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## On dynamic stability of drill strings in a supersonic gas flow

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In this work stability of the drill string nonlinear dynamics, complicated by the effect of an external axial load, initial curvature of the drill string, geometric nonlinearity and the influence of a supersonic gas flow as a circulating medium is studied. The drill string is modelled as a rotating elastic isotropic rod with constant cross-section. Pressure of the gas flow used to clean the borehole from drill cuttings and to transport them from the bottom to the surface is determined by the nonlinear dependences of the piston theory in the third approximation. Utilization of the Galerkin method allows to reduce the drill string mathematical model to an ordinary differential equation for the generalized time function, containing an asymmetric nonlinear characteristic, which is further eliminated by introducing the corresponding substitution. Considering a small perturbation to the system and applying the harmonic balance method, characteristic determinants are constructed. Equations describing boundaries of instability zones of basic resonance, which allow to determine the range of dangerous frequency regimes and to increase safety of the drilling process, are obtained.

**Key words:** drill string, stability, nonlinearity, gas flow.

### Дыбыстан газ ағынында бұрғылау бағанасының динамикалық тұрақтылығы туралы

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Осы жұмыста білікке түсетін сыртқы жүк-салмақтың әрекеті, бұрғылау бағанасының бастапқы қисықтығы, геометриялық сызықты еместік және айналыстағы орта түрінде дыбыстан газ ағынының әсерін ескере отырып бұрғылау бағанасы сызықты емес динамикасының тұрақтылығын зерттеледі. Бұрғылау бағанасы тұрақты көлденең қимасымен айналымы серпімді изотроптық білік түрінде қаралады. Ұңғыманы тазарту және бұрғылау шламын кенжарден жер бетіне тасымалдау үшін қолданылатын газ ағынының қысымы поршенді теорияның үшінші жуықтауда сызықты емес тәуелділіктерінен анықталады. Галеркин әдісін пайдалану уақыттық функцияны жай дифференциалдық теңдеуге ауысуға мүмкіндік береді. Тиісті ауыстыру енгізу жолымен бұл теңдеудің симметриялы емес сызықты емес сипаттамасын шығарылады. Шағын ұйтқу анықталай және гармоникалық баланс әдісі қолданылай, сипаттамалық анықтауыштар құрылады. Қауіпті жиіліктік режим ауқымы анықтауға және бұрғылау үдерісінің қауіпсіздігін көтеруге мүмкіндік беретін негізгі резонанс динамикалық орнықсыздығы аймақтарының шектерін теңдеулер табылады.

**Түйін сөздер:** бұрғылау бағанасы, тұрақтылық, сызықты еместік, газ ағыны.

**О динамической устойчивости бурильных колонн в сверхзвуковом потоке газа**  
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В работе изучается устойчивость нелинейной динамики бурильной колонны, осложненной действием внешней осевой нагрузки, начальной кривизной колонны, геометрической нелинейностью и влиянием сверхзвукового потока газа как циркулирующей среды. Бурильная колонна моделируется в виде вращающегося упругого изотропного стержня постоянного поперечного сечения. Давление потока газа, который применяется для очистки скважины и переноса бурового шлама с забоя на поверхность, определяется нелинейными зависимостями поршневой теории в третьем приближении. Использование метода Галеркина позволяет перейти к обыкновенному дифференциальному уравнению относительно обобщенной временной функции, содержащему несимметричную нелинейную характеристику, которую удастся исключить введением соответствующей замены. Задавая системе малое возмущение и применяя метод гармонического баланса, строятся характеристические определители, дающие уравнения границ зон динамической неустойчивости основного резонанса, которые позволят определить диапазон опасных частотных режимов и повысить безопасность процесса бурения скважин.

**Ключевые слова:** бурильная колонна, устойчивость, нелинейность, поток газа.

## 1 Introduction

Active development of oil and gas fields today makes the oil and gas extracting industry one of the largest and dynamically developing industries in the economy of modern Kazakhstan. Speed of driving and fail-safety of works when drilling oil and gas wells depend significantly on quality and modernization of drilling equipment, technological imperfections of drill strings, elaboration of the drilling modes for different types of soil rocks, influences of complicating external loads and factors of the environment, etc.

High-amplitude flexural vibrations, axial displacements and torsional stick-slip motions of drill strings caused by their difficult dynamic behaviour during the drilling process can result in serious technical failures of the drilling equipment (Nandakumar, 2013: 1), (Jansen, 1993). Therefore, there arises a problem of carrying out the dynamic analysis of the drill string stability, which is important for the increase in efficiency of drilling operations, protection of expensive components of the drilling equipment against undesirable vibrations and prevention of collapse of borehole walls while drilling.

Moreover, the drill string dynamics is highly nonlinear by its nature, and it is necessary to consider a system of nonlinear differential equations for its investigation (Al-Hiddabi, 2003: 1-2). It is caused generally by flexibility of drill strings due to their large length and the impact of the axial compressing load that can result in finite deformations of the drill strings (geometric nonlinearity).

This paper aims at dynamic stability of the drill string, modelled in the form of a rotating elastic rod, taking into account a supersonic gas flow circulating from the drill string outer side. A mathematical model studied in (Kudaibergenov, 2017) is complicated, in addition to the variable compressing load, initial curvature of the drill string and nonlinearity of the model, by the nonlinear influence of the gas flow in the third approximation that brings the dynamic analysis of stability of the drill string vibrations closer to real conditions of drilling.

## 2 Literature review

Research of quasistatic stability of a rotating drill string, carried out in (Gulyaev, 2006: 692-697), enabled to find critical rotary speeds of drill strings. The obtained buckling mode shapes, in turn, could be useful to determine points for installing centralizers to avoid contact

with borehole. In (Aarsnes, 2017: 2-3) importance of using the distributed models to analyze stability of the linearized axial-torsional dynamics of drill strings with subsequent determination of the normalized inverse of the system gain margin is shown. Coupled axial-torsional vibrations of a drill string taking into account state-dependent time delay and nonlinearity from dry friction and loss of contact are studied in (Liu, 2014: 1-8). The carried-out stability analysis established that self-excited oscillations of the drill string owing to the delay time effects could arise with high probability.

Amongst axial, lateral and torsional modes of vibration, the lateral mode is said to cause about 75% of drilling failures (Ghasemloonia, 2012: 948). In (Sahebkar, 2011: 743-759) the authors examine a nonlinear model of drill string lateral vibrations, obtained with application of Hamilton's principle, and employ the method of multiple scales to define the steady-state response and instability regions. Research of lateral dynamic behaviour of the drill string in a horizontal borehole taking into account continuous contact with borehole wells revealed that studied vibrations have snaking or whirling nature (Heisig, 2000). Simultaneous appearance of parametric resonance and whirl vibration phenomena, resulting in high-amplitude lateral vibrations of a rotating drill string, within the range of drilling operating conditions is shown in (Christoforou, 1997: 256-259).

Successful application of the Galerkin method to the analysis of drill string dynamics, particularly, to their dynamic stability in vertical holes was indicated in (Vaz, 1995: 437-440). Utilization of the Galerkin technique along with the harmonic balance and pseudo arc-length continuation methods to investigate steady-state frequency responses and stability branches of beam-like structures with polynomial-type nonlinearities were studied in (Bhattiprolu, 2016: 28-37). Such a combination of techniques allowed to quickly determine the harmonics needed for convergence of a periodic solution of the system.

At present gas drilling technology involving air, nitrogen or natural gas as circulating media is widely applied in oil and gas industry (Lian, 2015: 1412). In (Meng, 2014: 163-170) a simple and reliable theoretical model describing the propagation and attenuation of a pressure wave through pipes was created. It was established that static pressure had more influence on the attenuation factor than on wave speed.

At the same time, a large number of works are related to the nonlinear dynamics of cylindrical structures in a supersonic gas flow. Influences of the critical Mach number on the flutter emergence and limit cycle oscillations, existence of bifurcation and chaotic motions of a composite laminated plate under transversal aerodynamic pressure modelled by the first-order piston theory is given in (Chen, 2015: 1-8). The use of the third-order piston theory and shock wave aerodynamics is described in (Librescu, 2002: 802-810), whereas in (Kiiko, 2009: 135) the authors derive an expression for the excess pressure, essentially different from the piston theory formulae, and obtain purely investigated eigenvalue problems.

### 3 Materials and methods

#### 3.1 Mathematical model

Let us consider a nonlinear mathematical model of the drill string lateral vibrations (Kudaibergenov, 2017), based on Novozhilov's nonlinear elasticity theory (Novozhilov, 1999) taking into consideration the plane section hypothesis and obtained with application of Hamil-

ton's variation principle. The Cartesian coordinate system  $Oxyz$  ( $z$ -axis coincides with the drill string axis) is used. Vibrations of the drill string, presented as a rotating elastic isotropic rod with symmetric cross-section, in the  $Oyz$ -plane are studied.

In order to take into account the effect of a supersonic incompressible gas flow on the drill string dynamics relations of the piston theory (Volmir, 1967), connecting the aerodynamic pressure of the flow with speed of sound, are applied. According to them, we have the following:

$$P = P_0 \left( 1 - \frac{\kappa - 1}{2} \frac{U_n}{C_0} \right)^{\frac{2\kappa}{\kappa - 1}}, \quad (1)$$

where  $U_n$  is the normal projection of the gas flow speed on the drill string surface;  $C_0$  the sound speed for the unperturbed gas flow;  $P_0$  the pressure of the unperturbed flow;  $\kappa$  the polytropic exponent.

The gas is supposed to move in the upward direction, i.e. in the opposite direction to the drill string motion. Considering the stationary gas flow, when  $U_n = V_g \frac{\partial v}{\partial z}$  (Kudaibergenov, 2018: 570), and expanding (1) into a power series up to third order, we arrive at the following expression for the excessive pressure of the supersonic gas flow:

$$\Delta P = P - P_0 = P_0 \kappa \left( -\bar{M} \frac{\partial v}{\partial z} + \frac{\kappa + 1}{4} \bar{M}^2 \left( \frac{\partial v}{\partial z} \right)^2 - \frac{\kappa + 1}{12} \bar{M}^3 \left( \frac{\partial v}{\partial z} \right)^3 \right), \quad (2)$$

The use of nonlinear expression (2), containing the cubic power of  $\bar{M} \frac{\partial v}{\partial z}$  allows to get sufficiently exact values of pressure. At the same time, the process of perturbation distribution in gas can be considered as isentropic (Volmir, 1967).

Hence, the nonlinear mathematical model of plane lateral vibrations of the rotating drill string allowing for the nonlinear effect of the supersonic gas flow is written as follows

$$\begin{aligned} \rho A \frac{\partial^2 v}{\partial t^2} + EI_x \frac{\partial^4 v}{\partial z^4} - \rho I_x \frac{\partial^4 v}{\partial z^2 \partial t^2} + \frac{\partial}{\partial z} \left( N(z, t) \frac{\partial (v + v_0)}{\partial z} \right) - \frac{EA}{1 - \nu} \frac{\partial}{\partial z} \left( \frac{\partial v}{\partial z} \right)^3 \\ - \rho A \omega^2 v + h \Delta P = 0, \end{aligned} \quad (3)$$

where  $\rho$  is the mass density,  $A$  the cross-section area of the rod,  $v(z, t)$  the displacement of the flexural center of the cross-section along the  $y$ -axis owing to bending,  $E$  Young's modulus,  $I_x$  the axial inertia moment,  $v_0(z)$  the initial curvature of the rod,  $\nu$  Poisson's ratio,  $\omega$  the angular speed of rotation of the drill string,  $h$  the drill string wall thickness.

Boundary conditions for the simply supported rod are given by

$$v(z, t) = 0, \quad EI_x \frac{\partial^2 v(z, t)}{\partial z^2} \quad (z = 0, z = l). \quad (4)$$

The external loads having generally variable in time nature significantly influence the drill string dynamic stability. Accepting that the harmonic effect corresponds to the loading regime, the variable axial compressive load can be presented as

$$N = N_0 + N_t \cos \tilde{\Omega} t, \quad (5)$$

where  $N_0$  and  $N_t$  are the constant and variable in time components, respectively;  $\tilde{\Omega}$  is the frequency of the external effect.

When investigating natural and resonant oscillations of construction elements with nonlinear and elastic characteristics a sought solution is often approximated by a finite number of normal modes with subsequent reduction of an initial partial differential equation of motion to modal equations of motion, applying the Galerkin method. Studying oscillations of continuous systems with large amplitude such as rods, beams and shells the single-mode method which assumes independence of the modal form of oscillations from the influence of nonlinearity is effectively used (Szemplinska-Stupnicka, 1983).

Using the Galerkin method, consider the general form of the drill string nonlinear vibrations. In the given case, the lateral displacement  $v(z, t)$  and initial curvature  $v_0(z)$  can be approximated by periodic functions of the form:

$$v(z, t) = f(t) \sin\left(\frac{\pi z}{l}\right), \quad v_0(z) = f_0(t) \sin\left(\frac{\pi z}{l}\right). \quad (6)$$

On substituting expression (6) into governing equation (3), after integration under meeting the requirement of orthogonality of the substitution result to the basis function  $\sin\left(\frac{\pi z}{l}\right)$  and introducing the dimensionless time  $\tau = \Omega_0 t$ , the problem reduces to the nonlinear ordinary differential equation for the generalized function  $f(\tau)$ :

$$\frac{d^2 f}{d\tau^2} + (1 - 2\beta \cos \Omega\tau) f + \alpha_1 f^2 + \alpha_2 f^3 = F_0 + F_1 \cos \Omega\tau, \quad (7)$$

where

$$\begin{aligned} \beta &= \frac{\beta_2}{\beta_1}, \quad \Omega = \frac{\tilde{\Omega}}{\Omega_0}, \quad \alpha_i = \frac{\tilde{\alpha}_i}{\Omega_0^2}, \quad F_i = \frac{\tilde{F}_i}{\Omega_0^2}, \quad i = 1, 2; \\ \beta_1 &= \frac{1}{2\delta_1} \left( EJ \left(\frac{\pi}{l}\right)^4 - N_0 \left(\frac{\pi}{l}\right)^2 - \rho A \omega^2 \right), \quad \beta_2 = \frac{N_t \pi^2}{4l\delta_1}, \quad \Omega_0 = \sqrt{\frac{\beta_1}{\delta_1}}, \\ \delta_1 &= \frac{\rho l}{2} \left( A + J \left(\frac{\pi}{l}\right)^2 \right), \quad \tilde{\alpha}_1 = \frac{\bar{M}^2 P_0 \kappa (\kappa + 1) \pi h}{6l\delta_1}, \quad \tilde{\alpha}_2 = \frac{3EA\pi^4}{8(1-\nu)l^3\delta_1}, \\ \tilde{F}_0 &= f_0 \frac{N_0 \pi^2}{2l\delta_1}, \quad \tilde{F}_1 = f_0 \frac{N_t \pi^2}{2l\delta_1}. \end{aligned}$$

Here  $\Omega_0$  is the frequency of the drill string natural vibrations allowing for the constant component of the axial load,  $\beta$  the excitation coefficient.

It is worth noting that the influence of the supersonic gas flow is taken into account in the term with quadratic nonlinearity, in contrast to the authors' works (Kudaibergenov, 2014: 594), (Kudaibergenov, 2016: 496) where the flow of gas appeared in the term with cubic nonlinearity. Cubic term in equation (7) arises due to nonlinearity of the mathematical model of the drill string.

Moreover, the nonlinear characteristic in equation (7) is asymmetric because of existence of the quadratic term  $\alpha_1 f^2$ . Making the substitution  $f = \hat{f} - \frac{\alpha_1}{3\alpha_2}$  (Hayashi, 1986) we obtain the equation with symmetric nonlinear characteristic:

$$\frac{d^2 \hat{f}}{d\tau^2} + (\gamma - 2\beta \cos \Omega\tau) \hat{f} + \alpha \hat{f}^3 = \hat{F}_0 + \hat{F}_1 \cos \Omega\tau, \quad (8)$$

where

$$\gamma = 1 - \frac{\alpha_1^2}{3\alpha_2}, \quad \alpha = \alpha_2, \quad \hat{F}_0 = F_0 + \frac{\alpha}{3\alpha_2} - \frac{2\alpha_1^3}{27\alpha_2^2}, \quad \hat{F}_1 = F_1 - \frac{2\alpha_1\beta}{3\alpha_2}.$$

### 3.2 Stability analysis

For stability investigation let us suppose that  $\hat{f}_0$  is the periodic solution of equation (7), and introduce a small variation  $\delta f$ :

$$\hat{f} = \hat{f}_0 + \delta f. \quad (9)$$

If the variation  $\delta f$  rises indefinitely at  $\tau \rightarrow \infty$ , then the solution  $\hat{f}_0(\tau)$  is unstable. In the case when  $\delta f$  remains limited at  $\tau \rightarrow \infty$ , the solution  $\hat{f}_0(\tau)$  is stable.

On substituting (9) into equation (8), eliminating  $\hat{f}_0$ , and neglecting the powers of  $\delta f$  higher than one, the following linearized equation in terms of the variation  $\delta f$  is obtained:

$$\frac{d^2\delta f}{d\tau^2} + \left( \gamma - 2\beta \cos \Omega\tau + 3\alpha\hat{f}_0^2 \right) \delta f = 0. \quad (10)$$

Considering the case of basic resonance for the periodic solution  $\hat{f}_0(\tau)$ :

$$\hat{f}_0(\tau) = r_1 \cos(\Omega\tau - \phi_1), \quad (11)$$

where  $r_1$  is the amplitude,  $\phi_1$  the phase of the periodic solution; we arrive at the generalized Hill type equation in variations:

$$\frac{d^2\delta f}{d\tau^2} + \left( \theta_0 + \sum_{n=1}^2 (\theta_{nc} \cos n\Omega\tau + \theta_{ns} \sin n\Omega\tau) \right) \delta f = 0, \quad (12)$$

where

$$\theta_0 = \gamma + \frac{3}{2}\alpha r_1^2, \quad \theta_{1c} = -2\beta, \quad \theta_{1s} = 0, \quad \theta_{2c} = \frac{3}{2}\alpha r_1^2 \cos 2\phi_1, \quad \theta_{2s} = \frac{3}{2}\alpha r_1^2 \sin 2\phi_1.$$

In order to determine zones of instability, the particular solution of equation (12) is given by a spectrum of vibrations (Szemplinska-Stupnicka, 1983: 19):

$$\delta f = e^{\eta\tau} \sum_k \cos(k\Omega\tau - \psi_k), \quad k = 1, 3, 5, \dots, \infty \quad \text{or} \quad k = 0, 2, 4, \dots, \infty, \quad (13)$$

where  $\eta$  is the characteristic exponent that can take a real or imaginary value.

Hence, the behaviour of the quantity  $\delta f$  depends on the behaviour of the function  $e^{\eta\tau}$ , which, in turn, depends on the behaviour of the characteristic exponent  $\eta$ . Thus, if

1) the characteristic exponent has a negative real part, i.e.  $Re(\eta) < 0$ , then the solution  $\delta f \rightarrow 0$  at  $\tau \rightarrow \infty$ , that means the solution is stable;

2)  $Re(\eta) = 0$ , we get the solution on the boundary of stability and instability zones;

3)  $Re(\eta) > 0$ , then the solution  $\delta f$  is unstable since it increases indefinitely at  $\tau \rightarrow \infty$ .

The method of harmonic balance according to which coefficients at the corresponding cosines and sine are equated to zero is applied to find the zones of dynamic instability. At the same time, the order of the corresponding characteristic determinants depends on the number of harmonics retained in solution (13).

### 4 Results and discussion

Transition to the symmetric nonlinear characteristic in equation (8) allows to restrict ourselves by finding the instability zones of the odd order, which involve the terms corresponding to the odd harmonics in expression (13).

Assuming that the frequency of the small perturbation  $\delta f$  coincides with frequency of the periodic solution  $\tilde{f}_0$ , the first instability zone is determined. Then the solution of equation (12) can be written as

$$\delta f = e^{\eta\tau} b_1 \cos(\Omega\tau - \psi_1). \tag{14}$$

On substituting solution (14) into the equation of perturbed state (12) and applying the method of harmonic balance, we get the following characteristic determinant:

$$\Delta(\eta) = \begin{vmatrix} \eta^2 - \Omega^2 + \theta_0 + \frac{\theta_{2c}}{2} & 2\eta\Omega + \frac{\theta_{2s}}{2} \\ -2\eta\Omega + \frac{\theta_{2s}}{2} & \eta^2 - \Omega^2 + \theta_0 - \frac{\theta_{2c}}{2} \end{vmatrix}, \tag{15}$$

that provided  $\Delta(\eta = 0) = 0$ , defines the boundaries of the first instability zone of the basic resonance:

$$(A_0 - \Omega^2)^2 + (B_0 - B_1\Omega^2) r_1^2 + C_0 r_1^4 = 0, \tag{16}$$

where

$$A_0 = \gamma, \quad B_0 = 3\alpha\gamma, \quad B_1 = -3\alpha, \quad C_1 = \frac{27}{16}\alpha^2.$$

Existence of cubic nonlinearity in the model supposes finding the instability zones of higher order. When determining the third zone of instability of the basic resonance the expression for the small variation  $\delta f$  is given by

$$\delta f = e^{\eta\tau} (b_1 \cos(\Omega\tau - \psi_1) + b_3 \cos(3\Omega\tau - \psi_3)). \tag{17}$$

Similarly, on substituting (17) into (12), we apply the method of harmonic balance and take into consideration that the results of substitution have to be satisfied at any nontrivial values of  $b_k, \psi_k, k = 1, 3$ . Assuming that  $b_1 = b_3 = 1$ , we come to the fourth-order characteristic determinant of the form:

$$\Delta(\eta) = \begin{vmatrix} \eta^2 - \Omega^2 + \theta_0 + \frac{\theta_{2c}}{2} & 2\eta\Omega + \frac{\theta_{2s}}{2} & \frac{\theta_{2c}}{2} & \frac{\theta_{2s}}{2} \\ -2\eta\Omega + \frac{\theta_{2s}}{2} & \eta^2 - \Omega^2 + \theta_0 - \frac{\theta_{2c}}{2} & -\frac{\theta_{2s}}{2} & \frac{\theta_{2c}}{2} \\ \frac{\theta_{2c}}{2} & -\frac{\theta_{2s}}{2} & \eta^2 - 9\Omega^2 + \theta_0 & 6\eta\Omega \\ \frac{\theta_{2s}}{2} & \frac{\theta_{2c}}{2} & -6\eta\Omega & \eta^2 - 9\Omega^2 + \theta_0 \end{vmatrix}. \tag{18}$$

Then the boundaries of the third zone of instability can be described by the following equation, satisfying the condition  $\Delta(\eta = 0) = 0$ :

$$(A_0 - A_1\Omega^2 + A_2\Omega^4)^2 + (B_0 + B_1\Omega^2 + B_2\Omega^4 + B_3\Omega^6) r_1^2 + (C_0 + C_1\Omega^2 + C_2\Omega^4) r_1^4 + (D_0 + D_1\Omega^2) r_1^6 + E_0 r_1^8 = 0, \tag{19}$$

where

$$\begin{aligned} A_0 &= \gamma^2, & A_1 &= -10\gamma, & A_2 &= 9, \\ B_0 &= 6\alpha\gamma^3, & B_1 &= -90\alpha\gamma^2, & B_2 &= 354\alpha\gamma, & B_3 &= -270\alpha, \\ C_0 &= \frac{189}{16}\alpha^2\gamma^2, & C_1 &= -\frac{909}{8}\alpha^2\gamma, & C_2 &= \frac{3357}{16}\alpha^2, \\ D_0 &= \frac{135}{16}\alpha^3\gamma, & D_1 &= -\frac{567}{16}\alpha^3, & E_0 &= \frac{405}{256}\alpha^4. \end{aligned}$$

The third zone of instability allows to refine the solution of the problem and to determine the frequencies at which ultra-harmonic resonant oscillations resulting in loss of the drill string stability can arise.

## 5 Conclusion

The dynamic model of the drill string flat bending, developed by the authors, refines the known models and brings them closer to a real physical process. It becomes possible due to introducing to the model of the drill string vibrations their nonlinearity of geometric nature (finiteness of deformations, initial curvature of the drill string), and nonlinearity from pressure of the supersonic gas flow.

In this work, the methodology of the dynamic stability analysis of the systems without imposing limits on the magnitudes of their nonlinearity and nonautonomous terms is proposed. It is based on determining the instability zones of resonant vibrations and finding the corresponding characteristic determinants with application of the harmonic balance method. Completeness of the mathematical model allows to predict the drill string behaviour with high accuracy, eliminating dangerous resonant frequencies from the range of their operating regimes, thereby increasing efficiency and reliability of the drill string operation. Despite the fact that the proposed techniques were used to research stability of the system basic resonance, they can be successfully applied to the analysis of resonances on higher frequencies as well.

In future, the results of the work will be generalized to the case of spatial vibrations of the drill string with conducting their detailed numerical analysis.

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