Analytical Solution of the Problem About Bending of Annular Plates Subject to the Action of the Lateral Load

A.N. Tyurekhojayev[1], G.K. Kalzhanova[2], A.G. Ibrayev[3]

[1] K.I. Satpayev Kazakh National Technical University, Almaty, Kazakhstan
[2] Ilyas Zhansugurov Zhetysu State University, Taldykorgan, Kazakhstan
[3] L.N.Gumilev Eurasian National University, Astana, Kazakhstan
{tyurekhodja,kgk67,ibrayev.askar}@mail.ru

Abstract. The paper considers the problem of compound bending of non-uniform round flexible plate subjected to lateral load under temperature changes through the thickness of the plate with the influence of tension on bending. The problem is reduced to the study of an unrelated system of differential equations, analytical solution of which has not been possible using existing mathematical apparatus. The paper managed to decompose the related system of equations into two unrelated equations and to find a solution using the method of partial discretization of differential equations. Results are represented as formulae and graphs.

Keywords: Thin round plate, compound bending, lateral load, radial force, deflection of the median plane of the plate, system of linear differential equations with variable coefficients, method of partial discretization of differential equations.

In the mechanics of deformable solid bodies are of particular interest the problems associated with bends of flexible plates and of various flexible shells working in a inhomogeneous temperature field. Such problems commonly encountered in applied problems of construction, petroleum engineering, mechanical engineering, water and air transport.

In mathematical consideration of such problems one have to deal with systems of linear differential equations with variable coefficients and nonlinear terms, obtaining analytical solutions of which presents considerable mathematical difficulties. Analytical solutions of such problems can be obtained using the method of partial discretization, developed by Professor A.N. Tyurekhodzhaev based on the theory of generalized functions.

Consider the problem of thermoelasticity of inhomogeneous circular plates under axisymmetric temperature field allowing for the changes in the elastic properties of the plate material by its thickness. Complex bending of inhomogeneous flexible circular plate, exposed to the action of the lateral load, under the temperature change across the thickness of the plate is described by a system of connected differential equations [1]:

$$\begin{aligned} & a_{11} r \frac{d}{dr} \nabla^2 F + a_{13} r \frac{d}{dr} \nabla^2 u_z = 0, \\ & a_{13} r \frac{d}{dr} \nabla^2 F + a_{33} r \frac{d}{dr} \nabla^2 u_z + \frac{dF}{dr} \cdot \frac{du_z}{dr} = -\int q_z r dr + C, \end{aligned} \tag{1}$$

where

$$\begin{aligned} & a_{11} = kD_N,\ a_{13} = k(D_N D_\nu - D_{N\nu} D), \\ & a_{33} = D a_{14} + D_\nu a_{13} - D_M,\ a_{14} = k(D_N D - D_{N\nu} D_\nu), \end{aligned} \tag{2}$$

$$k = \frac{1}{D_N^2 - D_{N\nu}^2},\ \nabla^2 = \frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr},$$

$$D_N = \int\limits_{-h/2}^{h/2} \frac{E(z)}{1-\nu(z)^2} dz, \quad D_{N\nu} = \int\limits_{-h/2}^{h/2} \frac{E(z)\nu(z)}{1-\nu(z)^2} dz, \quad D = \int\limits_{-h/2}^{h/2} \frac{E(z)}{1-\nu(z)^2} z dz,$$

$$D_\nu = \int\limits_{-h/2}^{h/2} \frac{E(z)\nu(z)}{1-\nu(z)^2} z dz, \quad D_M = \int\limits_{-h/2}^{h/2} \frac{E(z)\nu(z)}{1-\nu(z)^2} z^2 dz, \quad D_{M\nu} = \int\limits_{-h/2}^{h/2} \frac{E(z)\nu(z)}{1-\nu(z)^2} z^2 dz.$$

Here F is the stress function, $u_z$is the deflection of the middle surface of the plate, h is the thickness of the plate, E is the modulus of elasticity, ν is the Poisson ratio, q_z is the external distributed lateral load per unit area of the middle surface, C is the constant of integration.

Solving these equations for $\frac{d}{dr}\nabla^2 F$ and $\frac{d}{dr}\nabla^2 u_z$ and considering the expression (2), we obtain

$$\begin{aligned} r\frac{d}{dr}\nabla^2 F = {} & \frac{D_{N\nu}D - D_N D_\nu}{D_N D_M - D^2} \cdot \frac{dF}{dr} \cdot \frac{du_z}{dr} + \\ & + \frac{D_{N\nu}D - D_N D_\nu}{D_N D_M - D^2} \left(\int q_z r dr - C \right), \\ r\frac{d}{dr}\nabla^2 u_z = {} & \frac{D_N}{D_N D_M - D^2} \cdot \frac{dF}{dr} \cdot \frac{du_z}{dr} + \\ & + \frac{D_N}{D_N D_M - D^2} \left(\int q_z r dr - C \right). \end{aligned} \tag{3}$$

Consider the problem of thermoelasticity of inhomogeneous circular plates under axisymmetric temperature field allowing for the influence of tension on the bending and the changes in the elastic properties of the plate material by its thickness.

It is generally accepted [1] that the system of equations (1) in the case of considering the influence of tension on the bending is not reduced to unconnected equations. In fact, the system of resolving equations (1) can be reduced to a system of differential equations with nonlinear terms, each of which includes only one resolving function:

$$\frac{d^3 u_z}{dr^3} + \frac{1}{r}\frac{d^2 u_z}{dr^2} - \left(\frac{BC_1}{2} + \frac{1+BC_2}{r^2} \right) \frac{du_z}{dr} - \frac{A}{r} \left(\frac{du_z}{dr} \right)^2 = \frac{B}{r} \left(\int q_z r dr - C \right), \tag{4}$$

$$\frac{d^3 F}{dr^3} + \frac{1}{r}\frac{d^2 F}{dr^2} + \left(\frac{BC_1}{2} - \frac{1-BC_2}{r^2} \right) \frac{dF}{dr} - \frac{B}{r} \left(\frac{dF}{dr} \right)^2 = \frac{A}{r} \left(\int q_z r dr - C \right), \tag{5}$$

where

$$A = \frac{D_{N\nu}D - D_N D_\nu}{D_N D_M - D^2}, \quad B = \frac{D_N}{D_N D_M - D^2}$$

And the functions F and u_z are related by

$$F = \frac{A}{B} u_z + \frac{C_1 r^2}{4} + C_2 \ln r + C_0. \tag{6}$$

The third order differential equations (4) to (5) relative to the normal force N_r, acting in the cylindrical section, and the normal turning angle ϑ_r, defined given an axisymmetric field by the relation

$$N_r = \frac{1}{r}\frac{dF}{dr}, \ \vartheta_r = -\frac{du_z}{dr},$$

are rewritten in the following form

$$\frac{d^2\vartheta_r}{dr^2}+\frac{1}{r}\frac{d\vartheta_r}{dr}-\left(\frac{BC_1}{2}+\frac{1+BC_2}{r^2}\right)\vartheta_r+\frac{A}{r}\vartheta_r^2=-\frac{B}{r}\left(\int q_z r dr-C\right), \tag{7}$$

$$\frac{d^2N_r}{dr^2}+\frac{3}{r}\frac{dN_r}{dr}+\left(\frac{BC_1}{2}-\frac{BC_2}{r^2}\right)N_r-BN_r^2=\frac{A}{r^2}\left(\int q_z r dr-C\right). \tag{8}$$

Radial force and deflection angle are related by

$$N_r=\frac{C_1}{2}+\frac{C_2}{r^2}-\frac{A}{B}\cdot\frac{\vartheta_r}{r}. \tag{9}$$

The exact solution of such equations using existing mathematical tools is not possible. Applying the method of partial discretization of differential equations, we determine the solution of these equations.

It should be noted that in this case it is sufficient to solve one of the equations (7)-(8). For instance, one can determine the turning angle of the normal, and then find the radial force by the formula (26).

Applying the method of partial discretization to the differential equation (7), we derive the following expression for the general solution of this equation

$$\begin{aligned}
\vartheta_r(r)=C_3r+\frac{C_4}{r}-\frac{A}{4}r\sum_{k=1}^{n}(r_k+r_{k+1})\left[\frac{\vartheta_r^2(r_k)}{r_k}H(r-r_k)-\frac{\vartheta_r^2(r_{k+1})}{r_{k+1}}H(r-r_{k+1})\right]+\\
+\frac{BC_1}{8}r\sum_{k=1}^{n}(r_k+r_{k+1})\left[\vartheta_r(r_k)H(r-r_k)-\vartheta_r(r_{k+1})H(r-r_{k+1})\right]+\\
+\frac{BC_2}{4}r\sum_{k=1}^{n}(r_k+r_{k+1})\left[\frac{\vartheta_r^2(r_k)}{r_k^2}H(r-r_k)-\frac{\vartheta_r^2(r_{k+1})}{r_{k+1}^2}H(r-r_{k+1})\right]+\\
+\frac{A}{4r}\sum_{k=1}^{n}(r_k+r_{k+1})\left[r_k\vartheta_r^2(r_k)H(r-r_k)-r_{k+1}\vartheta_r^2(r_{k+1})H(r-r_{k+1})\right]-\\
-\frac{BC_1}{8r}\sum_{k=1}^{n}(r_k+r_{k+1})\left[r_k^2\vartheta_r(r_k)H(r-r_k)-r_{k+1}^2\vartheta_r(r_{k+1})H(r-r_{k+1})\right]-\\
-\frac{BC_2}{4r}\sum_{k=1}^{n}(r_k+r_{k+1})\left[\vartheta_r(r_k)H(r-r_k)-\vartheta_r(r_{k+1})H(r-r_{k+1})\right]-\\
-\frac{B}{2}r\int\frac{1}{r}(\int q_z r dr-C)dr+\frac{B}{2r}\int r(\int q_z r dr-C)dr,
\end{aligned} \tag{10}$$

where $H(z)$ is the Heaviside unit function. Substituting the expression (10) into the formula (26), we derive

$$\begin{aligned}
N_r(r)=\frac{C_1}{2}+\frac{C_2}{r^2}-\frac{A}{B}\Bigg\{C_3+\frac{C_4}{r^2}-\frac{A}{4}\sum_{k=1}^{n}(r_k+r_{k+1})\left[\frac{\vartheta_r^2(r_k)}{r_k}H(r-r_k)-\right.\\
\left.-\frac{\vartheta_r^2(r_{k+1})}{r_{k+1}}H(r-r_{k+1})\right]+\frac{BC_1}{8}\sum_{k=1}^{n}(r_k+r_{k+1})[\vartheta_r(r_k)H(r-r_k)-
\end{aligned}$$

$$-\vartheta_r(r_{k+1})H(r-r_{k+1})]+\frac{BC_2}{4}\left[\frac{\vartheta_r(r_k)}{r_k^2}H(r-r_k)-\frac{\vartheta_r(r_{k+1})}{r_{k+1}^2}H(r-r_{k+1})\right]+$$

$$+\frac{A}{4r^2}\sum_{k=1}^{n}(r_k+r_{k+1})\left[r_k\vartheta_r^2(r_k)H(r-r_k)-r_{k+1}\vartheta_r^2(r_{k+1})H(r-r_{k+1})\right]-$$

$$-\frac{BC_1}{8r^2}\sum_{k=1}^{n}(r_k+r_{k+1})\left[r_k^2\vartheta_r(r_k)H(r-r_k)-r_{k+1}^2\vartheta_r(r_{k+1})H(r-r_{k+1})\right]-$$

$$-\frac{BC_2}{4r^2}\sum_{k=1}^{n}(r_k+r_{k+1})\left[\vartheta_r(r_k)H(r-r_k)-\vartheta_r(r_{k+1})H(r-r_{k+1})\right]-$$

$$\left.-\frac{B}{2}\int\frac{1}{r}(\int q_z r dr-C)dr+\frac{B}{2r^2}\int r(\int q_z r dr-C)dr\right\} \quad (11)$$

Consider the annular plate of constant thickness, the outer contour of which is rigidly clamped and inner one can shift in the axial direction of the plate, but it does not turn. The contours of the plate are free of radial forces. Then the constants are determined from the following boundary conditions

$$N_r\big|_{r=a}=0,\ \ N_r\big|_{r=b}=0,\ \vartheta_r\big|_{r=a}=0,\ \vartheta_r\big|_{r=b}=0. \quad (12)$$

According to (12) the constants C_1 and C_2 will be equal to zero. С учетом значений Given values of C_1 and C_2 the differential equation (7) becomes

$$\frac{d^2\vartheta_r}{dr^2}+\frac{1}{r}\frac{d\vartheta_r}{dr}-\frac{\vartheta_r}{r^2}+\frac{A}{r}\vartheta_r^2=-\frac{B}{r}\left(\int q_z r dr-C\right) \quad (13)$$

Discretizing the last term on the left hand side of the equation (13), we derive its following general solution

$$\vartheta_r(r)=C_3 r+\frac{C_4}{r}-\frac{A}{4}r\sum_{k=1}^{n}(r_k+r_{k+1})\left[\frac{\vartheta_r^2(r_k)}{r_k}H(r-r_k)-\frac{\vartheta_r^2(r_{k+1})}{r_{k+1}}H(r-r_{k+1})\right]+$$

$$[+\frac{A}{4r}\sum_{k=1}^{n}(r_k+r_{k+1})\left[r_k\vartheta_r^2(r_k)H(r-r_k)-r_{k+1}\vartheta_r^2(r_{k+1})H(r-r_{k+1})\right]-$$

$$-\frac{1}{2}r\int\frac{B}{r}\left(\int q_z r dr-C\right)dr+\frac{1}{2r}\int B\left(\int q_z r dr-C\right)rdr. \quad (14)$$

Consequently, for the deflection $u_z(r)$ will be

$$u_z(r)=-\frac{C_3 r^2}{2}+C_4\ln r+\frac{A}{8}r^2\sum_{k=1}^{n}(r_k+r_{k+1})\left[\frac{\vartheta_r^2(r_k)}{r_k}H(r-r_k)-\right.$$

$$\left.-\frac{\vartheta_r^2(r_{k+1})}{r_{k+1}}H(r-r_{k+1})\right]+\frac{A}{4}\sum_{k=1}^{n}(r_k+r_{k+1})[r_k\ln r_k\vartheta_r^2(r_k)H(r-r_k)-$$

$$-r_{k+1}\ln r_{k+1}\vartheta_r^2(r_{k+1})H(r-r_{k+1})]-\frac{A}{4}\left(\ln r+\frac{1}{2}\right)\times$$

$$\times\sum_{k=1}^{n}(r_k+r_{k+1})\left[r_k\vartheta_r^2(r_k)H(r-r_k)-r_{k+1}\vartheta_r^2(r_{k+1})H(r-r_{k+1})\right]+$$

$$+\frac{1}{2}\int r\left[\int\frac{B}{r}\left(\int q_z r dr-C\right)dr\right]dr-$$

$$\frac{1}{2}\int\frac{1}{r}\left[\int B\left(\int q_z r dr-C\right)rdr\right]dr+C_5. \qquad (15)$$

The constant C_5 is determined from the condition of rigid support of the plate's outer contour

$$u_z\big|_{r=b}=0. \qquad (16)$$

Relation (26), taking into account the values of the constants C_1, C_2 will take the form

$$N_r=-\frac{A}{B}\cdot\frac{\vartheta_r}{r} \qquad (17)$$

Substituting the expression (14) into (17) we obtain

$$N_r(r)=-\frac{A}{B}\left\{C_3+\frac{C_4}{r^2}-\frac{A}{4}\sum_{k=1}^{n}(r_k+r_{k+1})\left[\frac{\vartheta_r^2(r_k)}{r_k}H(r-r_k)-\frac{\vartheta_r^2(r_{k+1})}{r_{k+1}}H(r-r_{k+1})\right]+\right.$$

$$+\frac{A}{4r^2}\sum_{k=1}^{n}(r_k+r_{k+1})\left[r_k\vartheta_r^2(r_k)H(r-r_k)-r_{k+1}\vartheta_r^2(r_{k+1})H(r-r_{k+1})\right]-$$

$$\left.-\frac{1}{2}\int\frac{B}{r}\left(\int q_z r dr-C\right)dr+\frac{1}{2r^2}\int B\left(\int q_z r dr-C\right)rdr\right\} \qquad (18)$$

The constant C is determined from

$$rQ_r-rN_r\vartheta_r=-\int q_z r dr+C, \qquad (19)$$

where Q_r is the lateral force. Let the load be evenly distributed along the circumference with radius r_0 with intensity q_0

$$q_z=q_0\delta(r-r_0)$$

Then the constant would be equal to zero.

Taking into account the boundary conditions (12), the deflection angle of the plate can be written as

$$\vartheta_r(r)=-\frac{Bq_0r_0}{2}r\ln\frac{r}{r_0}H(r-r_0)+\frac{Bq_0r_0}{4r}(r^2-r_0^2)H(r-r_0)-\frac{b(r^2-a^2)}{r(b^2-a^2)}\times$$

$$\times\left\{\frac{A}{4b}\sum_{k=1}^{n}(r_k+r_{k+1})\left[r_k\vartheta_r^2(r_k)-r_{k+1}\vartheta_r^2(r_{k+1})\right]-\right.$$

$$\left.-\frac{A}{4}b\sum_{k=1}^{n}(r_k+r_{k+1})\left[\frac{\vartheta_r^2(r_k)}{r_k}-\frac{\vartheta_r^2(r_{k+1})}{r_{k+1}}\right]-\frac{Bq_0r_0b}{2}\ln\frac{b}{r_0}+\frac{Bq_0r_0}{4b}(b^2-r_0^2)\right\}+$$

$$+\frac{A}{4r}\sum_{k=1}^{n}(r_k+r_{k+1})\left[r_k\vartheta_r^2(r_k)H(r-r_k)-r_{k+1}\vartheta_r^2(r_{k+1})H(r-r_{k+1})\right]-$$

$$-\frac{A}{4}r\sum_{k=1}^{n}(r_k+r_{k+1})\left[\frac{\vartheta_r^2(r_k)}{r_k}H(r-r_k)-\frac{\vartheta_r^2(r_{k+1})}{r_{k+1}}H(r-r_{k+1})\right] \qquad (20)$$

And the analytic expression of the turning angle at points r_k is determined as follows

$$\vartheta_r(r_k)=\frac{-1\pm\sqrt{1-A(r_{k+1}-r_{k-1})\left(\frac{r_k}{b}-\frac{b}{r_k}\right))f(r_k)G}}{\frac{A}{2}(r_{k+1}-r_{k-1})\left(\frac{r_k}{b}-\frac{b}{r_k}\right)f(r_k)},$$

where

$$G=\left(\frac{Bq_0r_0(b^2-r_0^2)}{4b}-\frac{Bq_0r_0b}{2}\ln\frac{b}{r_0}\right)f(r_k)-\frac{Bq_0r_0(r_k^2-r_0^2)}{4r_k}H(r_k-r_0)+$$

$$+\frac{Bq_0r_0r_k}{2}\ln\frac{r_k}{r_0}H(r_k-r_0)+\frac{A}{4}(r_1+r_2)\left(\frac{r_1}{b}-\frac{b}{r_1}\right)f(r_1)\vartheta_r^2(r_1)-$$

$$-\frac{A}{4}(r_1+r_2)\left(\frac{r_1}{r_k}-\frac{r_k}{r_1}\right)\vartheta_r^2(r_1)+\frac{A}{4}\sum_{j=2}^{k-1}(r_{j+1}-r_{j-1})f(r_k)\left(\frac{r_j}{r_k}-\frac{r_k}{r_j}\right)\vartheta_r^2(r_j)-$$

$$-\frac{A}{4}\sum_{j=2}^{k-1}(r_{j+1}-r_{j-1})\left(\frac{r_j}{r_k}-\frac{r_k}{r_j}\right)\vartheta_r^2(r_j),\quad f(r_k)=\frac{b(r_k^2-a^2)}{r_k(b^2-a^2)}.$$

The deflection of the middle surface of the plate is equal to

$$u_z=-\frac{Bq_0r_0}{4}(r^2-r_0^2)H(r-r_0)+\frac{Bq_0r_0}{4}(r^2+r_0^2)\ln\frac{r}{r_0}H(r-r_0)-$$

$$-\frac{b}{(b^2-a^2)}\left(a^2\ln\frac{r}{b}+\frac{b^2-r^2}{2}\right)\left\{\frac{A}{4b}\sum_{k=1}^{n}(r_k+r_{k+1})\left[r_k\vartheta_r^2(r_k)-r_{k+1}\vartheta_r^2(r_{k+1})\right]-\right.$$

$$-\frac{A}{4}b\sum_{k=1}^{n}(r_k+r_{k+1})\left[\frac{\vartheta_r^2(r_k)}{r_k}-\frac{\vartheta_r^2(r_{k+1})}{r_{k+1}}\right]-\frac{Bq_0r_0b}{2}\ln\frac{b}{r_0}+$$

$$\left.+\frac{Bq_0r_0}{4b}(b^2-r_0^2)\right\}+\frac{A}{4}\left(\ln r+\frac{1}{2}\right)\sum_{k=1}^{n}(r_k+r_{k+1})[r_k\vartheta_r^2(r_k)H(r-r_k)-$$

$$-r_{k+1}\vartheta_r^2(r_{k+1})H(r-r_{k+1})]+\frac{A}{4}\sum_{k=1}^{n}(r_k+r_{k+1})[r_k\ln r_k\vartheta_r^2(r_k)H(r-r_k)-$$

$$-r_{k+1}\vartheta_r^2(r_{k+1})H(r-r_{k+1})]+\frac{A}{8}r^2\sum_{k=1}^{n}(r_k+r_{k+1})\left[\frac{\vartheta_r^2(r_k)}{r_k}H(r-r_k)-\right.$$

$$-\frac{\vartheta_r^2(r_{k+1})}{r_{k+1}}H(r-r_{k+1})\Bigg]+\frac{A}{4}\left(\ln b+\frac{1}{2}\right)\sum_{k=1}^{n}(r_k+r_{k+1})[r_k\vartheta_r^2(r_k)-$$

$$-r_{k+1}\vartheta_r^2(r_{k+1})]-\frac{A}{4}\sum_{k=1}^{n}(r_k+r_{k+1})\left[r_k\ln r_k\vartheta_r^2(r_k)-r_{k+1}\ln r_{k+1}\vartheta_r^2(r_{k+1})\right]-$$

$$-\frac{A}{8}b^2\sum_{k=1}^{n}(r_k+r_{k+1})\times\left[\frac{\vartheta_r^2(r_k)}{r_k}-\frac{\vartheta_r^2(r_{k+1})}{r_{k+1}}\right]+$$

$$+\frac{Bq_0r_0(b^2-r_0^2)}{4}-\frac{Bq_0r_0(b^2+r_0^2)}{4}\ln\frac{b}{r_0} \qquad (21)$$

Radial force in the case of the plate, exposed to the lateral load, distributed evenly along the circumference with radius r_0 with intensity q_0, is expressed by the formula

$$N_r(r)=\frac{Aq_0r_0}{2}\ln\frac{r}{r_0}H(r-r_0)-\frac{Aq_0r_0(r^2-r_0^2)}{4r^2}H(r-r_0)+$$

$$+\frac{A}{B}\frac{b(r^2-a^2)}{r^2(b^2-a^2)}\Bigg\{\frac{A}{4b}\sum_{k=1}^{n}(r_k+r_{k+1})\left[r_k\vartheta_r^2(r_k)-r_{k+1}\vartheta_r^2(r_{k+1})\right]-$$

$$-\frac{A}{4}b\sum_{k=1}^{n}(r_k+r_{k+1})\left[\frac{\vartheta_r^2(r_k)}{r_k}-\frac{\vartheta_r^2(r_{k+1})}{r_{k+1}}\right]-\frac{Bq_0r_0b}{2}\ln\frac{b}{r_0}+\frac{Bq_0r_0}{4b}(b^2-r_0^2)\Bigg\}-$$

$$-\frac{A^2}{4Br^2}\sum_{k=1}^{n}(r_k+r_{k+1})\left[r_k\vartheta_r^2(r_k)H(r-r_k)-r_{k+1}\vartheta_r^2(r_{k+1})H(r-r_{k+1})\right]+$$

$$+\frac{A^2}{4B}\sum_{k=1}^{n}(r_k+r_{k+1})\left[\frac{\vartheta_r^2(r_k)}{r_k}H(r-r_k)-\frac{\vartheta_r^2(r_{k+1})}{r_{k+1}}H(r-r_{k+1})\right] \qquad (22)$$

Performing numerical calculations of the turning angle and deflection of annular plate of constant thickness, exposed to the lateral load evenly distributed along the circumference r_0, given the elasticity modulus and coefficient of varying by the plate thickness according to the linear law given the certain values of the parameters, we plot down the curves. Figures 1 and 2 show the curves for changes in the turning angle of the normal and deflection for different cases of the lateral load distribution [1].

The considered method allows to find a solution to the equations (4) – (5) for virtually any laws of change both in the elastic modulus and the Poisson ratio.

References

1. Kovalenko A.D. Fundamentals of thermoelasticity. Kiev: "Naukova Dumka", (1970) [in Russian]
2. Tyurekhodzhaev A.N., Kalzhanova G.K. The problem of axisymmetric nonlinear bending of inhomogeneous flexible circular plate in the inhomogeneous temperature field. "Reports of the National Academy of Sciences of Kazakhstan". №3. pp.23-33. (in Russian) (2005)

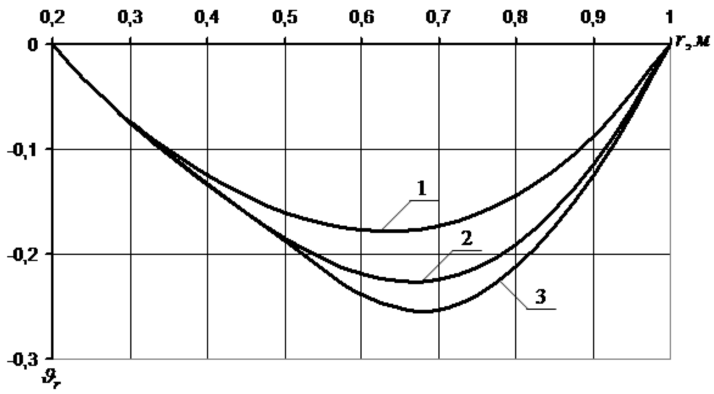

Fig. 1. The curves of changes in the angle of turning for the plate with an orifice, subject to the lateral load distributed: 1 – evenly over the plate surface; 2 – evenly over the area of the annular with radius and width ; 3 – evenly along the circumference of radius r_0 $(c < r_0 < d)$.

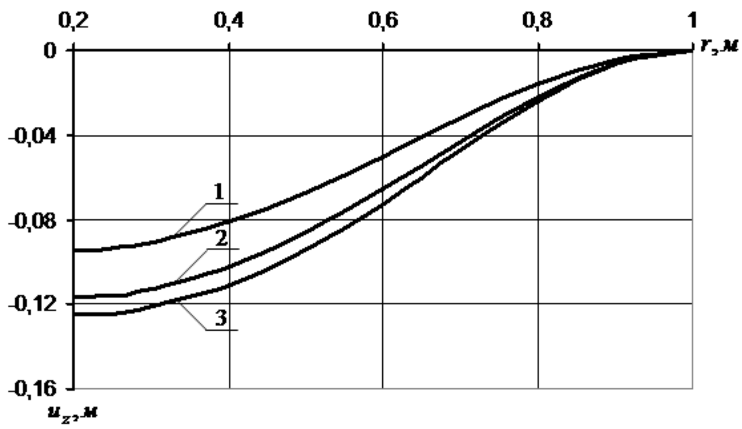

Fig. 2. The curves of changes in the deflection for the plate with an orifice, subject to the lateral load distributed: 1 – evenly over the plate surface; 2 – evenly over the area of the annular with radius and width; 3 – evenly along the circumference of radius r_0 $(c < r_0 < d)$.