence of the vertical pressure and causes tangential stresses, the magnitude of which also depends on the pressure of the drilling fluid. They lead to buckling of rocks, narrowing of the wellbore and the occurrence of collapses.

In drilling, the rock pressure always exceeds the hydrostatic pressure of the drilling fluid and results in the destruction of the walls of the well if the rock strength is insufficient or it is weakened by the action of the drilling fluid. The most significant deformation of the rock directly at the walls of the well, where there is no balancing of the lateral pressure by the hydrostatic one and the forces of cohesion of the rock. The main negative effect of the drilling fluid on the strength of rocks is manifested in the physico-chemical change in their structure under the effect of the filtrate. The process is also enhanced by the mechanical action of the drill string on the walls of the wells.

When the drilling mud interacts with the rock, the following types of disturbances in the integrity of the well walls are identified: collapses (screes); swelling; plastic flow (creep);

chemical dissolution; erosion (Oil and Gas Mining. URL: http://oilloot.ru/78-tekhnika-i-tekhnologii-stroitelstva-skvazhin/167-burovye-promyvochnye-zhidkosti).

For this reason, studying the composition of drilling fluids, assessing their impact on rocks and the entire drilling process are a relevant problem and require more in-depth studies to prevent mentioned unwanted effects.

2 Literature review

A number of studies have been devoted to the problem of drilling of oil and gas wells taking into account the effect of drilling fluid. It is well-known about the works of (Gulyayev et al., 2009: 141-142), (Gulyayev et al., 2011: 760-761), which provide a derivation of linear mathematical models of the drill string lateral vibrations, taking into account the internal flow of the drilling fluid as an added mass. In the work (Ritto et al., 2009), the influence of drilling fluid on the drill string dynamics was studied using direct well flushing when the fluid moves down inside the drill string and then is directed upward in the annulus of the well. The authors used the Timoshenko beam model and the finite element method for system discretization. The impact of the internal and backward external fluid flows on the vibrations of the flexible cantilever pipe, which can be regarded as a drill string, has also been studied (Moditis et al., 2016: 120-138). It was found that the external flow of fluid could cause a loss of stability of the divergence type system. The stability problems of the drill string motion are studied in the work (Liu et al., 2015), where the linear model of coupled longitudinal and transverse vibrations of the "Bottom-hole assembly - upper drill pipe"system was investigated, and (Kudaibergenov et al., 2017: 93-99), in which the study of resonant phenomena and dynamic stability of drill columns was complicated by taking into account geometric nonlinearity. The results of the research showed that the geometric nonlinearity in the models made significant corrections to the results of the dynamic analysis of the drill string stability. The effect of mud properties, flow velocity and angular speed of rotation on the nonlinear behavior of drill strings in an unconventional horizontal well was studied by (Wilson et al., 2016). It was shown that the correct modelling of the fluid forces acting on the drill string allows explaining the process of safe operation of BHAs within resonance frequencies.

As it is known, gas or compressed air is also widely applied as the circulating medium in the rotary method of well drilling. This technology, called gas drilling, allows in a number of cases to increase the rate of drilling and opening of oil bearing horizons, ensure the integrity of rocks (Lian et al., 2015: 1412), reduce drilling costs and increase the life of bits (Meng et al., 2015: 163). However, unlike drilling with fluid, the gas drilling leads to much more serious vibrations of the drill string due to the low damping effect of the gas as drilling mud (Li et al., 2007). At the same time it was found that the correct choice of gas parameters not only preserves the stability of the drilling system, but also enables to reduce the amplitude of lateral vinrations of the drill string (Khajiyeva et al., 2018: 573-578). In addition, it was established that, assuming the smallness of the drill string deformations in the absence of the gas flow and in its presence, instability of the solution for the considered value of the axial load was observed. The refinement of the model by removing restrictions on the magnitude of the deformations leaded to stabilization of the solution, as well as in the case of nonlinearity taking into account the gas flow. This confirmes the fact that nonlinear models give a more accurate description of the studied processes, whereas linear models in most cases result in a rough approximation.

The aim of this paper is modelling of the nonlinear dynamics of a drill string under the action of an internal incompressible fluid flow. The focus is on the lateral vibrations of the drill string, because they can exhibit their destructive effects leading to the destruction of the borehole walls and their distortion without appearing on the surface (Saldivar et al., 2014). The obtained model generalizes the model of (Liang et al., 2018: 65-69) to the case of geometric nonlinearity and allowance for the effect of external forces.

3 Material and methods

3.1 Elastic potentials

For modelling of the drill string nonlinear dynamics allowing for the influence of the drilling fluid flow two right-handed coordinate systems are introduced: a global system $OX_1X_2X_3$ related to the drill string lateral displacements and the local one $Ox_1x_2x_3$ to include the drill string rotation. The drill string is considered in the form of an isotropic elastic rod of length l. The rod is rotated along the x_3-axis with the angular speed Ω and is under the effect of a compressing force $N(x_3, t)$ and a torque $M(x_3, t)$. The direction of the X_3 - and x_3-axes coincide with that of the rod axis.

Let $\tilde{U}_0$ be the elastic energy per unit volume of the rod. The condition according to that the elastic energy $\tilde{U}_0$ represents the stress potential is written as follows (Rabotnov, 1988):

$$\sigma_{ij} = \frac{\partial \tilde{U}_0}{\partial \varepsilon_{ij}}, \quad i, j = 1, 2, 3. \tag{1}$$

Applying the Legendre transformation, i.e. assuming that $\Phi = \sigma_{ij}\varepsilon_{ij} - \tilde{U}_0 = \Phi(\sigma_{ij})$, we have

$$\varepsilon_{ij} = \frac{\partial \Phi}{\partial \sigma_{ij}}, \quad i, j = 1, 2, 3. \tag{2}$$

From the mechanical point, the potential $\tilde{U}_0$ is the potential energy of elastic strain accumulated in the rod, whereas the quantity Φ, referred to as the strain potential, is the additional work. As the linear elastic material is considered in the paper, the potential Φ numerically equals $\tilde{U}_0$ and hence we can write

$$\sigma_{ij} = \frac{d\Phi(\varepsilon_{ij})}{d\varepsilon_{ij}}, \quad \varepsilon_{ij} = \frac{d\Phi(\sigma_{ij})}{d\sigma_{ij}}, \quad i, j = 1, 2, 3. \tag{3}$$

The strain potential can be also rewritten in the form:

$$\Phi = \frac{1}{2}\Phi\sigma_{ij}\varepsilon_{ij}. \tag{4}$$

The equations of generalized Hooke's law for a linear elastic homogeneous isotropic solid body in terms of Lame parameters are given as

$$\sigma_{ij} = 2G\varepsilon_{ij} + \lambda\delta_{ij}\varepsilon_{kk}, \quad i, j, k = 1, 2, 3, \tag{5}$$

where $G = \dfrac{E}{2(1+\lambda)}$ is the shear modulus, $\lambda = \dfrac{E\nu}{(1+\nu)(1-2\nu)}$, E Young's modulus, ν Poisson's ratio, $\delta_{ij} = \begin{cases} 1, & i = j, \\ 0, & i \neq j \end{cases}$ the Kronecker delta.

Substituting stress components (5) into expression for the strain potential (4), we have

$$\Phi = \left(1 + \frac{\lambda}{2}\right)(\varepsilon_{11}^2 + \varepsilon_{22}^2 + \varepsilon_{33}^2) + \lambda(\varepsilon_{11}\varepsilon_{22} + \varepsilon_{22}\varepsilon_{33} + \varepsilon_{11}\varepsilon_{33}) + 2G(\varepsilon_{12}^2 + \varepsilon_{23}^2 + \varepsilon_{13}^2). \quad (6)$$

3.2 General relations underlying the model

The research performed in (Khajiyeva et al., 2018: 573-578) showed the benefit of using nonlinear models instead of linear ones that mostly give a rough approximation of studied processes. Therefore, the modelling of the drill string dynamics under the effect of the drilling fluid flow will be carried out with taking into account geometric nonlinearity.

According to the second system of simplifications of Novozhilov's nonlinear elasticity theory (Novozhilov, 1999), the strain components are given as follows:

$$\varepsilon_{ii} = e_{ii} + \frac{1}{2}\left(\omega_j^2 + \omega_k^2\right), \quad \varepsilon_{ij} = e_{ij} - \omega_i\omega_j, \quad i, j, k = 1, 2, 3, \quad (7)$$

where

$$e_{ii} = \frac{\partial U_i}{\partial x_i}, \quad e_{ij} = \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}, \quad \omega_i = \frac{(-1)^i}{2}\frac{\partial U_j}{\partial x_k} - \frac{\partial U_k}{\partial x_j},$$
$$i, j, k = 1, 2, 3, \ (i \neq j \neq k, j < k). \quad (8)$$

Here e_{ii} mean the relative elongations, e_{ii} shears, and ω_i the rotation angles of the rod fibre.

Acceptance of the displacement hypothesis for the case of spatial lateral vibrations (more general form can be found in (Filippov, 1970) and (Bobrovnitskij et al., 1983: 428)) yields

$$\begin{cases} U_1(x_1, x_2, x_3, t) = u_1(x_3, t), \\ U_2(x_1, x_2, x_3, t) = u_2(x_3, t), \\ U_3(x_1, x_2, x_3, t) = -\dfrac{\partial u_1(x_3, t)}{\partial x_3} - \dfrac{\partial u_1(x_3, t)}{\partial x_3}. \end{cases} \quad (9)$$

Then the position vector of an arbitrary point $P(x_3)$ of the rod is defined as

$$\vec{r}(x_3, t) = u_1(x_3, t)\vec{i} + u_2(x_3, t)\vec{j} - \left(\frac{\partial u_1(x_3, t)}{\partial x_3}x_1 + \frac{\partial u_2(x_3, t)}{\partial x_3}x_2\right)\vec{k}, \quad (10)$$

wherein the unit vectors $\vec{i} = \vec{i}(t), \vec{j} = \vec{j}(t), \vec{k} = \vec{k}$.

Given that $\dfrac{\partial \vec{i}}{dt} = \omega\vec{j}, \dfrac{\partial \vec{j}}{dt} = -\omega\vec{i}, \dfrac{\partial \vec{k}}{dt} = 0$, from (10), we can find the velocity of the rod:

$$\vec{v} = \frac{\partial \vec{r}}{\partial t} = (\frac{\partial u_1}{\partial t} - \Omega u_2)\vec{i} + (\frac{\partial u_2}{\partial t} + \Omega u_1)\vec{j} - (\frac{\partial^2 u_1}{\partial x_3 \partial t}x_1 + \frac{\partial^2 u_2}{\partial x_3 \partial t}x_2)\vec{k}. \quad (11)$$

The velocity of the drilling fluid, according to (Liang et al., 2018: 67), is expressed as

$$\vec{v_f} = \vec{v} + V_f \vec{\tau}, \tag{12}$$

where

$$\vec{\tau} = \frac{\partial \vec{r}}{\partial x_3} = \frac{\partial u_1}{\partial x_3}\vec{i} + \frac{\partial u_2}{\partial x_3}\vec{j} - (\frac{\partial^2 u_1}{\partial x_3^2}x_1 + \frac{\partial^2 u_2}{\partial x_3^2}x_2)\vec{k}. \tag{13}$$

Here $\vec{\tau}$ is the direction vector tangential to the rod axis, V_f the fluid speed.

On substituting the displacement projections over the corresponding coordinate axes (9) into the strain components (8), we can find new relations for (7), which, using (6), give the expression of the elastic potential Φ for the rod spatial lateral vibrations. After that, introducing the following inertial characteristics

$$S_{x_1} = \int\limits_A x_2 dA,\ S_{x_2} = \int\limits_A x_1 dA,\ I_{x_1} = \int\limits_A x_2^2 dA,\ I_{x_1 x_2} = \int\limits_A x_1 x_2 dA,\ I_{x_2} = \int\limits_A x_1^2 dA, \tag{14}$$

one can determine the expression for the potential energy of the rod deformation.

3.3 Ostrogradsky-Hamilton's variation method

For derivation of the governing equations of the drill string dynamics with the drilling fluid flow, Ostrogradsky-Hamilton's variation principle is utilized:

$$\delta = \int\limits_{t_1}^{t_2} (T - U_0 + \Pi)\, dt = 0, \tag{15}$$

where T is the kinetic energy of the system, U_0 the potential energy, Π the potential of external forces, δ the variation, t_1, t_2 arbitrary time moments.

The potential energy of the rod can be found from the Clapeyron formula:

$$U_0 = \iiint\limits_{V_0} \Phi dV_0 = \int\limits_0^l \int\limits_A \Phi dA dx_3,, \tag{16}$$

where V_0 is the rod volume, A the cross-section area of the rod. Finally, we have

$$\begin{aligned} U_0 = \left(1 + \frac{\lambda}{2}\right) \int\limits_0^l \left[\frac{A}{2} \left(\left(\frac{\partial u_1}{\partial x_3}\right)^4 + \left(\frac{\partial u_2}{\partial x_3}\right)^4 + \left(\frac{\partial u_1}{\partial x_3}\right)^2 \left(\frac{\partial u_2}{\partial x_3}\right)^2 \right) + \right. \\ \left. + I_{x_2} \frac{\partial^2 u_1}{\partial x_3^2} + I_{x_1} \frac{\partial^2 u_2}{\partial x_3^2} \right] dx_3 + \frac{\lambda A}{4} \int\limits_0^l \left[\left(\left(\frac{\partial u_1}{\partial x_3}\right)^4 + \left(\frac{\partial u_2}{\partial x_3}\right)^4 + \right.\right. \\ \left.\left. +3 \left(\frac{\partial u_1}{\partial x_3}\right)^2 \left(\frac{\partial u_2}{\partial x_3}\right)^2 \right) \right] dx_3 + 2GA \int\limits_0^l \left(\frac{\partial u_1}{\partial x_3}\right)^2 \left(\frac{\partial u_2}{\partial x_3}\right)^2 dx_3. \end{aligned} \tag{17}$$

It is worth noting that the x_3-axis coincides with the rod neutral line, and thereby the static moments $S_{x_1} = S_{x_2} = 0$. Moreover, due to the fact that the x_1- and x_2-axes are the symmetry axes of the rod, the centrifugal moment is also zero, i.e. $I_{x_1 x_2} = 0$.

Kinetic energy of the system is given as the sum of those of the rod motion T_0 and the drilling fluid motion T_f (Liang et al., 2018: 68):

$$T = T_0 + T_f, \tag{18}$$

where

$$T_0 = \frac{1}{2}\rho \int\limits_0^l \int\limits_A (\vec{v}, \vec{v})\, dA dx_3, \tag{19}$$

$$T_f = \frac{1}{2}\rho_f \int\limits_0^l \int\limits_{A_f} (\vec{v_f}, \vec{v_f})\, dA_f dx_3. \tag{20}$$

Here ρ_f the flow density, A_f the internal cross-section area of the drill string.

Substitution of expressions for the rod and drilling fluid velocities (11)-(12) into (18)-(19) yields

$$\begin{aligned} T_0 = \frac{1}{2}\rho \int\limits_0^l \Bigg[& A\left(\left(\frac{\partial u_1}{\partial t}\right)^2 + \left(\frac{\partial u_2}{\partial t}\right)^2 + 2\Omega\left(u_1\frac{\partial u_2}{\partial t} - u_2\frac{\partial u_1}{\partial t}\right) + \Omega^2\left(u_1^2 + u_2^2\right)\right) + \\ & + I_{x_2}\left(\frac{\partial^2 u_1}{\partial x_3 \partial t}\right)^2 + I_{x_1}\left(\frac{\partial^2 u_2}{\partial x_3 \partial t}\right)^2\Bigg] dx_3, \end{aligned} \tag{21}$$

$$\begin{aligned} T_0 = \frac{1}{2}\rho_f \int\limits_0^l \Bigg[& A_f\left(\left(\frac{\partial u_1}{\partial t}\right)^2 + \left(\frac{\partial u_2}{\partial t}\right)^2 + 2\Omega\left(u_1\frac{\partial u_2}{\partial t} - u_2\frac{\partial u_1}{\partial t}\right) + \Omega^2\left(u_1^2 + u_2^2\right)\right) + \\ & + 2V_f\left(\frac{\partial u_1}{\partial x_3}\left(\frac{\partial u_1}{\partial t} - \Omega u_2\right) + \frac{\partial u_2}{\partial x_3}\left(\frac{\partial u_2}{\partial t} - \Omega u_1\right)\right) + V_f^2\left(\left(\frac{\partial u_1}{\partial x_3}\right)^2 + \left(\frac{\partial u_2}{\partial x_3}\right)^2\right) + \\ & + I_{x_2}\left(\frac{\partial^2 u_1}{\partial x_3 \partial t}\right)^2 + 2\frac{\partial^2 u_1}{\partial x_3 \partial t}\frac{\partial^2 u_1}{\partial x_3^2} + \left(\frac{\partial^2 u_1}{\partial x_3^2}\right)^2 + \\ & I_{x_1}\left(\frac{\partial^2 u_2}{\partial x_3 \partial t}\right)^2 + 2\frac{\partial^2 u_2}{\partial x_3 \partial t}\frac{\partial^2 u_2}{\partial x_3^2} + \left(\frac{\partial^2 u_2}{\partial x_3^2}\right)^2\Bigg] dx_3, \end{aligned} \tag{22}$$

In the process of drilling the drill string is subject to the action of the axial load $N(x_3, t)$, resulting in the rod compression in the axial direction, and the torque $M(x_3, t)$, causing the torsion deformation. In order to take into account their influence and the gravitation energy of the drill string and the fluid flow pressure in the model, the potential of external forces Π, consisting of the sum of three external potentials, is introduced:

$$\Pi = \Pi_l + \Pi_t + \Pi_g, \tag{23}$$

where Π_l is the potential allowing for the effect of the axial load, Π_t brings the action of the torque, and Π_g contains the gravitation energy of the system. Their expressions read as follows:

$$\Pi_l = -\frac{1}{2}\int\limits_0^l N(x_3, t)\left[\left(\frac{\partial u_1}{\partial x_3}\right)^2 + \left(\frac{\partial u_2}{\partial x_3}\right)^2 + I_{x_2}\left(\frac{\partial u_1}{\partial x_3}\right)^2 + I_{x_1}\left(\frac{\partial u_2}{\partial x_3}\right)^2\right] dx_3, \tag{24}$$

$$\Pi_t = \frac{1}{2}\int\limits_0^l M(x_3,t)\left(\frac{\partial^2 u_1}{\partial x_3^2}\frac{\partial u_2}{\partial x_3} + \frac{\partial^2 u_2}{\partial x_3^2}\frac{\partial u_1}{\partial x_3}\right)dx_3, \tag{25}$$

$$\Pi_g = \frac{1}{2}\left(\rho A + \rho_f A_f\right) g \int\limits_0^l (l - x_3)\left[\left(\frac{\partial u_1}{\partial x_3}\right)^2 + \left(\frac{\partial u_2}{\partial x_3}\right)^2\right]dx_3. \tag{26}$$

4 Results and discussion

The nonlinear mathematical model of the drill string lateral vibrations, taking into account the influence of the drilling fluid flow and the external loadings, is finally obtained by substituting (21)-(23) into (13) with turn to Young's modulus and using the integration by parts technique while calculating the variation of the energies and the potential of external forces. It is worth noting that the axial load $N(x_3,t)$ and the torque $M(x_3,t)$ are assumed to be distributed along the rod length. Thus, we have the following mathematical model:

$$\begin{aligned}
&EI_{x_2}\frac{\partial^4 u_1}{\partial x_3^4} - \rho I_{x_2}\frac{\partial^4 u_1}{\partial x_3^2 \partial t^2} + \frac{\partial^2}{\partial x_3^2}\left(M(x_3,t)\frac{\partial u_2}{\partial x_3}\right) + \frac{\partial}{\partial x_3}\left(N(x_3,t)\frac{\partial u_1}{\partial x_3}\right) - \\
&-\frac{EA}{1-\nu}\frac{\partial}{\partial x_3}\left(\frac{\partial u_1}{\partial x_3}\right)^3 - \frac{EA(5-6\nu)}{2(1-\nu)}\frac{\partial}{\partial x_3}\left(\frac{\partial u_1}{\partial x_3}\left(\frac{\partial u_2}{\partial x_3}\right)^2\right) + \\
&+\left(\rho A + \rho_f A_f\right)\left(\frac{\partial^2 u_1}{\partial t^2} - 2\Omega\frac{\partial u_2}{\partial t} - \Omega^2 u_1\right) - \rho_f\left(\frac{\partial^4 u_1}{\partial x_3^4} + 2\frac{\partial^4 u_1}{\partial x_3^3 \partial t} + \frac{\partial^4 u_1}{\partial x_3^2 \partial t^2}\right) + \\
&+\rho_f A_f\left(V_f^2\frac{\partial^2 u_1}{\partial x_3^2} + 2V_f\frac{\partial^2 u_1}{\partial x_3 \partial t} - 2V_f\Omega\frac{\partial u_2}{\partial x_3}\right) - \\
&-\left(\rho A + \rho_f A_f\right) g\left(\frac{\partial u_1}{\partial x_3} - (l - x_3)\left(\frac{\partial u_1}{\partial x_3}\right)^2\right) = 0, \\
&EI_{x_1}\frac{\partial^4 u_2}{\partial x_3^4} - \rho I_{x_1}\frac{\partial^4 u_2}{\partial x_3^2 \partial t^2} - \frac{\partial^2}{\partial x_3^2}\left(M(x_3,t)\frac{\partial u_1}{\partial x_3}\right) + \frac{\partial}{\partial x_3}\left(N(x_3,t)\frac{\partial u_2}{\partial x_3}\right) - \\
&-\frac{EA}{1-\nu}\frac{\partial}{\partial x_3}\left(\frac{\partial u_2}{\partial x_3}\right)^3 - \frac{EA(5-6\nu)}{2(1-\nu)}\frac{\partial}{\partial x_3}\left(\frac{\partial u_2}{\partial x_3}\left(\frac{\partial u_1}{\partial x_3}\right)^2\right) + \\
&+\left(\rho A + \rho_f A_f\right)\left(\frac{\partial^2 u_2}{\partial t^2} + 2\Omega\frac{\partial u_1}{\partial t} - \Omega^2 u_2\right) - \rho_f\left(\frac{\partial^4 u_2}{\partial x_3^4} + 2\frac{\partial^4 u_2}{\partial x_3^3 \partial t} + \frac{\partial^4 u_2}{\partial x_3^2 \partial t^2}\right) + \\
&+\rho_f A_f\left(V_f^2\frac{\partial^2 u_2}{\partial x_3^2} + 2V_f\frac{\partial^2 u_2}{\partial x_3 \partial t} + 2V_f\Omega\frac{\partial u_1}{\partial x_3}\right) - \\
&-\left(\rho A + \rho_f A_f\right) g\left(\frac{\partial u_2}{\partial x_3} - (l - x_3)\left(\frac{\partial u_2}{\partial x_3}\right)^2\right) = 0,
\end{aligned} \tag{27}$$

Each of equations of the mathematical model (27) includes terms containing the geometric nonlinearity and the effects of the axial load and the torque. Neglecting these terms, we arrive at the model derived in (Liang et al., 2018: 68-69). Retaining the influence of the external forces and at the same time neglecting the geometric nonlinearity and not considering the energy of gravitation, the model of (Gulyayev et al., 2011: 760-761) can be obtained. As a result, the derived nonlinear mathematical model of the drill string dynamics is more complete compared to the mentioned linear ones and is more adapted to the real conditions of oil and gas well drilling with the use of the drilling fluid.

5 Conclusion

A nonlinear dynamic model of drill string lateral vibrations taking into account the effect of the drilling fluid pressure was developed. Lifting restrictions on magnitudes of the drill string displacements, allowing for the influence of the drilling fluid on the drill string dynamics, and representation of the drill string deformation as spatial that of a rod result in the refinement and generalization of the model. It is verified by the comparison with known linear models of the drill string lateral vibrations.

Presented in this work methodology of development of the drill string motion mathematical model will enable to solve problems of the drilling equipment nonlinear dynamics at qualitatively new and mathematically valid level, and can be applied for dynamic modelling of various rod elements. To further our research, we intend to carry out the numerical analysis of the derived model and generalize it for the case of spatial drill string vibrations of different types.

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