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Nonlinear differential equation with first order partial derivatives

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The asymptotic behavior of solutions of a nonlinear differential equation with first-order partial derivatives solved with respect to one of the derivatives is investigated. Each first-order partial differential equation under certain conditions has a fundamental system of integrals or an integral basis. We note that for a general linear partial differential equation of the first order there can be no nontrivial integral. For a linear homogeneous first-order partial differential equation, where the coefficients of the equation are given on an unbounded set and have continuous first-order partial derivatives, with the first coefficient equal to one, an integral basis exists. In this paper, a nonlinear partial differential equation of the first order, which is solved with respect to one of the derivatives, is estimated from two sides by first-order partial differential equations. Using differential inequalities it is proved that a nonlinear differential equation with first-order partial derivatives solved with respect to one of the derivatives has a solution that tends to zero as one tends to infinity to one of the independent variables. At present, the theory of partial differential equations finds its application in various fields of natural science.

Key words:equation, first order partial derivatives.

Сызықты бірінші ретті дербес туындылы теңдеулер туралы

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Туындылардың біреуіне байланысты шешілген бірінші ретті дербес туындылы сызықты емес дифференциалдық теңдеудің шешімдерінің асимптотикалық мінезі зерттеледі. Бірінші ретті дербес туындылы дифференциалдық теңдеудің әрқайсысының қандайда бір шарттарда фундаменталды интегрладар жүйесі немесе интегралдық базисі болады. Айта кететіні, жалпы бірінші ретті сызықты дербес туындылы дифференциалдық теңдеудің тривиалды емес интегралы болмауы да мүмкін. Бірінші ретті сызықты дербес туындылы дифференциалдық теңдеу үшін, оның коэффициенттері шенелмеген жиында беріліп, үзіліссіз бірінші ретті дербес туындылары болса және бірінші коэффициенті бірге тең болса, интегралды базис бар болады. Бұл жұмыста туындылардың біреуіне байланысты шешілген бірінші ретті дербес туындылы сызықты емес дифференциалдық теңдеу екі жағынан бірінші ретті дербес туындылы дифференциалдық теңдеулермен бағаланады. Дифференциалдық теңсіздіктерді пайдалана отырып, туындылардың біреуіне байланысты шешілген бірінші ретті дербес туындылы сызықты емес теңдеудің тәуелсіз айнымалыларнының біреуі плюс шексіздікке ұмтылған жағдайда нөлге ұмтылатын шешімі бар болатыны дәлелденген. Қазіргі таңда дербес туындылы дифференциалдық теңдеулер теориясы жаратылыс танудың түрлі салаларында өз қолданыстарын табуда.

Түйін сөздер: теңдеу, бірінші ретті дербес туындылар.

О нелинейном дифференциальном уравнении с частными производными первого порядка

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Рассматривается асимптотическое поведение решений нелинейного дифференциального уравнения, с частными производными первого порядка разрешенное относительно одной из производных. Каждое дифференциальное уравнение с частными производными первого порядка при некоторых условиях имеет фундаментальную систему интегралов или интегральный базис. Заметим, для общего линейного дифференциального уравнения с частными производными первого порядка может не существовать нетривиального интеграла. Для линейного однородного дифференциального уравнения с частными производными первого порядка, где коэффициенты уравнения заданы на неограниченном множестве и имеют непрерывные частные производные первого порядка, причем первый коэффициент равен единице, интегральный базис существует. В работе нелинейное дифференциальное уравнение с частными производными первого порядка, разрешенное относительно одной из производных, оцениваются с двух сторон дифференциальными уравнениями с частными производными первых порядков. Использованием дифференциальных неравенств доказано, что нелинейное дифференциальное уравнение, с частными производными первого порядка разрешенное относительно одной из производных имеет решение стремящейся к нулю при стремлении на плюс бесконечность одной из независимой переменной. В настоящее время теория дифференциальных уравнений с частными производными находит свое применение в различных областях естествознании.

Ключевые слова: уравнение, частные производные первого порядка.

1 Introduction

The Cauchy problem for a nonlinear partial differential equation of the first order solved with respect to one of the derivatives, as is well known, under certain conditions has a unique solution in a small neighborhood. The paper deals with a nonlinear partial differential equation of the first order solved with respect to one of the derivatives, and the solution of the Cauchy problem is assumed extending to the right to plus infinity. A nonlinear differential equation with first-order partial derivatives solved with respect to one of the derivatives was estimated from two sides by partial differential equations of the first order, the behavior of the solutions of which are known. Using differential inequalities, the asymptotic behavior of the solution of a first-order partial differential equation solved with respect to one of the derivatives was studied and was proved that the nonlinear differential equation with first-order partial derivatives solved with respect to one of the derivatives has a solution that tends to zero while one of the independent variables tends to plus infinity.

2 Literature review

The general theory is presented in the books [1-10]. The domain of existence of solutions was investigated by Kamke and data is contained in the reference books [11, 12]. The domain of existence of solutions was investigated in the works [13-15]. Non-analytic equations are considered in the papers[16-18]. In work of Kruzhkov generalized solutions was considered [19]. Kovalevskaya's theorem was published in [20-22]. An example of nonexistence of a solution constructed in [23-26]. Differential inequalities are considered by Nagumo [27-29].

3 Materials and research methods

Let us consider a nonlinear partial differential equation of the first order with n + 1 independent variables solved with respect to one of the derivatives

$$\frac{\partial u}{\partial t} + H\left(u, t, y_1, \ldots, y_n, \frac{\partial u}{\partial y_1}, \ldots, \frac{\partial u}{\partial y_n}\right) = 0 \tag{1}$$

where
$H\left(u, t, y_1, \ldots, y_n, \frac{\partial u}{\partial y_1}, \ldots, \frac{\partial u}{\partial y_n}\right) = \left[\frac{1}{2}\sum\limits_{k=1}^{n} p_{1k}(t)y_k + f\left(u, t, y_1, \ldots, y_n, \frac{\partial u}{\partial y_1}, \ldots, \frac{\partial u}{\partial y_n}\right)\right]\frac{\partial u}{\partial y_1} + \left[\sum\limits_{k=1}^{n} p_{2k}(t)y_k\right]\frac{\partial u}{\partial y_2} + \ldots + \left[\sum\limits_{k=1}^{n} p_{nk}(t)y_k\right]\frac{\partial u}{\partial y_n}$

$$u(o, y_1, \ldots, y_n) = \varphi(y_1, \ldots, y_n). \tag{2}$$

We define the (t, y) set B as follows

$$B = \{(t, y) : 0 \le t < +\infty,\ c_k - L_k t \le y \le d_k + L_k t, k = 1,\ \ldots, n\}$$

where $L_k > 0$, $c_k < 0 < d_k$. Function $H(u, t, y, q)$ is defined in $E \subseteq \mathbb{R}^{2+2n}$, whose projection onto the $(t, y)-$space contains B. $(\varphi(0), 0, 0, \varphi_y(0)) \in E$ and $\varphi_y \in C^2$. The problem (1), (2) for small $|t|$, $\|y\|$, has a unique solution $u(t, y)$ of class C^2.[12, p.173]. We take sufficiently small $|t_0|$, $\|y_0\|$, where $(t_0, y_0) \in B,\ t_0 > 0$ and we assume that the solution $u(t, y)$ satisfying the condition (2) defining on the point $(t_0, y_0) \in B$ and continuing on$t > t_0$. For definiteness, we denote this solution of equation (1) with $u(t, y; t_0, y_0)$.

Theorem 1. Suppose that the following conditions hold on the set $E \in \mathbb{R}^{2+2n}$ whose projection onto the $(x, y)-$space contains B:

A) $|H\left(u, t, y_1, \ldots, y_n, \frac{\partial u}{\partial \bar{y}_1}, \ldots, \frac{\partial u}{\partial \bar{y}_n}\right) - H\left(u, t, y_1, \ldots, y_n, \frac{\partial u}{\partial \bar{\bar{y}}_1}, \ldots, \frac{\partial u}{\partial \bar{\bar{y}}_n}\right)| \le \sum\limits_{k=1}^{n} L_k |\frac{\partial u}{\partial \bar{y}} - \frac{\partial u}{\partial \bar{\bar{y}}}|$;

B) The inequality is fulfilled:

$$f\left(u, t, y_1, \ldots, y_n, \frac{\partial u}{\partial y_1}, \ldots, \frac{\partial u}{\partial y_n}\right) < \frac{1}{2}\sum_{k=1}^{n} p_{1k}(t)y_k;$$

$p_{ik}(t) \in \mathbb{C}^2(I),\ i = 1, \ldots, n,\ k = 1, \ldots, n,\ I \equiv [0, +\infty)$ satisfy next conditions:

$a_1) p_{k-1,k-1}(t) - p_{kk}(t) \ge \alpha_1\psi(t),\ t \in I,\ k = 2, \ldots, n.\ \alpha_1 > 0$,

$\psi(t) \in \mathbb{C}(I),\ \psi(t) > 0,\ \int\limits_{t_0}^{+\infty} \psi(s)ds = +\infty$;

$b_1) \lim\limits_{t\to+\infty} \frac{|p_{ik}(t)|}{\psi(t)} = 0,\ i \ne k,\ i = 1, 2, \ldots, n,\ k = 1, 2, \ldots, n$;

$c_1) \lim\limits_{t\to+\infty} \frac{1}{\nu(t)} \int\limits_{t_0}^{t} (-p_{kk}(s))ds = \beta_k,\ k = 1, 2, \ldots, n$. Where $\nu = \int\limits_{t_0}^{t} \psi(s)ds \uparrow +\infty$ and the inequality performs: $\beta_1 < 0$;

M) next inequalities are true:

$p_{ik}(t) \ge b_{ik}(t),\ b_{ik}(t) \in \mathbb{C}^2(I),\ i = 1, \ldots, n,\ k = 1, \ldots, n,$ where $b_{ik}(t)\, i = 1, \ldots, n,\ k =$

$1, \ldots, n$ satisfy next conditions:
$a_2) b_{k-1,k-1}(t) - b_{kk}(t) \geq \alpha_2 \psi(t),\ t \in I,\ k = 2, \ldots, n.\ \alpha_2 > 0,$
$b_2) \lim\limits_{t\to+\infty} \frac{|b_{ik}(t)|}{\psi(t)} = 0,\ i \neq k,\ i = 1, 2, \ldots, n,\ k = 1, 2, \ldots, n;$
$c_2) \lim\limits_{t\to+\infty} \frac{1}{\nu(t)} \int\limits_{t_0}^{t} b_{kk}(s) ds = \mu_1,\ k = 1, 2, \ldots, n.$ Where $\mu_1 < 0$.
Inequality is true:

$$f(u, t, y, q) \geq h(u, t, y)$$

Where $h \in \mathbb{C}^2(D)$, $D \in E$, $t \in [t_0, +\infty)$, $|h(\theta, t, y)| \leq \delta(t)\|y\|$ and

$$\delta(t) \in \mathbb{C}(I),\ \lim_{t\to+\infty} \frac{\delta(t)}{\psi(t)} = 0;$$

Then for the solution $u(t, y; t_0, y_0)$ of equation (1) there exists a limit

$$\lim_{t\to+\infty} u(t, y; t_0, y_0) = 0$$

Proof. On the set $E \in \mathbb{R}^{2+2n}$ whose projection onto $(x, y)-$ space contains B, we consider the equation

$$\frac{\partial\vartheta}{\partial t} - f_1\left(\vartheta, t, y, \frac{\partial\vartheta}{\partial y}\right) = 0 \tag{3}$$

where $f_1\left(\vartheta, y, t, \frac{\partial\vartheta}{\partial y}\right) = \left(\sum\limits_{k=1}^{n} p_{1k}(t) y_k\right) \frac{\partial\vartheta}{\partial y_1} + \left(\sum\limits_{k=1}^{n} p_{2k}(t) y_k\right) \frac{\partial\vartheta}{\partial y_2} + \ldots + \left(\sum\limits_{k=1}^{n} p_{nk}(t) y_k\right) \frac{\partial\vartheta}{\partial y_n}$.
For equation (3), the characteristic system of differential equations has the form

$$\frac{dy_1}{dt} = -\sum_{k=1}^{n} p_{1k}(t) y_k,\ \frac{dy_i}{dt} = -\sum_{k=1}^{n} p_{ik}(t) y_k\ i = 2, \ldots, n. \tag{4}$$

The characteristic system (4) is considered for initial values

$$y_k|_{t=t_0} = \bar{y}_k^0,\ k = 1, \ldots, n.$$

The solution of the characteristic system (4) exists

$$y_k = \varphi_k(t, t_0, \bar{y}_1^0, \ldots, \bar{y}_n^0),\ k = 1, \ldots, n \tag{5}$$

for arbitrary initial values $\bar{y}_k^0, (k = 1, \ldots, n)$.
Let $(t_0, \bar{y}_k^0) \in B$. (5) are solvable with respect to $\bar{y}_1^0, \ldots, \bar{y}_n^0$ and holds

$$\bar{y}_k^0 = \varphi_k(t_0, t, y_1, \ldots, y_n),\ k = 1, \ldots, n \tag{6}$$

(6) forms an integral basis of equation (3). By B), inequality

$$\vartheta_t > H(u, t, y, \vartheta_y)$$

Indeed
$\vartheta_t - H(u, t, y, \vartheta_y) = \left(\sum\limits_{k=1}^{n} p_{1k}(t) y_k\right) \frac{\partial\vartheta}{\partial y_1} + \left(\sum\limits_{k=1}^{n} p_{2k}(t) y_k\right) \frac{\partial\vartheta}{\partial y_2} + \ldots + \left(\sum\limits_{k=1}^{n} p_{nk}(t) y_k\right) \frac{\partial\vartheta}{\partial y_n} -$

$\left(\frac{1}{2}\sum\limits_{k=1}^{n} p_{1k}(t)y_k + f(\nu,t,y,\vartheta_y)\right)\frac{\partial\vartheta}{\partial y_1} - \left(\sum\limits_{k=1}^{n} p_{2k}(t)y_k\right)\frac{\partial\vartheta}{\partial y_2} - \ldots - \left(\sum\limits_{k=1}^{n} p_{nk}(t)y_k\right)\frac{\partial\vartheta}{\partial y_n} =$
$\left(\frac{1}{2}\sum\limits_{k=1}^{n} p_{1k}(t)y_k - f(\nu,t,y,\vartheta_y)\right)\frac{\partial\vartheta}{\partial y_1} > 0$
Let $\vartheta(t_0,t,y) = \varphi_k(t_0,t,y_1,...,y_n), k \in 1,...,n$ be a solution of the equation $(\vartheta,t,y,\vartheta_y) \in E$ satisfying the condition $\vartheta(t_0,t_0,y) = \omega_1(y)$, $\omega_1(y) \in C^2$ and such that $\omega_1(y) > \varphi(y)$. Then everywhere on B next inequality is true:

$$\vartheta(t_0,t,y) > u(t,y,t_0,y_0),\ t \in I. \tag{7}$$

In fact, if inequality (7) is not true, then there is a point $(t_1,y_1) \in B$, where $t_1 > t_0$ is such that inequality (7) is true (t_0,t_1), and at (t_1,y_1) will have equality, i.e. $\vartheta(t_0,t_1,y_1) = u(t_1,y_1;t_0,y_0)$. Integrating equations (1) and (3), we obtain

$$u(t_1,y_1;t_0,y_0) - u(t_0,y_0;t_0,y_0) + \int\limits_{t_0}^{t_1} H ds = 0$$

and

$$\vartheta(t_0,t_1,y_1) - \vartheta(t_0,t_0,y_0) - \int\limits_{t_0}^{t_1} f_1 ds = 0$$

This implies

$$u(t_0,y_0;t_0,y_0) + \int\limits_{t_0}^{t_1}(-H - f_1)ds - \vartheta(t_0,t_0,y_0) = 0$$

The difference $\vartheta - u$ is positive for $t_0 < t < t_1$ and is zero for $t = t_1$. Hence, the derivative of the difference $\vartheta - u$ at the point $t = t_1$ is nonpositive, i.e. $((\vartheta - u)_t)_{t=t_1} = 0$, then this implies the inequality

$$u(t_0,y_0;t_0,y_0) - \vartheta(t_0,t_0,y_0) \geq 0.$$

This contradicts the inequality $\omega_1(y) > f(y)$. Consequently, we have (7). Consider the equation

$$\frac{\partial\theta}{\partial t} - f_2\left(\theta,t,y,\frac{\partial\theta}{\partial y}\right) = 0 \tag{8}$$

where
$f_2\left(\theta,t,y,\frac{\partial\theta}{\partial y}\right) = \left(\sum\limits_{k=1}^{n} b_{1k}(t)y_k + h(\theta,t,y)\right)\frac{\partial\theta}{\partial y_1} + \left(\sum\limits_{k=1}^{n} b_{2k}(t)y_k\right)\frac{\partial\theta}{\partial y_2} + \ldots + \left(\sum\limits_{k=1}^{n} b_{nk}(t)y_k\right)\frac{\partial\theta}{\partial y_n}.$
For the equation (8), the characteristic system has the form

$$\frac{dy_1}{dt} = -\left(\sum_{k=1}^{n} b_{1k}(t)y_k + h(\theta,t,y)\right),\ \frac{dy_i}{dt} = -\sum_{k=1}^{n} b_{ik}(t)y_k\ i = 2,\ldots,n.$$

Let

$$y_i = \theta_i(t,t_0,\bar{\bar{y}}_1^0,\ldots,\bar{\bar{y}}_n^0),\ i = 1,\ldots,n;\ t \in (t_0,+\infty)$$

be a solution of the characteristic system, where $(t_0, \bar{\bar{y}}_0) \in B$. This system of solutions is solvable with respect to $\bar{\bar{y}}_1^0, \dots, \bar{\bar{y}}_n^0$, therefore

$$\theta_i(t_0, t, y) = \theta_i(t_0, t, y_1, ..., y_n), k \in 1, ..., n$$

forms an integral basis of equation (8), for which in the whole domain the functional determinant

$$\frac{\partial(\theta_1, \dots, \theta_n)}{\partial(y_1, \dots, y_n)} > 0$$

By condition M), inequality

$$\theta_t \le H(\theta, t, y, \theta_y).$$

Indeed,
$\theta_t - H(\theta, t, y, \theta_y) = \left(\sum_{k=1}^n b_{1k}(t)y_k + h(\theta, t, y)\right) \frac{\partial\theta}{\partial y_1} + \left(\sum_{k=1}^n b_{2k}(t)y_k\right) \frac{\partial\theta}{\partial y_2} + \dots + \left(\sum_{k=1}^n b_{nk}(t)y_k\right) \frac{\partial\theta}{\partial y_n} - \left(\sum_{k=1}^n p_{1k}(t)y_k + f(\theta, t, \theta, \theta_y)\right) \frac{\partial\theta}{\partial y_1} - \left(\sum_{k=1}^n p_{2k}(t)y_k\right) \frac{\partial\theta}{\partial y_2} - \dots - \left(\sum_{k=1}^n p_{nk}(t)y_k\right) \frac{\partial\theta}{\partial y_n} = \left(\sum_{k=1}^n (b_{1k}(t) - p_{1k}(t))y_k + h(\theta, t, y) - f(\theta, t, \theta, \theta_y) +\right) \frac{\partial\theta}{\partial y_1} + \left(\sum_{k=1}^n (b_{2k}(t) - p_{2k}(t))y_k\right) \frac{\partial\theta}{\partial y_2} + \dots + \left(\sum_{k=1}^n (b_{nk}(t) - p_{nk}(t))y_k\right) \frac{\partial\theta}{\partial y_n} \le 0.$
Let $\theta(t_0, t, y) = \theta_i(t_0, t, y), i \in 1, \dots, n$ be a solution of the equation (8) $(\theta, t, y, \theta_y) \in E$ satisfying the condition $\theta(t_0, t_0, y) = \omega_2(y)$, where $\omega_2(y) \in \mathbb{C}^2$ and such that $\omega_2(y) < f(y)$. We have the inequality

$$u(t, y, t_0, y_0) > \theta(t_0, t, y) \ t \in I. \tag{9}$$

Indeed, if inequality (9) is not true, then there is a point $(t_1, y_1) \in B$, where $t_1 > t_0$ is such that inequality (9) is true in the interval (t_0, t_1), and at (t_1, y_1) have the equality $u(t_1, y_1; t_0, y_0) = \theta(t_0, t_1, y)$. Integrating equations (1) and (8), we obtain

$$u(t_1, y_1; t_0, y_0) - u(t_0, y_0; t_0, y_0) + \int_{t_0}^{t_1} H ds = 0$$

and

$$\theta(t_0, t_1, y_1) - \theta(t_0, t_0, y_0) - \int_{t_0}^{t_1} f_2 ds = 0$$

This implies

$$-u(t_0, y_0; t_0, y_0) + \int_{t_0}^{t_1} (H + f_2) ds + \theta(t_0, t_0, y) = 0$$

The difference $u - \theta$ is positive for $t_0 < t < t_1$ and is zero for $t = t_1$. Therefore, the derivative of the difference $u - \theta$ at the point $t = t_1$ is nonpositive, i.e. we have the inequality

$((u-\theta)_t)_{t=t_1} \le 0$. From this and the inequality $\theta_t = H(\theta, t, y, \theta_y)$ it follows that equality $((u-\theta)_t)_{t=t_1} = 0$. Then equality

$$-u(t_0, y_0; t_0, y_0) + \theta(t_0, t_0, y)ds = 0$$

This contradicts the inequality $\omega_2(y) < f(y)$. Therefore, (9) holds. By assumption, conditions A) and the integrals $\vartheta(t_0, t, y), \theta(t_0, t, y)$ of equations (3), (8) with initial values $\vartheta(t_0, t_0, y) = \omega_1(y)$, $\theta(t_0, t_0, y) = \omega_2(y)$ belong to B classC^1 and satisfy the following conditions:
1)$(\vartheta, t, y, \vartheta_y) \in E, (\theta, t, y, \theta_y) \in E$;
2)$\vartheta_t > H(u, t, y, \vartheta_y)$, $\theta_t = H(\theta, t, y, \theta_y) inE$;
3)$\omega_1(y) > f(y) > \omega_2(y)$ Everywhere on B, inequality

$$\vartheta(t_0, t, y) > u(t, y; t_0, y_0) > \theta(t_0, t, y),\ t \in I \tag{10}$$

By condition a_1, b_1, c_1 of B), the characteristic system (4) has a generalized upper central exponent equal to $\beta_1 < 0$. Therefore system (4) is asymptotically stable in the sense of Lyapunov as $t \to +\infty$. From which it follows that

$$\lim_{t_0 \to +\infty} \vartheta(t_0, t, y) = 0,\ \ t > t_0 \tag{11}$$

A linear homogeneous system of differential equations

$$\frac{dy_i}{dt} = -\sum_{k=1}^{n} b_{ik}(t) y_k, i = 1, \dots, n.$$

due to the condition a_2, b_2, c_2 of M) has a generalized upper central exponent equal to $\mu_1 < 0$. Therefore, the system is asymptotically stable in the sense of Lyapunov on $t \to +\infty$. Moreover, system (8)

$$\frac{dy_1}{dt} = -\left(\sum_{k=1}^{n} b_{1k}(t) y_k + h(\theta, t, y)\right),\ \frac{dy_i}{dt} = -\sum_{k=1}^{n} b_{ik}(t) y_k,\ i = 2, \dots, n.$$

by the condition on $h(\theta, t, y)$ has an asymptotically stable zero solution. Hence we will have

$$\lim_{t_0 \to +\infty} \theta(t_0, t, y) = 0,\ \ t > t_0 \tag{12}$$

Consequently, it follows from (10), (11), (12) that the solution $u(t, y; t_0, y_0)$ of equation (1) has a limit

$$\lim_{t \to +\infty} u(t, y; t_0, y_0) = 0.$$

The theorem is proved.

4 Results and discussion

The paper deals with a nonlinear differential equation with partial derivatives of the first order, solved with respect to one of the derivatives and the asymptotic behavior of the solution. Using differential inequalities it is proved that a nonlinear differential equation with first-order partial derivatives solved with respect to one of the derivatives has a solution tending to zero as one of the independent variables tending to infinity.

5 Conclusion

A condition for a nonlinear differential equation with first-order partial derivatives solved with respect to one of the derivatives was found, for which the equation has a solution that tends to zero as one of the independent variables tends to infinity.

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