Complex Software for Numerical Simulation of Convective Flow of Viscous Incompressible Fluid in a Curvilinear Coordinate System

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Abstract. In this paper, the development of a complex software for numerical simulation of convective flow of viscous incompressible fluid in a doubly connected areas in a curvilinear coordinate system is considered. For the discretization of the physical domain, the technology of construction of curvilinear structure difference grids using the transfinite interpolation method, the equidistribution method, and the method of Thompson are presented. In order to test the software, the calculations for various configurations of the cavity and temperature conditions at the border are conducted.

Keywords: incompressible fluid, curvilinear coordinate system, convective flow, numerical simulation.

1 Introduction

In recent years, it is often required to solve problems in complex areas with complex geometry. For modeling in complex areas, in the first place it is required to discretize the physical domain, that is, to conduct the step of modeling the physical geometry using a set of cells grids. It should be noted that the use of nonuniform grids can cause the appearance of non-physical sources of mass and momentum of impulse, as well as may be accompanied by the loss of important properties inherent approximated differential equations. Equation models recorded in curvilinear coordinates are more complicated than the original equations, in particular, they contain variable coefficients, additional terms, non-zero right-hand sides, etc. Therefore, the question of approximating equations on curvilinear grids is urgent and requires close attention. In addition, the diverse requirements imposed on the difference grid make curvilinear grid a complex mathematical problem. In this regard, the development of theoretical concepts and methodological approaches to the use of new information technologies in the hydrodynamic studies that takes into account the specific features of the subject area, development, adaptation of tools and testing them in the process of modeling the natural and man-made objects that are important for the national economy, are very relevant. In this paper, the development of a complex of software for numerical simulation of convective flow of viscous incompressible fluid in a doubly connected areas in a curvilinear coordinate system is considered. Discretization of the physical domain is represented by the technology of creating curvilinear difference structure grids using the methods of transfinite interpolation, equidistribution, and the method of Godunov-Thompson [1-3]. For modeling the convective flow, an incompressible fluid equation is used in the vorticity ω, stream function ψ and temperature θ under appropriate initial and boundary conditions [3] in curvilinear coordinate systems.

2 Statement of the problem and the computational algorithm

General transformations of the viscous incompressible fluid equation

In the construction of finite-difference schemes, it is convenient to write the equations of fluid dynamics in a compact vector form. For example, the Navier-Stokes equations for an incompressible fluid in a Cartesian coordinate system can be written as follows:

$$\frac{\partial U}{\partial t}+\frac{\partial E}{\partial x}+\frac{\partial F}{\partial y}=\frac{\partial E_v}{\partial x}+\frac{\partial F_v}{\partial y}+\overrightarrow{f} \tag{1}$$

Here, the vectors $U, E, F, E_v, F_v, \overrightarrow{f}$ are defined as follows:

$$U[0,\ u,\ v,\ \theta],\ \ E[u,\ u^2+\pi,\ uv,\ u\theta],\ \ F[v,\ uv,\ v^2+\pi,\ v\theta], \tag{2}$$

$$E_v=\mu_x\left[0,\ \frac{\partial u}{\partial x},\ \frac{\partial v}{\partial x},\ \frac{\partial \theta}{\partial x}\right]F_v=\mu_y\left[0,\ \frac{\partial u}{\partial y},\ \frac{\partial v}{\partial y},\ \frac{\partial \theta}{\partial y}\right]. \tag{3}$$

Boundary conditions:
$u=0,\ v=0,\ \theta=\theta^0;\ t=0,$
$u(0,y,t)=u(X,y,t),\ v(0,y,t)=v(X,y,t),\ \theta(0,y,t)=\theta(X,y,t);\ x=0,\ 0\le y\le Y,$
$u(X,y,t)=u(0,y,t),\ v(X,y,t)=v(0,y,t),\ \theta(X,y,t)=\theta(0,y,t);\ x=X,\ 0\le y\le Y,$

$u=0,\ v=0,\ -\frac{x_\eta}{J}\frac{\partial\theta}{\partial\xi}+\frac{x_\xi}{J}\frac{\partial\theta}{\partial\eta}=0\ (\theta=\varphi_1);\ y=0,\ 0\le x\le X,$

$u=0,\ v=0,\ \theta=\varphi_2\left(-\frac{x_\eta}{J}\frac{\partial\theta}{\partial\xi}+\frac{x_\xi}{J}\frac{\partial\theta}{\partial\eta}=0\right);\ y=Y,\ 0\le x\le X.$

Let us consider the coordinate transformation of the general form, which represents the (x, y) physical plane into the (ξ, η) computing plane and providing opportunity to solve problems in a uniform grid

$$\xi=\xi(x,y), \eta=\eta(x,y) \tag{4}$$

Applying the chain rule, we obtain the following expressions:

$$\frac{\partial}{\partial x}=\xi_x\frac{\partial}{\partial\xi}+\eta_x\frac{\partial}{\partial\eta};\quad \frac{\partial}{\partial y}=\xi_y\frac{\partial}{\partial\xi}+\eta_y\frac{\partial}{\partial\eta}; \tag{5}$$

To find the metric coefficients $\xi_x, \eta_x, \xi_y, \eta_y$, we write expressions for the differentials

$$d\xi=\xi_x dx+\xi_y dy,\quad d\eta=\eta_x dx+\eta_y dy. \tag{6}$$

By comparing the corresponding elements of two matrices with the above equations, we obtain the following metric coefficients

$$\xi_x=\frac{1}{J}y_\eta,\quad \eta_x=-\frac{1}{J}y_\xi,\quad \xi_y=-\frac{1}{J}x_\eta,\quad \eta_y=-\frac{1}{J}x_\xi. \tag{7}$$

where J - Jacobian of the transformation.

We apply the coordinate transformation to the general form of the Navier-Stokes equations for an incompressible fluid written in vector form to obtain the following equation converted:

$\frac{\partial U}{\partial t}+\frac{\partial E}{\partial x}+\frac{\partial F}{\partial y}=\frac{\partial E_v}{\partial x}+\frac{\partial F_v}{\partial y}+\overrightarrow{f}$

Applying the chain rule, we obtain the following expressions:

$$\frac{\partial U}{\partial t}+\xi_x\frac{\partial E}{\partial \xi}+\eta_x\frac{\partial E}{\partial \eta}+\xi_y\frac{\partial F}{\partial \xi}+\eta_y\frac{\partial F}{\partial \eta}=\xi_x\frac{\partial E_v}{\partial \xi}+\eta_x\frac{\partial E_v}{\partial \eta}+\xi_y\frac{\partial F_v}{\partial \xi}+\eta_y\frac{\partial F_v}{\partial \eta}+\overrightarrow{f}. \quad (8)$$

We multiply the transformed equation in the Jacobian, group the similar terms and from the equation (8) we get:

$J\frac{\partial U}{\partial t}+J\xi_x\frac{\partial E}{\partial \xi}+J\eta_x\frac{\partial E}{\partial \eta}+J\xi_y\frac{\partial F}{\partial \xi}+J\eta_y\frac{\partial F}{\partial \eta}=J\xi_x\frac{\partial E_v}{\partial \xi}+J\eta_x\frac{\partial E_v}{\partial \eta}+J\xi_y\frac{\partial F_v}{\partial \xi}+$

$+J\eta_y\frac{\partial F_v}{\partial \eta}+J\overrightarrow{f},$

$J\frac{\partial U}{\partial t}+J\left[\xi_x\frac{\partial E}{\partial \xi}+\xi_y\frac{\partial F}{\partial \xi}\right]+J\left[\eta_x\frac{\partial E}{\partial \eta}+\eta_y\frac{\partial F}{\partial \eta}\right]=J\left[\xi_x\frac{\partial E_v}{\partial \xi}+\xi_y\frac{\partial F_v}{\partial \xi}\right]+$

$+J\left[\eta_x\frac{\partial E_v}{\partial \eta}+\eta_y\frac{\partial F_v}{\partial \eta}\right]+J\overrightarrow{f},$

$J\frac{\partial U}{\partial t}+\frac{\partial}{\partial \xi}(J\xi_xE+J\xi_yF)+\frac{\partial}{\partial \eta}(J\eta_xE+J\eta_yF)-\frac{\partial}{\partial \xi}(J\xi_xE_v+J\xi_yF_v)-$

$-\frac{\partial}{\partial \eta}(J\eta_xE_v+J\eta_yF_v)-J\overrightarrow{f}-E\left[\frac{\partial J\xi_x}{\partial \xi}+\frac{\partial J\eta_x}{\partial \eta}\right]-F\left[\frac{\partial J\xi_y}{\partial \xi}+\frac{\partial J\eta_y}{\partial \eta}\right]+$

$+E_v\left[\frac{\partial J\xi_x}{\partial \xi}+\frac{\partial J\eta_x}{\partial \eta}\right]+F_v\left[\frac{\partial J\xi_y}{\partial \xi}+\frac{\partial J\eta_y}{\partial \eta}\right]=0.$

Considering the ratio of metric coefficients, last terms in the square brackets are zero because

$E\left[\frac{\partial y_\eta}{\partial \xi}-\frac{\partial y_\xi}{\partial \eta}\right]=E((y_\eta)_\xi)-(y_\xi)_\eta)=0$

$F\left[-\frac{\partial \xi_\eta}{\partial \xi}+\frac{\partial x_\xi}{\partial \eta}\right]=F((x_\xi)_\eta-(x_\eta)_\xi)=0.$

Common transformed equation will have the following canonical form:

$J\frac{\partial U}{\partial t}+\frac{\partial}{\partial \xi}(J\xi_xE+J\xi_yF)+\frac{\partial}{\partial \eta}(J\eta_xE+J\eta_yF)-\frac{\partial}{\partial \xi}(J\xi_xE_v+J\xi_yF_v)-$
$\frac{\partial}{\partial \eta}(J\eta_xE_v+J\eta_yF_v)-J\overrightarrow{f}.$

Using the metric coefficients, we obtain

$$E_v=\mu_x\frac{\partial U}{\partial x}=\frac{\mu_x}{J}\left[y_\eta\frac{\partial U}{\partial \xi}-y_\xi\frac{\partial U}{\partial \eta}\right], F_v=\mu_y\frac{\partial U}{\partial y}=\frac{\mu_x}{J}\left[-x_\eta\frac{\partial U}{\partial \xi}+x_\xi\frac{\partial U}{\partial \eta}\right] \quad (9)$$

Substituting the metric coefficients in the general canonical form, we have

$J\frac{\partial U}{\partial t}+\frac{\partial}{\partial \xi}(y_\eta E-x_\eta F)+\frac{\partial}{\partial \eta}(-y_\xi E+x_\xi F)=\frac{\partial}{\partial \xi}(y_\eta E_v-x_\eta F_v)+$

$+\frac{\partial}{\partial\eta}(y_\xi E_v + x_\xi F_v) + J\overrightarrow{f}.$

Substitute the values of E_v and F_v from (9) in the general canonical form:

$$J\frac{\partial U}{\partial t} + \frac{\partial}{\partial\xi}(y_\eta E - x_\eta F) + \frac{\partial}{\partial\eta}(-y_\xi E + x_\xi F) = \frac{\partial}{\partial\xi}\left(\frac{y_\eta^2}{J}\mu_x\frac{\partial U}{\partial\xi} - \frac{y_\eta y_\xi}{J}\mu_x\frac{\partial U}{\partial\eta}\right) +$$
$$\frac{\partial}{\partial\xi}\left(\frac{x_\eta^2}{J}\mu_y\frac{\partial U}{\partial\xi} - \frac{x_\eta x_\xi}{J}\mu_y\frac{\partial U}{\partial\eta}\right) - \frac{\partial}{\partial\eta}\left(\frac{y_\eta y_\xi}{J}\mu_x\frac{\partial U}{\partial\xi} - \frac{y_\xi^2}{J}\mu_x\frac{\partial U}{\partial\eta}\right) -$$
$$\frac{\partial}{\partial\eta}\left(\frac{x_\eta x_\xi}{J}\mu_y\frac{\partial U}{\partial\xi} - \frac{x_\xi^2}{J}\mu_y\frac{\partial U}{\partial\eta}\right).$$

further group the similar terms

$$J\frac{\partial U}{\partial t} + \frac{\partial}{\partial\xi}(y_\eta E - x_\eta F) + \frac{\partial}{\partial\eta}(-y_\xi E + x_\xi F) =$$
$$= \frac{\partial}{\partial\xi}\left[\frac{y_\eta^2\mu_x + x_\eta^2\mu_y}{J}\frac{\partial U}{\partial\xi} - \frac{y_\eta y_\xi\mu_x + x_\eta x_\xi\mu_y}{J}\mu_x\frac{\partial U}{\partial\eta}\right] -$$
$$-\frac{\partial}{\partial\eta}\left[\frac{y_\eta y_\xi\mu_x + x_\eta x_\xi\mu_y}{J}\mu_x\frac{\partial U}{\partial\xi} - \frac{x_\xi^2\mu_y + y_\xi^2\mu_x}{J}\frac{\partial U}{\partial\eta}\right] + \overrightarrow{f}.$$

As a result, we obtain the Navier-Stokes equations in curvilinear coordinates of the form:

$$\frac{1}{J}\frac{\partial U}{\partial t} + \frac{\partial}{\partial\xi}(y_\eta E - x_\eta F) + \frac{\partial}{\partial\eta}(-y_\xi E + x_\xi F) =$$
$$= \frac{\partial}{\partial\xi}\left[(J\mu_x y_\eta^2 + J\mu_y x_\eta^2)\frac{\partial U}{\partial\xi}\right] + \frac{\partial}{\partial\eta}\left[(J\mu_x y_\xi^2 + J\mu_y x_\xi^2)\right] -$$
$$-\frac{\partial}{\partial\xi}\left[(J\mu_x y_\xi y_\eta + J\mu_y x_\xi x_\eta)\frac{\partial U}{\partial\eta}\right] - \frac{\partial}{\partial\eta}\left[(J\mu_x y_\xi y_\eta + J\mu_y x_\xi x_\eta)\frac{\partial U}{\partial\xi}\right] + \frac{\overrightarrow{f}}{J}. \quad (10)$$

The obtained Navier-Stokes equations for a viscous incompressible fluid in general curvilinear coordinates are convenient for the numerical solution of the Navier-Stokes equations in domains with complex geometry using the method of curvilinear grids.

The final form of the equations of fluid dynamics in divergent form is as follows:

$$\frac{\partial}{\partial\xi}(y_\eta u - x_\eta v) + \frac{\partial}{\partial\eta}(-y_\xi u + x_\xi v) = 0. \quad (11)$$

$$\frac{1}{J}\frac{\partial u}{\partial t} + \frac{\partial}{\partial\xi}(y_\eta u^2 - x_\eta uv) + \frac{\partial}{\partial\eta}(-y_\xi u^2 + x_\xi uv) + \frac{\partial}{\partial\xi}(y_\eta\pi) - \frac{\partial}{\partial\eta}(y_\xi\pi) =$$
$$= \frac{\partial}{\partial\xi}\left(a_{11}\frac{\partial u}{\partial\xi}\right) + \frac{\partial}{\partial\eta}\left(a_{22}\frac{\partial u}{\partial\eta}\right) - \frac{\partial}{\partial\xi}\left(a_{12}\frac{\partial u}{\partial\eta}\right) - \frac{\partial}{\partial\eta}\left(a_{12}\frac{\partial u}{\partial\xi}\right) + \frac{\overrightarrow{f}}{J}. \quad (12)$$

$$\frac{1}{J}\frac{\partial v}{\partial t} + \frac{\partial}{\partial\xi}(y_\eta uv - x_\eta v^2) + \frac{\partial}{\partial\eta}(-y_\xi uv + x_\xi v^2) - \frac{\partial}{\partial\xi}(x_\eta\pi) + \frac{\partial}{\partial\eta}(x_\xi\pi) =$$
$$= \frac{\partial}{\partial\xi}\left(a_{11}\frac{\partial v}{\partial\xi}\right) + \frac{\partial}{\partial\eta}\left(a_{22}\frac{\partial v}{\partial\eta}\right) - \frac{\partial}{\partial\xi}\left(a_{12}\frac{\partial v}{\partial\eta}\right) - \frac{\partial}{\partial\eta}\left(a_{12}\frac{\partial v}{\partial\xi}\right) + \frac{\overrightarrow{f}}{J}. \quad (13)$$

$$\frac{1}{J}\frac{\partial\theta}{\partial t} + \frac{\partial}{\partial\xi}(y_\eta u\theta - x_\eta v\theta) + \frac{\partial}{\partial\eta}(-y_\xi u\theta + x_\xi v\theta) =$$

$$= \frac{\partial}{\partial \xi}\left(a_{11}\frac{\partial \theta}{\partial \xi}\right) + \frac{\partial}{\partial \eta}\left(a_{22}\frac{\partial \theta}{\partial \eta}\right) - \frac{\partial}{\partial \xi}\left(a_{12}\frac{\partial \theta}{\partial \eta}\right) - \frac{\partial}{\partial \eta}\left(a_{12}\frac{\partial \theta}{\partial \xi}\right) + \frac{\overrightarrow{f}}{J}. \tag{14}$$

where $a_{11} = J(y_\eta^2\mu_x + x_\eta^2\mu_y)$, $a_{22} = J(y_\xi^2\mu_x + x_\xi^2\mu_y)$, $a_{12} = J(y_\xi y_\eta\mu_x + x_\xi x_\eta\mu_y)$.

Expanding the brackets in the equation of continuity (11) and in the convection terms of the equations (12) - (14) and applying the chain rule, the equations (11) - (14) can be written as follows:

$$y_\eta\frac{\partial u}{\partial \xi} - x_\eta\frac{\partial v}{\partial \xi} - y_\xi\frac{\partial u}{\partial \eta} + x_\xi\frac{\partial v}{\partial \eta} = 0 \tag{15}$$

$$\frac{1}{J}\frac{\partial u}{\partial t} + (y_\eta u - x_\eta v)\frac{\partial u}{\partial \xi} + (-y_\xi u + x_\xi v)\frac{\partial u}{\partial \eta} + y_\eta\frac{\partial \pi}{\partial \xi} - y_\xi\frac{\partial \pi}{\partial \eta} =$$

$$= \frac{\partial}{\partial \xi}\left(a_{11}\frac{\partial u}{\partial \xi}\right) + \frac{\partial}{\partial \eta}\left(a_{22}\frac{\partial u}{\partial \eta}\right) - \frac{\partial}{\partial \xi}\left(a_{12}\frac{\partial u}{\partial \eta}\right) - \frac{\partial}{\partial \eta}\left(a_{12}\frac{\partial u}{\partial \xi}\right) + \frac{\overrightarrow{f}}{J}. \tag{16}$$

$$\frac{1}{J}\frac{\partial v}{\partial t} + (y_\eta u - x_\eta v)\frac{\partial v}{\partial \xi} + (-y_\xi u + x_\xi v)\frac{\partial v}{\partial \eta} - x_\eta\frac{\partial \pi}{\partial \xi} + x_\xi\frac{\partial \pi}{\partial \eta} =$$

$$= \frac{\partial}{\partial \xi}\left(a_{11}\frac{\partial v}{\partial \xi}\right) + \frac{\partial}{\partial \eta}\left(a_{22}\frac{\partial v}{\partial \eta}\right) - \frac{\partial}{\partial \xi}\left(a_{12}\frac{\partial v}{\partial \eta}\right) - \frac{\partial}{\partial \eta}\left(a_{12}\frac{\partial v}{\partial \xi}\right) + \frac{\overrightarrow{f}}{J}. \tag{17}$$

$$\frac{1}{J}\frac{\partial \theta}{\partial t} + (y_\eta u - x_\eta v)\frac{\partial \theta}{\partial \xi} + (-y_\xi u + x_\xi v)\frac{\partial \theta}{\partial \eta} =$$

$$= \frac{\partial}{\partial \xi}\left(a_{11}\frac{\partial \theta}{\partial \xi}\right) + \frac{\partial}{\partial \eta}\left(a_{22}\frac{\partial \theta}{\partial \eta}\right) - \frac{\partial}{\partial \xi}\left(a_{12}\frac{\partial \theta}{\partial \eta}\right) - \frac{\partial}{\partial \eta}\left(a_{12}\frac{\partial \theta}{\partial \xi}\right) + \frac{\overrightarrow{f}}{J}. \tag{18}$$

Such an approach of the transformation of equations will help to avoid computing second derivatives of the metric coefficients and the emergence of non-linear terms. And besides strictly divergent form of the equations is useful in the development of difference schemes.

For the numerical modeling of convective flows in curvelinear doubly connected domain, we consider the formulation of the problem in curvilinear coordinate systems of the form

$$\frac{\partial}{\partial \xi}(y_\eta u - x_\eta v) + \frac{\partial}{\partial \eta}(-y_\xi u + x_\xi v) = 0 \tag{19}$$

$$\frac{1}{J}\frac{\partial u}{\partial t} + \frac{\partial u}{\partial \xi}(y_\eta(u^2 + \pi) - x_\eta uv) + \frac{\partial}{\partial \eta}(-y_\xi u^2 + x_\xi uv) =$$
$$= \frac{\partial}{\partial \xi}\left[(J\mu_x y_\eta^2 + J\mu_y x_\eta^2)\frac{\partial U}{\partial \xi}\right] + \frac{\partial}{\partial \eta}\left[(J\mu_x y_\xi^2 + J\mu_y x_\xi^2)\frac{\partial U}{\partial \eta}\right] -$$

$$= \frac{\partial}{\partial \xi}\left[J\mu_x y_\xi y_\eta + J\mu_y x_\xi x_\eta\frac{\partial U}{\partial \eta}\right] - \frac{\partial}{\partial \eta}\left[J\mu_x y_\xi y_\eta + J\mu_y x_\xi x_\eta\frac{\partial U}{\partial \xi}\right] + \frac{\overrightarrow{f}}{J}. \tag{20}$$

In numerical constructing of curvilinear grids in doubly connected domains using the equidistribution method and the method of Godunov-Thompson, as well as the numerical implementation of incompressible fluid equations, an implicit scheme and the method of fractional steps are used. In the direction of the external and internal borders, the cyclic sweep is used, and in the direction of the normal the scalar sweep is used.

One of the most common traditional methods of constructing curvilinear grids from the considered class of methods is the method of equidistribution, that is the class in which the grid are obtained by mapping the computational domain to the physical domain. The idea of the equidistribution method is to find a non-degenerate mapping carrying the fixed (uniform rectangular) grid on the computational domain to the adaptive mesh refinement on the physical domain which satisfies the principle of equidistribution:

$$wJ = const$$

where J is Jacobian of this transformation, w is the function of the grid density.

The value of the Jacobian is proportional to the measure of the grid cell. The term of measure refers to the length of the cell in a one dimensional domain, area in two-dimensional domain, and volume in the three-dimensional case. Therefore, the meaning of the ratio is that the larger the value of the density function at the grid cell, the smaller measure of this cell. When using the equidistribution method, it is required to find a direct mapping from the computational domain to the physical domain by solving a complex non-linear equation with variable coefficients in the computational domain of a simple shape, square. Now we have to look for the reverse transformation by solving simple equations in the physical domain whose boundaries are generally curvelinear. Curvilinear boundary complicates the numerical solution of the problem. Therefore, the direct conversion is preferably provided that the inverse transform is the solution of equations. This approach was proposed in the works of S.K. Godunov and J.F. Thompson and colleagues [3], so the method of constructing curvilinear grids called the Godunov-Thompson method (GT-method).

For the discretization of the physical domain, the technology of construction of difference curvilinear structure grids using the method of transfinite interpolation is presented. Transfinite interpolation is implemented in two stages. In the first stage, values of grid nodes from the left and right borders are interpolated. In the second stage, grid nodes from lower and upper boundaries of the domain are interpolated.

There are several algorithms for constructing curvilinear grids and results of the application of these algorithms for the domains of simple form. Unfortunately, the quality of the obtained mesh can be judged only visually. When using curvilinear grids in the calculation of mathematical physics problems, there are some objective characteristics of grids:

- Orthogonality,
- Locally uniformly,
- Undrawn cells, etc.

With these features, one can evaluate the quality of the grid and its suitability for the calculations. Quantitative information on the grid given by quality criteria may be useful for preliminary assessment of grids allowing certainly reject unsuitable grids even before solving the main problem. Thus, there is a need for automatic determination of the characteristics of the grid. Some approaches for the analysis of the quality of grids are given in [1].

In order to select the most appropriate grids for the main problem, the quality of difference grids are considered on the following evaluation criteria:

– Convexity of the cells;
– Orthogonality of coordinate grid lines;
– Elongation of the cells;
– Adaptation of the grid to a predetermined control function, proposed in [1,4].

3 Simulation results

The calculations for various configurations of the cavity and temperature conditions at the border are conducted. To obtain the graphs of numerical calculations, we use the graphical editor Tecplot.

In this paper, the numerical algorithm is used for the doubly connected domain with curvelinear boundaries.

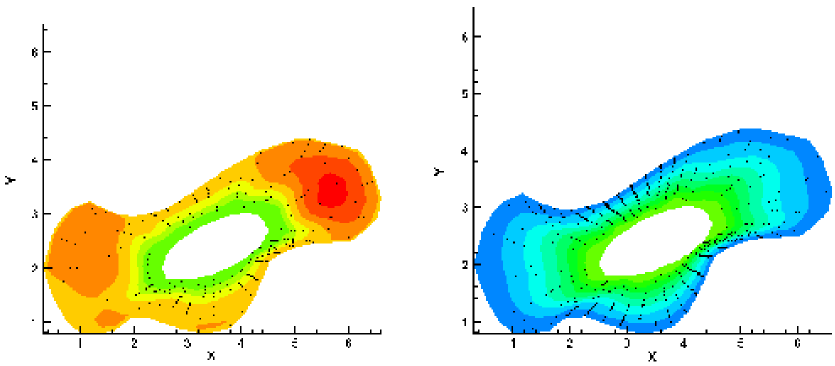

Fig. 1. The changing of the vector ψ

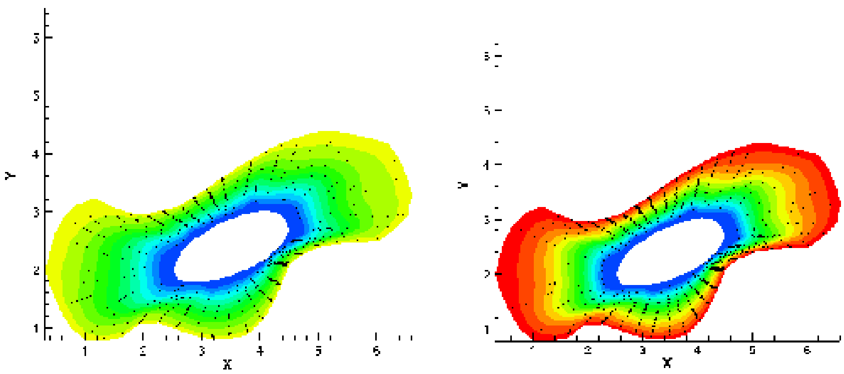

Fig. 2. The changing of the vector ψ at the iteration of 5x5000

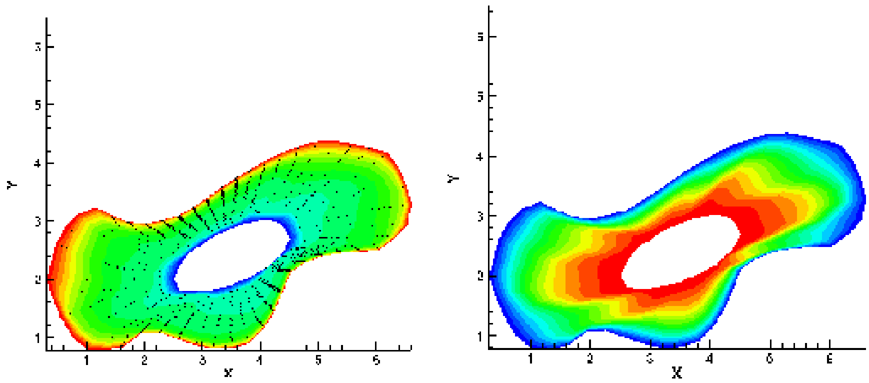

Fig. 3. Changes in temperature at the iteration of 50x20000

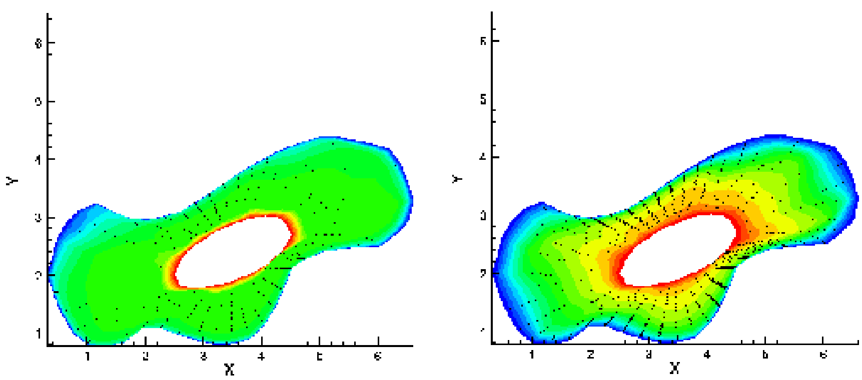

Fig. 4. Changes in temperature at the iteration of 5x5000

4 Conclusion

Modern requirements to the reliability of the numerical results and the reliability of methodical software requires careful testing and verification of the developed software. Testing of the developed methods, algorithms and software complex for the problems is performed on the development of a viscous incompressible fluid flow [5]. On the basis of the proposed methods, methods for constructing curvilinear grids of this class for the doubly-connected domains are developed. A complex of software for the automated construction of curvilinear grids is developed, as well as the quality criteria of the grid is considered. Experimental studies of the proposed methods are conducted. As a result of testing, it is revealed that the quality of the resulting grid meets the generally accepted criteria.

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