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On the existence of a conditionally periodic solution of one quasilinear differential system in the critical case

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In the theory of nonlinear oscillations one often encounters conditionally periodic oscillations resulting from the superposition of several oscillations with frequencies incommensurable with each other. When finding a solution to a resonant quasilinear differential system in the form of a conditionally periodic function, the problem of a small denominator arises. Consequently, the proof of the existence, and even more the construction of such a solution is not an easy task. In this article, drawing on the work of VI. Arnold, I. Moser, and other researchers proved the existence and constructed a conditionally periodic solution of a second-order quasilinear differential system in the critical case. Accelerated convergence method by N.N. Bogolyubova, Yu.A. Mitropolsky, A.M. Samoylenko. The result can be applied to construct a conditionally periodic solution of specific differential systems.
Key words: conditionally periodic, accelerated convergence, frequency, resonance.

Квазисызықтық дифференциалдық жүйенің сындарлы жағдайдағы шартты-периодты шешімінің бар болуы

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Сызықтық емес тербелістер теориясында жиілігі өлшемдес емес бірнеше тербелістердің қабаттасып келуі нәтижесінде пайда болатын шартты-периодты тербелістермен жиі кездесуге тура келеді. Осындай резонанстық жағдайдағы квазисызықтық жүйенің шартты-периодты шешімін табу үдерісі "кішкене бөлім"мәселесін туындатады. Бұл мәселе шешімнің бар болуын дәлелдеу мен оны құру есебін қиындата түседі. Біздің ұсынып отырған мақаламызда В.И. Арнольдтің, И. Мозердің және басқа да зерттеушілердің жұмыстары негізінде екінші ретті бір сындарлық жағдайдағы квазисызықтың дифференциалдық жүйенің шартты периодты шешімінің бар болатыны дәлелденіп, оны құру жолы көрсетілді. Шешімді құру барысындағы жуықтау тізбегі Боголюбов, Ю.А. Митропольский, А.М. Самойленколар ұсынған үдемелі әдіске сүйеніп берілді. Жұмыстың нәтижесін нақты дифференциалдық жүйелердің шартты-периодты шешімдерін құру үшін пайдалануға болады.
Түйін сөздер: шартты-периодты, үдемелі жинақтылық, жиілік, резонанс.

О существовании условно-периодического решения одной квазилинейной дифференциальной системы в критическом случае

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В теории нелинейных колебаний приходится часто встречаться с условно-периодическими колебаниями, возникающими в результате наложения нескольких колебаний с несоизмеримыми между собой частотами. При отыскании решения резонансной квазилинейной дифференциальной системы в виде условно-периодической функции возникает проблема малого знаменателя. Вследствие этого, доказательство существования, а тем более построения такого решения является нелегкой задачей. В данной статье опираясь на работы В.И. Арнольда, И. Мозера и других исследователей доказано существование и построено условно-периодическое решение одной квазилинейной дифференциальной системы второго порядка в критическом случае. Методом построения последовательности приближения выбран метод ускоренный сходимости Н.Н. Боголюбова, Ю.А. Митропольского, А.М. Самойленко. Результат может быть применен для построения условно-периодического решения конкретных дифференциальных систем.
Ключевые слова: условно-периодическое, ускоренная сходимость, частота, резонанс.

1 Introduction

Conditionally periodic functions are functions that are representable by trigonometric polynomials or series of the form

$$\sum_{|k_1|+...+|k_n|\geq 0} A^{(k_1...,k_n)}\cos(k_1w_1+...,+k_nw_n)t+B^{(k_1...,k_n)}\sin(k_1w_1+...+k_nw_n)t$$

where t – argument, the summation is over all possible integer values $k_1, k_2..., k_n$; numbers $w_1, w_2, ..., w_n$ – fixed real, and rationally incommensurable numbers i.e. such that for any integers $k_1, k_2..., k_n$

$$k_1w_1+k_2w_2..., k_nw_n\neq 0$$

$A^{(k_1...,k_n)}$, $B^{(k_1,...,k_n)}$–constant coefficients, от $k_1, k_2, ..., k_n$ Set of numbers $w_1, w_2, ..., w_n$ is called the frequency basis or the frequency spectrum of the conditionally periodic function. It is characteristic that the frequency basis of the conditionally periodic function is finite.

The name "conditionally - periodic"was introduced by O. Staude.

The conditionally periodic function is often written for the convenience of the operations used in a complex form

$$\sum A^{(k_1,...,k_n)}e^{i(k_1w_1+...+k_nw_n)t}$$

If termwise integrating a conditionally periodic function written in the form of the indicated series without a free term, then this series may converge or may diverge. With convergence, we obtain a conditionally periodic function. If the series diverges, then one cannot integrate term by term.

Based on the ideas of A.N. Kolmogorov, V.I. Arnold managed to beat this difficulty and achieve serious results. In the present paper, we have produced a proof of the existence of a conditionally periodic solution of one quasilinear system.

2 Literature review

As already noted in celestial mechanics, nonlinear oscillations are often encountered, expressed by conditionally periodic functions. But the problem of a small denominator, arising as a result of the integration of such functions, made it difficult to carry out deep and diverse studies of the existence and construction of the conditionally periodic solution of differential equations. At the same time, KL's estimates were obtained in the theory of real numbers. Seidel [2], under which integrated conditionally-periodic functions are admissible: for the majority (in the sense of Lebesgue measure) frequencies belonging to the region n of the dimensional number space Ω with a bounded sum of modules of any n numbers.

The original literature is the works of A.N. Kolmogorov [1], V.I. Arnold [3-4], who developed a method of proving the existence and built conditionally periodic solutions of Hamiltonian systems, I. Moser [5-7], who considered a common system characteristic of the problems of the theory of nonlinear oscillations and proved the existence of conditionally periodic solutions with a smaller number of basic frequencies than the number of degrees of freedom of the system. G.A. Merman [9] investigated the divergence of conditionally periodic series in powers of a small parameter. Yu.A. Ryabov, E.A. Grebennikov, L.K. Lika [10-11] addressed the issues of constructing conditionally-periodic solutions of canonical systems.

The method of constructing a sequence of approximate solutions was chosen by the method of accelerated convergence N.N. Bogolyubova, Yu.A. Mitropolsky, A.M. Samoylenko [12].

In recent years, A. Bari, H. Brezis, E. Feireist, H.P. have been investigating the existence of periodic and conditionally periodic solutions of nonlinear differential equations. Pelyukh, Suvak O.A. [13-15].

3 Materials and methods

Take quasi-linear system of differential equations

$$\frac{dx}{dt} = Ax + \varepsilon f(t,x), \quad (1)$$

Where

$$x = colon(x_1, x_2),\ A = (a_{jk}),\ j = k = 1,2,\ f(t,x) = colon(f_1(t,x_1,x_2), f_2(t,x_1,x_2))$$

conditionally-periodic by t with frequency basis $\omega_1, \omega_2, ..., \omega_n$; analytical by t and x in the domain $= \{(t,x) \subset C^3 :\ \|x\| \le h,\ \|Im\omega t\| \le q\}$ function, $\det|A - \lambda E| = 0$ has purely imaginary roots $i\sigma_1, i\sigma_2$, and rational numbers σ_1, σ_2 non-co-measurable with $\omega_1, \omega_2, ..., \omega_n$, $\varepsilon-$is a small parameter.

Let $S-$be a matrix, making the matrix A to Jordan form:

$$J = \begin{pmatrix} \sigma_1 i & 0 \\ 0 & \sigma_2 i \end{pmatrix}.$$

With conversion$x = Sy$, of the system (1) reduces to the form

$$\frac{dy}{dt} = Jx + S^{-1}\varepsilon f(t, Sy) \quad (2)$$

Without loss of generality, can assume that the system (1) has the form (2), i.e. A has the form:$J := diag(\sigma_1 i, \sigma_2 i)$.

3.1 The method of successive approximations

In order to find a conditionally-periodic solutions of the method of accelerated convergence [12]is applied. As an initial approximate conditionally-periodic solutions of the system

(1)$x^{(0)}(t,\varepsilon)=0:=colon(0;0)$is chosen. Its residual denoted by $x^{(1)}(t,\varepsilon)$ and take this function as a first approximation to the original conditionally-periodic solutions of the system (1).Then the system relatively to $x^{(1)}(t,\varepsilon)$ will look like:

$$\frac{dx^{(1)}(t,\varepsilon)}{dt}=(J+\varepsilon \mathrm{P}^{(0)}(t))x^{(1)}(t,\varepsilon)+\varepsilon\chi^{(1)}(t,x^{(0)}),$$

where $\mathrm{P}^{(0)}(t) := f'_x(t,0) := \left(\frac{\partial f_j}{\partial x_k}\right)_{(t;0)}$, $j,k=1,2;$ $\chi^1(t,x^{(0)}) := f(t,0)$.Amendment to $x^{(1)}(t,\varepsilon)$ denote as $y^{(1)}(t,\varepsilon)=colon(y_1^{(1)}(t,\varepsilon),y_2^{(1)}(t,\varepsilon))$. Then it has the system:

$$\frac{dy^{(1)}(t,\varepsilon)}{dt}=(J+\varepsilon f^1_x(t,x^{(1)})y^{(1)}(t,\varepsilon)+\varepsilon Y^{(1)}(t,x^{(1)}), \quad (3)$$

where $Y^{(1)}(t,x^{(1)}) := f(t,x^{(1)})-f(t,0)-f'_x(t,0)x^{(1)}$.

The second approximation is determined by the formula $x^{(2)}(t,\varepsilon) := x^{(1)}(t,\varepsilon)+y^{(1)}(t,\varepsilon)$, and
the amendment is denoted by $y^{(2)}(t,\varepsilon)$ etc. Then to determine$x^{(j)}(t,\varepsilon) := colon(x_1^{(j)}(t,\varepsilon),x_2^{(j)}(t,\varepsilon))$ and $y^{(j)}(t,\varepsilon) := colon(y_1^{(j)}(t,\varepsilon),y_2^{(j)}(t,\varepsilon))$, $j=1,2,...,$ the following system of equations is obtained

$$\frac{dx^{(j)}(t,\varepsilon)}{dt}=(J+\varepsilon \mathrm{P}^{(j-1)}(t))x^{(j)}+\varepsilon\chi^{(j)}(t,x^{(j-1)}), \quad (4)$$

$$\frac{dy^{(j)}(t,\varepsilon)}{dt}=(J+\varepsilon P^{(j)}(t))y^{(j)}+\varepsilon Y^{(j)}(t,x^{(j-1)},y^{(j-1)}), \quad (5)$$

where $P^{(j-1)},\chi^{(j)},Y^{(j)},j=2,3,...-$ are defined similarly to $P^{(0)},\chi^{(1)},Y^{(1)}$.

3.2 Integrating Model Equation

The systems (4) and (5) have the same structure and are linear non-homogeneous systems of the form:

$$\frac{dz}{dt}=(J+\varepsilon \mathrm{P}(t))z+\varepsilon q(t) \quad (6)$$

where $J=diag(\sigma_1,\sigma_2)$, $P(t)=(p_{jk}(t))$, $j,k=1,2;$ $q(t) := colon(q_1(t),q_2(t))$. Matrix $P(t)$ and vector-function $q(t)$are considered to be analytical and conditionally periodic by t with frequency basis $\omega_1,\omega_2,...,\omega_n$. Let $B=(b^1,b^2)=\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22}\end{pmatrix}$ be a constant, and$R(t)=(r^1(t) \;\; r^2(t))$ is purely conditionally-periodic parts of the matrix $P(t)$, satisfying the equation $P(t)=B+R(t)$. Denote$T(t) := \int R(t)dt$ and let $TR=RT,\ BT=TB$. Making a replacement in the equation(6)with $z=e^{\varepsilon T(t)}\vartheta$, have

$$\frac{d\vartheta}{dt} = (J + \varepsilon B)\vartheta + \varepsilon g(t), \quad (7)$$

where $g(t) := \exp(-\varepsilon T(t))q(t),\ g(t) := colon(g_1(t), g_2(t))$.

Let $g(t)$ be the following

$$g(t) = \sum_{\|k\| \geq 0} C^k \exp(i(k,\omega)t), \quad (8)$$

where$k := (k_1, ..., k_n),\ \omega = (\omega_1, ..., \omega_n),\ \|k\| := |k_1| + ... + |k_n|,\ C^k := colon(C_1^k, C_2^k),\ k\omega := k_1\omega_1 + ... + k_n\omega_n$.

Conditionally-periodic solution of the system (7) is defined by the same formof series

$$\vartheta(t,\varepsilon) = \sum_{\|k\| \geq 0} d^k \exp(i(k,\omega)t), \quad (9)$$

where $d^k = colon(d_1^k, d_2^k)-$undetermined coefficients. Using (9) in the system (7) considering(8) the
formal solution of the system(7)is the following :

$$\vartheta(t,\varepsilon) = \sum_{\|k\| \geq 0} \varepsilon \begin{bmatrix} i((k,\omega) - \sigma_1) - \varepsilon b_{11};\ -\varepsilon b_{12} \\ -\varepsilon b_{21};\ i((k,\omega) - \sigma_2) - \varepsilon b_{22} \end{bmatrix}^{-1} C^k \exp(i(k,\omega)t). \quad (10)$$

It is actual conditionally-periodic solution if the series (10)converges uniformly by t.

Let: $k_{n+1} := -1,\ \omega_{n+1} := \sigma_1,\ k_{n+2} := -1,\ \omega_{n+2} := \sigma_2,\quad k^* := (k_1, ..., k_{n+2}),\ \omega^* := (\omega_1, ..., \omega_{n+2}),\quad (k^*, \omega^*) := k_1\omega_1 + ... + k_n\omega_n + k_{n+2}\omega_{n+2}$.

Then for most of the frequencies (meaning Lebega) $\{\omega_1, ..., \omega_n, \omega_{n+2}\} \subset Q^*$ and for integers$k_1, ..., k_{n+2}$ the following estimation is true:

$$|(k^*, \omega^*)| \geq K\left(\|k\| + 2\right)^{-(n+2)}, \quad (11)$$

where $K \succ 0-$is some fixed number smaller than [1] :

In the strip $\|Jm\omega t\| \leq q$ lets define $M_0 := \|q(t)\|_0 = \sup_t \|q(t)\|,\quad N_0 := \|R(t)\|_0 = \sup_t \|R(t)\|$, lets define $\|\cdot\|_0$is the normof the strip $\|Jm\omega t\| \leq q$.

Then for the coefficients of the series $r^m(t) = \sum_{\|k\| \geq 0} \rho^{mk} e^{i(k,\omega)t},\quad \rho^{mk} := colon(\rho_1^{mk}, \rho_2^{mk}),\quad m = 1, 2$; and the following estimation is true: $\left\|\rho^{mk}\right\|_0 \leq N_0 e^{-\|k\|q},\ m = 1, 2$.

Therefore, for $T(t)$ in the strip $\|Jm\omega t\| \leq q - 2\delta_1;\ 2\delta_1 \leq q$ there is $T(t) = \sum_{\|k\| \geq 0} \frac{1}{i(k,\omega)} \rho^{mk} e^{i(k,\omega)t},\quad \|T(t)\|_1 \leq \frac{QN_0}{\delta_1^{2n}},\ Q := \frac{1}{K}\left(\frac{4n}{e}\right)^n,\ \forall\omega \in \bar{G}$,where $\bar{G}-$ is a set where the following satisfies $|(k,\omega)| \geq K\|k\|^{-n}$,where $\|\cdot\|_1$ is the norm of the strip $\|Jm\omega t\| \leq q-2\delta_1$. In general, $\|\cdot\|_j$is taken as the norm of the strip $\|Jm\omega t\| \leq q-2(\delta_1+\cdots+\delta_j)$. Then $\|g(t)\|_1 \leq M_0 e^{\varepsilon N_0 Q \delta_1^{-2n}} =: M_1,\ \left\|C^k\right\| \leq M_1 e^{-\|k\|(q-2\delta_1)}$.

Therefore in the strip $\|Jm\omega t\| \leq q-2\delta_1-2\delta_2,\ 2\delta_2 \prec q-2\delta_1$ for conditionally-periodic solution of the system (6) have

$$\|x(t,\varepsilon)\|_2 \leq \frac{\varepsilon M_1}{\sqrt{2}\delta_2^{2n+1}}(1+Q_1)\exp(2\varepsilon N_0 Q\delta_1^{-2n}),\ Q_1 := \frac{1}{4K}\left(\frac{4(n+2)}{e}\right)^{n+2}.$$

Lets prove convergence of the series of the estimated solutions $x^{(j)}(t,\varepsilon)$ to solution of system (1).

Return to equations (4) and (5). Let in equation (2)matrix $P^{(0)}(t) := f'_x(t,0)$ has a form:

$$P^{(0)}(t) = B^{(0)} + iC^{(0)} + R^{(0)}(t),\ B^{(0)} := (b_{lm}),\ C^{(0)} := (c_{lm}),\ l,m=1,2;$$

where $R^{(0)}(t)-$ pure conditionaly-periodic part of matrix $P^{(0)}(t)$. Then

$$\left\|x^{(1)}(t,\varepsilon)\right\| \leq \frac{\varepsilon M}{\sqrt{2}\delta_2^{2n+1}}(1+Q_1)\exp(2\varepsilon N_0 Q\delta_1^{-2n})$$

where $N_0 := \left\|R^{(0)}(t)\right\|_0$. If $x^{(1)}(t,\varepsilon)$ stays in the domain D, then $f(t,x^{(1)})$ and $Y^{(1)}(t,x^{(1)})$ also

will be analytical by t and $x^{(1)}$. If $N_1 := \left\|R^{(1)}(t)\right\|,\ R^{(1)}(t)-$ is the pure conditionally- periodic part of matrix $P^{(1)}(t)$, then

$$\left\|y^{(1)}(t,\varepsilon)\right\| \leq \frac{\varepsilon M_2}{\sqrt{2}\delta_4^{2n+1}}(1+Q_1)\exp(2\varepsilon N_1 Q\delta_3^{-2n})$$

For $\omega = (\omega_1,...,\omega_{n+1},\omega_{n+2})$, $\quad \omega_{n+i} := \sigma_i + \varepsilon(c_{11}^{(i)} + c_{22}^{(i)}),\ i=1,2$, satisfying the inequality (11), where $c_{lm}^{(1)}$ from the following expansion

$$P^{(1)}(t) = B^{(1)} + iC^{(1)} + R^{(1)}(t),\ B^{(1)} := (b_{lm}^{(1)}),\ C^{(1)} := (c_{lm}^{(1)}),\ R^{(1)} := (r_{lm}^{(1)}),\ l,m=1,2.$$

Similar estimation is obtained for $y^{(j)}(t,\varepsilon),\ j \geq 2$.

Let in the region $D_{the following conditions hold}$:

$$\|f'_x(t,x)\| \leq \frac{1}{2}L,\ \|f''_{x^2}(t,x)\| \leq 2P,\ f''_{x^2} := \left(\frac{\partial^2 f_l}{\partial x_m^2}\right),\ m,l=1,2.$$

Then under the condition that all the approximations $x^{(j)}(t,\varepsilon)$ stay in the region $\|x\| \leq h$ we have

$$\left\|P^{(j)}(t)\right\| \leq L, \ \left\|Y^{(j)}(t, x^{(j-1)}, y^{(j-1)})\right\| \leq P\left\|y^{(j-1)})\right\|^2.$$

Therefore,

$$\left\|x^{(j)}(t,\varepsilon)\right\|_2 \leq \frac{\varepsilon Q_0}{\sqrt{2}\delta_2^{2n+1}}(1+Q_1)\exp(2\varepsilon LQ\delta_1^{-2n}), \quad (12)$$

$$\left\|y^{(j)}(t,\varepsilon)\right\|_{2j+2} \leq \frac{\varepsilon P(1+Q)}{\sqrt{2}\delta_2^{2n+1}}\exp(2\varepsilon LQ\delta_{2j+1}^{-2n})\left\|y^{(j-1)}(t,\varepsilon)\right\|_{2j}^2, \ j \geq 1, \quad (13)$$

where $Q_0 := \|f(t,0)\|_0$.

3.3 Convergence of the sequence of approximation

Solution of the system (1) is the sum of the series

$$x(t,\varepsilon) = \sum_{j=1}^{\infty} y^{(j)}(t,\varepsilon). \quad (14)$$

Then using the inequalities (12), (13)we obtain that this series is majored by the series

$$m_0 + \sum_{j=1}^{\infty} m_j, \ m_0 := \frac{\varepsilon Q_0}{\sqrt{2}\delta_2^{2n+1}}(1+Q_1)\exp(\varepsilon Q^*\delta_1^{-2n}), \ Q^* := 2LQ,$$

$$m_j := \frac{\varepsilon P}{\sqrt{2}\delta_{2j+2}^{2n+1}}(1+Q_1)\exp(\varepsilon Q^*\delta_{2j+1}^{-2n})m_{j-1}^2. \quad (15)$$

Consider the series

$$\sum_{j=0}^{\infty} \bar{m}_j = \sum_{j=0}^{\infty} \varepsilon_0 m_j, \ \varepsilon_0 := \frac{\varepsilon P}{\sqrt{2}}(1+Q_1). \quad (16)$$

instead of (15).

Choose $\delta_1 \prec 1, \ \delta_2 \prec 1$, so that

$$\delta_2^{r(2n+1)}\exp(-r\varepsilon Q^*\delta_1^{-2n}) =: \bar{m}_0,$$

$$2\delta_2^{r(2n+1)}\exp(-r\varepsilon Q^*\delta_1^{-2n}) = \left(\varepsilon(1+Q_1)\right)^2 PQ_0\exp(\varepsilon Q^*\delta_1^{-2n})\delta_2^{-(2n+1)}$$

$$2\delta_2^{(r+1)(2n+1)} = (\varepsilon(1+Q_1))^2 \exp((r+1)\varepsilon Q^* \delta_1^{-2n}).$$

Let $(2-\alpha)r = \alpha,\ 1 \prec \alpha \prec 2$.

The other δ_j choose so that $\delta_{2j+2} = \delta_{2j}^{\alpha},\ \delta_{2j+1} = \delta_{2j-1}\alpha^{-1} = \delta_1\alpha^{-j},\ j = 1, 2, ...$

Then the series (16) has the form $\sum\limits_{j=0}^{\infty} \bar{m}_j = \sum\limits_{j=0}^{\infty} \bar{m}_0^{\alpha^j}$. This series converges for $\bar{m}_0 \prec 1$. And if $\bar{m}_0^{\alpha-1} \prec \frac{1}{2}$, then its sum S_1 satisfies the inequality $S_1 \leq 2\bar{m}_0$. The series $\sum\limits_{j=0}^{\infty} \delta_j$ also converges due to the relationship between its members, its sum satisfies the inequality $2S_\delta \leq q$, if

$$2\left[\delta_2^{2n+1} \exp(-\varepsilon Q^* \delta_1^{-2n})\right]^{\alpha-1} \prec 1. \quad (17)$$

Thus we have derived the convergence conditions (15) and (16). Setting $2\alpha = 3$for definiteness, we get $r = 3$. Then from (16) and $\bar{m}_0^{\alpha-1} \prec \frac{1}{2}$ it follows that

$$\varepsilon \prec \frac{1}{16\sqrt{PQ_0}(1+Q_1)} =: \varepsilon_1.$$

Besides from the inequality $2S_\delta \leq q$ we obtain

$$\varepsilon \prec \frac{q^2}{\sqrt{PQ_0}(1+Q_1)} =: \varepsilon_2.$$

The condition that the approximations $x^{(j)}(t, \varepsilon)$ stay in the region $\|x\| \leq h$ yields $S_2 \leq h$, here S_2 is a sum of the series dominating the series (14). By (15): $S_1 = \varepsilon_0 S_2$. Consequently, $\varepsilon_0 S_2 \leq 2\bar{m}_0$. Hence, under the assumption $2\bar{m}_0 \leq \varepsilon_0 h$ the approximations stay in the region $\|x\| \leq h$. This condition implies the inequality

$$\varepsilon \prec \frac{h^2\sqrt{P}}{Q_0^{3/2}(1+Q_1)} =: \varepsilon_3.$$

Thereby, with $\varepsilon \prec \min(\varepsilon_1, \varepsilon_2, \varepsilon_3)$the series (14)uniformly converges on the real axis t to the
conditionally-periodic solution of the system (1).

4 Results and reasoning.

The existence of a conditionally periodic solution of system (1) is proved. For this purpose, a sequence has been constructed, rapidly converging to the conditionally periodic solution of system (1).

First, a linear inhomogeneous system is investigated in detail, which is a model equation for the terms of the sequence of approximations. The research results are used to build a sequence of approximate solutions and in their evaluation. The conclusions of the work have scientific and practical value. Simultaneously with the proof of the existence of a solution, the construction of its construction is given. Unlike previous researchers, a critical case was considered and the method of accelerated convergence was applied.

5 Conclusion

The question of the existence and construction of a conditionally periodic solution is one of the most important in celestial mechanics. In problems of celestial mechanics, a conditionally periodic solution is a solution in which positional variables (semi-major axis, eccentricity, inclination, etc.) are expressed as conditionally-periodic functions, and angular variables (pericenter longitude, angle longitude, etc.) are expressed as a formula:

nt– conditionally periodic function,

where n – average angular variation for a given variable. Movement in an orbit corresponding to a conditionally periodic solution occurs in a limited region of space, and after a certain period of time, the celestial body returns to as close to any point within this region as you like. A characteristic feature of the series representing a conditionally periodic function is that their members are arranged according to increasing degrees of one or several small parameters, as in ordinary power series. However, the question of the convergence of these series in a strict mathematical sense has long remained open. Moreover, it was proved that a considerable number of these series diverges.

Thus, the assumption that conditionally periodic solutions in the problems of celestial mechanics exist and that the real motions of celestial bodies should be described by conventionally periodic functions required proof. G.A. Merman's paper first obtained a rigorous proof of the existence of a conditionally periodic solution in the plane bounded three-body problem.

V.I. Arnol'd developed a method for proving the existence and constructing conditionally periodic solutions of Hamiltonian systems of differential equations of a satisfied general form [3-4]. This method of A.N. Kosmogorov-V.I. Arnold was disclosed in terms of their practical application by Yu.A.Rabov and E.G. Grebennikov for the same Hamiltonian systems.

In recent years, A.Bari, H. Brezis, Feireist E., Pelyukh H.P., Syvak Q.A. have become involved in the problems described above or close to them [13-15]. In some international conferences, the author of this article made reports of particular details of the formulation of the question and the election of the method [16-17], theses of which were published.

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