# 2-бөлім | Раздел 2 | Section 2

# Информатика | Информатика | Computer Science

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## Stabilization of one non-liner system with coefficients depending on the condition of the control object

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For the mathematical model of a three-sector economic cluster, an optimal control problem is posed on an infinite time interval. An optimal stabilization problem is considered for a single class of nonlinear systems with coefficients depending on the state of the control object with constraints on control. A non-linear stabilizing control has been found taking into account constraints to the control, which depends on the state of the system and the current point in time. The results obtained for a nonlinear system are used in the construction of control parameters for a three-sector economic cluster over an infinite time interval. For the considering example, the optimal distribution of labor and investment resources has been determined, which satisfy the balance ratios. The given example illustrates the use of the proposing control method of a nonlinear system.
**Key words**: effective management accounting, three- sector economic cluster, Lagrange multipliers method, nonlinear systems, quadratic functional.

### Коэффициентері басқару объектісінің қалып - күйінен тәуелді болатын бір сызықты емес жүйені тұрақтандыру

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Үш секторлы экономикалық кластердің математикалық моделі үшін уақыттың шексіз интервалында тиімді басқару есебі қойылады. Коэффициенттері басқару объектісінің қалып - күйінен тәуелді сызықты емес жүйенің бір класы үшін тиімді тұрақтандыру есебі қарастырылады. Жүйенің қалып - күйінен және ағымдағы уақыттан тәуелді болатын басқаруға қойылған шектеуді ескеріп сызықты емес тұрақтандырушы басқару табылды. Сызықты емес жүйелер үшін алынған нәтижелер шексіз уақыт интревалында үш секторлы экономикалық кластерлерге арналған басқарушы параметрлерді құрастыру барысында қолданылады. Қарастырылып отырған мысал үшін баланстық қатынасты қанағаттандыратын еңбек және инвестициялық ресурстардың тиімді үлестірімі анықталған. Келтірілген мысал сызықты емес жүйелерді басқару әдістерінің тиімді қолданылуын сипаттайды.
**Түйін сөздер**: тиімді басқару есебі, үш секторлы экономикалық кластер, Лагранж көбейткіштер әдісі, сызықты емес жүйелер, квадраттық функционал.

**Стабилизация одной нелинейной системы с коэффициентами, зависящими от состояния объекта управления**

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Для математической модели трехсекторного экономического кластера ставится задача оптимального управления на бесконечном интервале времени. Рассматривается задача оптимальной стабилизации для одного класса нелинейных систем с коэффициентами, зависящими от состояния объекта управления с ограничениями на управления. Найдено нелинейное стабилизирующее управление с учетом ограничений на управление, которое зависит от состояния системы и текущего момента времени. Полученные результаты для нелинейной системы, используются при конструировании управляющих параметров для трехсекторного экономического кластера на бесконечном интервале времени. Для рассматриваемого примера определены оптимальное распределение трудовых и инвестиционных ресурсов, которые удовлетворяют балансовым соотношениям. Приведенный пример иллюстрирует использование предлагаемого метода управления нелинейной системы.
**Ключевые слова**: задача оптимального управления, трехсекторный экономический кластер, метод множителей Лагранжа, нелинейная система, квадратичный функционал

## 1 Introduction

In control theory, much attention is given to the problem of studying stability in nonlinear systems and the problem of stabilizing nonlinear control systems. The theoretical basis for solving linear-quadratic problems in some cases can be applied in the synthesis of control actions for nonlinear systems. One of the promising and rapidly developing methods for designing nonlinear regulators is a method based on the application of the matrix Riccati equation. Recently, new control algorithms for nonlinear systems have appeared, based on the use of the Riccati equations with coefficients depending on the state of the system. The ambiguity of the representation of a nonlinear system as a system of linear structure and the absence of sufficiently universal algorithms for solving the Riccati equation, whose parameters also depend on the state, generate a set of possible suboptimal solutions. Therefore, it is relevant to study nonlinear systems whose coefficients depend on the states of the system.

## 2 Literature review

To determine the place of our researches, briefly consider the known methods of synthesis of nonlinear controlled mechanical systems. The existing methods for the synthesis of systems can be divided into the following two groups: the Pontryagin maximum principle [1], the Bellman - Krotov method (optimal control theory) [2,4]. The theory and methods of solving this group of methods for the synthesis of systems are well known from the monographs of L. S. Pontryagin and others [1], R. Bellman [2], V. F. Krotov, and V. I. Gurman [3]-[5]. Undoubtedly, these methods are valuable and solve the problems of synthesis, when all their prerequisites take place. Essentially, the maximum principle reduces the solution of the original problem to a boundary problem for a system of ordinary differential equations with two times more order than the order of the system itself[6]-[9]. Solving the latter is generally quite a difficult task. Moreover, software control is determined, which is not always acceptable for solving the problem of practice. The R. Bellman method of dynamic programming in the case of the smoothness of the so-called Bellman function reduces the original problem to solving a partial differential equation of the first order[8]-[11]. In the general case, solving a partial differential equation is a difficult task.

In technical and economic systems, there are many different types of nonlinearities; therefore, different approaches to constructing control laws that are rational with respect to a given quality criterion [12] - [15].

In practice, there are a large number of optimal control problems for economic systems that are nonlinear systems with coefficients depending on the state of the control object [16, 17].

It should be noted that the controllability criteria for nonlinear systems were obtained in the work of J. Kleimka [19]. The stability studies of nonlinear systems were performed by A.P. Afanasyev and others [12] and S.M. Lobanov and others [13]. M.G. Dmitriev and others [14] investigate optimal stabilization for one class of nonlinear systems with coefficients depending on the state of the control object over an infinite time interval. V.N. Afanasyev and P.V. Orlov [15] consider a class of nonlinear systems for which there exists a coordinate representation (diffeomorphism) that transforms the original system into a system with a linear part and nonlinear feedback. In works [20,21], optimal control problems with using of Lagrange multipliers for technical systems and a linearized system of the economic cluster were considered.

## 3 Research method

This paper deals an economic system, which, by means of transformations, come down to an optimal stabilization problem for one class of nonlinear systems with coefficients depending on the state of the control object. Conversion is in progress of initial nonlinear differential equation, which describes the initial control system, into a system with a linear structure, but with parameters depending on the state of the control object. The use of a nonlinear quadratic quality functional allows at the synthesis of control to carry out the construction of the matrix Riccati equation with parameters not dependent of the state of the control object. This approach is the basis for the synthesis of optimal nonlinear control systems. It is proposed to use a combined method based on the construction of a nonlinear feedback, which allows present the sought-for control in the form of a stabilizing control, dependent on the state of the nonlinear system and the current point in time. In addition, this method gives a chance to take into account the existing restrictions on the values of controls. The obtained results for nonlinear systems are used in the construction of control parameters for a three-sector economic cluster over an infinite time interval.

### 3.1 Three-sector economic model of the cluster

Consider the optimal control problem for the economic model of a cluster consisting of three $i = 0$ (material sector), $i = 1$ (fund-generating sector), $i = 2$ (consumer sector). The considered mathematical model consists of [16]:

a) three specific release functions of the Cobba-Douglas type:

$$x_i = \theta_i A_i k_i^{\alpha_i}, \quad A_i > 0, \quad 0 < \alpha_i < 1, \quad (i = 0,\, 1,\, 2); \tag{1}$$

b) three differential equations describing the dynamics of the capital-labor ratio:

$$\dot{k}_i = -\lambda_i k_i + (s_i/\theta_i)x_i, \quad k_i(0) = k_i^0, \quad \lambda_i > 0, \quad (i = 0,\, 1,\, 2); \tag{2}$$

c) three balance ratios:

$$s_0 + s_1 + s_2 = 1, \quad s_0 > 0, \quad s_1 > 0, \quad s_2 > 0; \tag{3}$$

$$\theta_0 + \theta_1 + \theta_2 = 1, \quad \theta_0 > 0, \quad \theta_1 > 0, \quad \theta_2 > 0; \tag{4}$$

$$(1 - \beta_0)x_0 = \beta_1 x_1 + \beta_2 x_2, \quad \beta_0 \geq 0, \quad \beta_1 \geq 0, \quad \beta_2 \geq 0; \tag{5}$$

Here, the state of the economic system (capital-labor ratio) is described by a vector $(k_0,\ k_1,\ k_2)$, and $(s_0,\ s_1,\ s_2,\ \theta_0,\ \theta_1,\ \theta_2)$ - the vector of control $(s_0,\ s_1,\ s_2)$ - the share of sectors in the distribution of investment resources, $(\theta_0, \theta_1, \theta_2)$ - the share of sectors in the distribution of labor resources); $x_i$ - specific output (the number of manufactured products in the $i$ - sector is counting for one worker); $\beta_i$ - direct material costs at production output in the $i$ - sector; $(i = 0,\, 1,\, 2)$. The initial state of the system is $k_0^0,\, k_1^0,\, k_2^0$ where is $k_i^0 = k_i(0)$ the capital-sector ratio $i$, $(i = 0,\, 1,\, 2)$ at $t = 0$. The task of transferring a nonlinear system from a given initial state $(k_0^0,\, k_1^0,\, k_2^0)$ to desired state $k_0^s,\, k_1^s,\, k_2^s$ over an infinite time interval is considered $[0, \infty)$. The system's equilibrium state $k_0^s,\, k_1^s,\, k_2^s$ is chosen as the desired final state:

$$k_1^s = \left(\frac{s_1 A_1}{\lambda_1}\right)^{\frac{1}{1-\alpha_1}}, \quad k_0^s = \frac{s_0 \theta_1 A_1 (k_1^s)^{\alpha_1}}{\lambda_0 \theta_0}, \quad k_2^s = \frac{s_2 \theta_1 A_1 (k_1^s)^{\alpha_1}}{\lambda_2 \theta_2};$$

The values of the capital-labor ratio $k_i^s \quad (i = 0,\, 1,\, 2)$ in equilibrium state depend on the controls $(s_0,\, s_1,\, s_2,\, \theta_0,\, \theta_1,\, \theta_2)$ for which the values $(s_0^s,\, s_1^s,\, s_2^s,\, \theta_0^s,\, \theta_1^s,\, \theta_2^s)$ are determined in the work [18].

### 3.2 Statement of the problem of optimal stabilization for a class of nonlinear systems with coefficients depending on the state of the control object.

The task of transferring a nonlinear system from a given initial state to a desired state in an infinite time interval is considered $[t_0, \infty)$.

The mathematical model of the control object (2), we write in the form of a system of differential equations in vector form:

$$\dot{y}(t) = Ay(t) + BD(y)u(t) + B(D(y) - D(k^s))v^s, \quad y(t_0) = y_0, \quad [t_0, \infty); \tag{6}$$

using next designations:

$$y_1 = k_1 - k_1^s, \quad y_2 = k_2 - k_2^s, \quad y_3 = k_0 - k_0^s,$$

$$u_1 = s_1 - v_1^s, \quad u_2 = \frac{s_2 \theta_1}{\theta_2 - v_2^s}, \quad u_3 = \frac{s_0 \theta_1}{\theta_0 - v_3^s}, \quad v_1^s = s_1^s, \quad \frac{s_2^s \theta_1^s}{\theta_2^s} = v_2^s, \quad \frac{s_0^s \theta_1^s}{\theta_0^s} = v_3^s,$$

$$f_1(y_1) = (y_1 + k_1^s)^{\alpha_1}, \quad f_2(y_2) = (y_2 + k_2^s)^{\alpha_2}, \quad f_3(y_3) = (y_3 + k_0^s)^{\alpha_0},$$

$$A = \begin{pmatrix} -\lambda_1 & 0 & 0 \\ 0 & -\lambda_1 & 0 \\ 0 & 0 & -\lambda_1 \end{pmatrix}, \quad B = \begin{pmatrix} A_1 & 0 & 0 \\ 0 & A_1 & 0 \\ 0 & 0 & A_1 \end{pmatrix},$$

$$D(y) = \begin{pmatrix} (y_1+k_1^s)^{\alpha_1} & 0 & 0 \\ 0 & (y_1+k_1^s)^{\alpha_1} & 0 \\ 0 & 0 & (y_1+k_1^s)^{\alpha_1} \end{pmatrix}, \quad D(k^s) = \begin{pmatrix} (k_1^s)^{\alpha_1} & 0 & 0 \\ 0 & (k_1^s)^{\alpha_1} & 0 \\ 0 & 0 & (k_1^s)^{\alpha_1} \end{pmatrix},$$

$$Ak^s + BD(k^s)v^s = 0$$

Here $y = (y_1, \quad y_2, \quad y_3)^*$ means vector the state of the object, $u = (u_1, \quad u_2, \quad u_3)^*$ means the control vector.

Using the differential equation (6) and the balance sheet relations (3) - (5) describe the control object in the following form:

$$\dot{y}(t) = Ay(t) + BD(y)v(t), \quad y(t_0) = y_0, \quad t \in [t_0, \infty); \tag{7}$$

$$v(t) \in V(t) = \{v | \gamma_1(t) \leq v(t) - (E - D^{-1}(y)D(k^s))v^s \leq \gamma_2(t), t \in [t_0, \infty); \gamma_1, \gamma_2 \in C[t_0, \infty]\}$$

$$u(t) = v(t) - (E - D^{-1}(y)D(k^s))v^s;$$

$$g(u, y, s, \theta) = 0.$$

We will assume that system (7) is controllable. Matrices satisfy the condition of controllability, i.e. condition is performed $ranq[B, AB, \ldots, A^{n-1}B] = n$. Let a functional be installed that depends on the control and state of the object:

$$J(u) = \frac{1}{2} \int_{t_0}^{\infty} [y^*(t)Q(y)y(t) + v^*(t)Rv(t)] \, dt \tag{8}$$

where $Q(y) = KBD(y)R^{-1}D^*(y)B^*K - KBD(k^s)R^{-1}D^*(k^s)B^*K + Q_1$ – a positive semidefinite matrix, and $R, D(y)$- a positive definite matrix.

Task is set. It is required to find a stabilizing control $u(y, t)$ that translates system (7) from a given initial state $y(t_0) = y_0$ to the desired equilibrium state $y(\infty) = 0$ over a time interval $[t_0, \infty)$, minimizing functional (8).

For the optimal control problem (7)-(8), a search is made for such a control $v(y, t)$, so that the equilibrium position in a closed system is asymptotically stable according to Lyapunov. For this purpose, a method based on using of a special type of Lagrange multipliers was used [20].

### 3.3 Solution of task of optimal stabilization.

To solve this assigned task let's add to the expression for the functional (8) a system of differential equations (7) with a multiplier $\lambda = Ky$, and also the following expression

$$\lambda_1^*(t)[\gamma_1 - u(t)] + \lambda_2^*(t)[u(t) - \gamma_2],$$

where $\lambda_1 \geq 0$, $\lambda_2 \geq 0$. As a result, we get the following functionality:

$$L(y, v) = \int_{t_0}^{\infty} \{\frac{1}{2}y^*Q(y)y + \frac{1}{2}v^*Rv + (Ky)^*(Ay + BD(y)v - \dot{y}) + \lambda_1^*(t)[\gamma_1 - v(t) + \\ + (E - D^{-1}(y)D(k^s))v^s] + \lambda_2^*(t)[v(t) - (E - D^{-1}(y)D(k^s))v^s - \gamma_2]\} \, dt \tag{9}$$

where $K$- is the symmetric positive definite constant matrix.

We introduce the following functions:

$$V(y,t) = \frac{1}{2} y^* K y \tag{10}$$

$$\begin{aligned} M(y,v,t) = \frac{1}{2} y^*(t) Q(y) y(t) + \frac{1}{2} v^*(t) R v(t) + (Ky)^*(Ay(t) + \\ + BD(y)v(t)) + \lambda_1^*(t)[\gamma_1 - v(t) + (E - D^{-1}(y)D(k^s))v^s] + \\ + \lambda_2^*(t)[v(t) - (E - D^{-1}(y)D(k^s))v^s - \gamma_2] \end{aligned} \tag{11}$$

From the form of function (10) and (11), rightly the following representation of the functional (9)

$$L(y,v) = V(y_0,t_0) + \int_{t_0}^{\infty} M(y,u,t)\, dt \tag{12}$$

from the stationarity conditions for the functional (12), we obtain the control from the relation

$$Rv(t) = -D^*(y)B^*Ky + (\lambda_1 - \lambda_2) \tag{13}$$

where the constant matrix K satisfies the algebraic matrix equation:

$$KA + A^*K - KBD(k^s)R^{-1}D^*(k^s)B^*K + Q_1 = 0 \tag{14}$$

Using the following designations:

$$\begin{aligned} A_1(y,t) = A - BD(y)R^{-1}D^*(y)B^*K, \qquad \varphi(y,t) = R^{-1}[\lambda_1 - \lambda_2], \\ \lambda_1(y,t) = R\max\{0;\, \gamma_1 - \omega(y,t)\} \geq 0, \quad \lambda_2(y,t) = R\max\{0;\, \omega(y,t) - \gamma_2\} \geq 0, \\ \omega(y,t) = -(E - D^{-1}(y)D_s)v_s - R^{-1}D^*(y)B^*Ky \end{aligned} \tag{15}$$

the differential that determines the law of motion of the system, we will present in the following form

$$\dot{y} = A_1(y,t)y(t) + BD(y)\varphi(y,t), \quad y(t_0) = y_0 \tag{16}$$

Note that the choice of multipliers $\lambda_1(t) \geq 0$, $\lambda_2(t) \geq 0$ of the form (15) provides that the conditions for complementary slackness are met

$$\lambda_1^*(t)[\gamma_1 - u(t)] = 0, \quad \lambda_2^*(t)[u(t) - \gamma_2] = 0$$

The results established for the optimal control problem (7) – (8) are formulated as the following statement.

**Theorem.** Let $Q(y)$- be a positive semidefinite matrix, and $R$, $D(y)$- a positive definite matrices in the interval $[t_0, \infty)$. Suppose that system (7) is quite manageable at the moment of time $t_0$. Then, for the optimality of the pair $(y(t), v(t))$ in problem (7) – (8), it suffices to performance the following conditions:

1) the trajectory satisfies the differential equation

$$\dot{y} = A_1(y,t)y(t) + BD(y)\varphi(y,t), \quad y(t_0) = y_0 \tag{17}$$

2) management $v(t)$ is defined as follows:

$$v(y,t) = \omega(y,t) + \varphi(y,t). \tag{18}$$

The matrix $K$ satisfies the matrix equation (14), the vector function $\varphi(y(t),t)$ is determined by the formula (15) in such a way as to ensure that the constraints on the control (7) are satisfied.

## 4 Results and reasoning.

### 4.1 Algorithm for solving the problem of optimal stabilization of three-sector economic model of cluster.

We describe convenient for implementation on computer an algorithm for solving an optimal control problem (6) – (8).

1. Solve the system of algebraic equations (14) to determine the matrix $K$.

2. Integrate the system of differential equations (17) in the interval $[t_0, \infty)$ under the initial conditions $y(t_0) = y_0$. In the process of integrating the system (17), it is necessary to print a graph of the optimal trajectory and optimal control $v(t)$.

3. Let the system state $y(t)$ and optimal control $v(t)$ be found, then

$$u(t) = v(t) - (E - D^{-1}(y)D(k^s))v^s, \quad f_i(y_i) = (y_i + k_i^s)^{\alpha_i},$$

$$\omega = \frac{\beta_1 A_1 f_1(y_1) + \beta_2 A_2 f_2(y_2)(1-u_1-v_1^s)/(u_2+v_2^s)}{(1-\beta_0)A_0 f_3(y_3)(1-u_1-v_1^s)/(u_3+v_3^s) + \beta_2 A_2 f_2(y_2)(1-u_1-v_1^s)/(u_2+v_2^s)} \tag{19}$$

ensure the fulfillment of condition (5);

$$s_1 = u_1 + v_1^s, \; s_2 = (1-\omega)(1-u_1-v_1^s), \; s_0 = \omega(1-u_1-v_1^s)$$

ensure the fulfillment of condition (3);

$$\theta_1 = \frac{1}{1 + s_0/(u_3+v_3^s) + s_2/(u_2+v_2^s)}, \quad \theta_2 = \frac{(1-\omega)(1-s_1)\theta_1}{(u_2+v_2^s)}, \quad \theta_0 = \frac{\omega(1-s_1)\theta_1}{(u_3+v_3^s)}$$

ensure the fulfillment of condition (4);

Example. Were carried out numerical calculations on a computer with the following values of the parameters (table 1):

Table 1: Parameter values for a three - sector economic cluster

| I | $\alpha_i$ | $\beta_i$ | $\lambda_i$ | $A_i$ | $s_i^*$ | $\theta_i^*$ | $k_i^*$ |
|---|---|---|---|---|---|---|---|
| 0 | 0.46 | 0.39 | 0.05 | 6.19 | 0.2763 | 0.3944 | 966.4430 |
| 1 | 0.68 | 0.29 | 0.05 | 1.35 | 0.4476 | 0.2562 | 2410.1455 |
| 2 | 0.49 | 0.52 | 0.05 | 2.71 | 0.2761 | 0.3494 | 1090.1238 |

The optimal control problem is solved for the values of the initial state of the system $y(t_0)$ which are given in the following form:

$$y(t_0) = (-800, \quad -400, \quad 400)^* \tag{20}$$

and matrix $R$, $Q_1$, $K$, have a form:

$$R = \begin{pmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{pmatrix}, \quad Q_1 = \begin{pmatrix} 16 \cdot 10^{-4} & 0 & 0 \\ 0 & 8 \cdot 10^{-4} & 0 \\ 0 & 0 & 8 \cdot 10^{-4} \end{pmatrix},$$

$$K = \begin{pmatrix} 0.2033 \cdot 10^{-2} & 0 & 0 \\ 0 & 0.1094 \cdot 10^{-2} & 0 \\ 0 & 0 & 0.1090 \cdot 10^{-2} \end{pmatrix}$$

The results of the calculations of the state of the system are presented in Figure 1(a). From Figure 1(b), it can seen that the optimal controls do not exceed the region defing by the constraints.

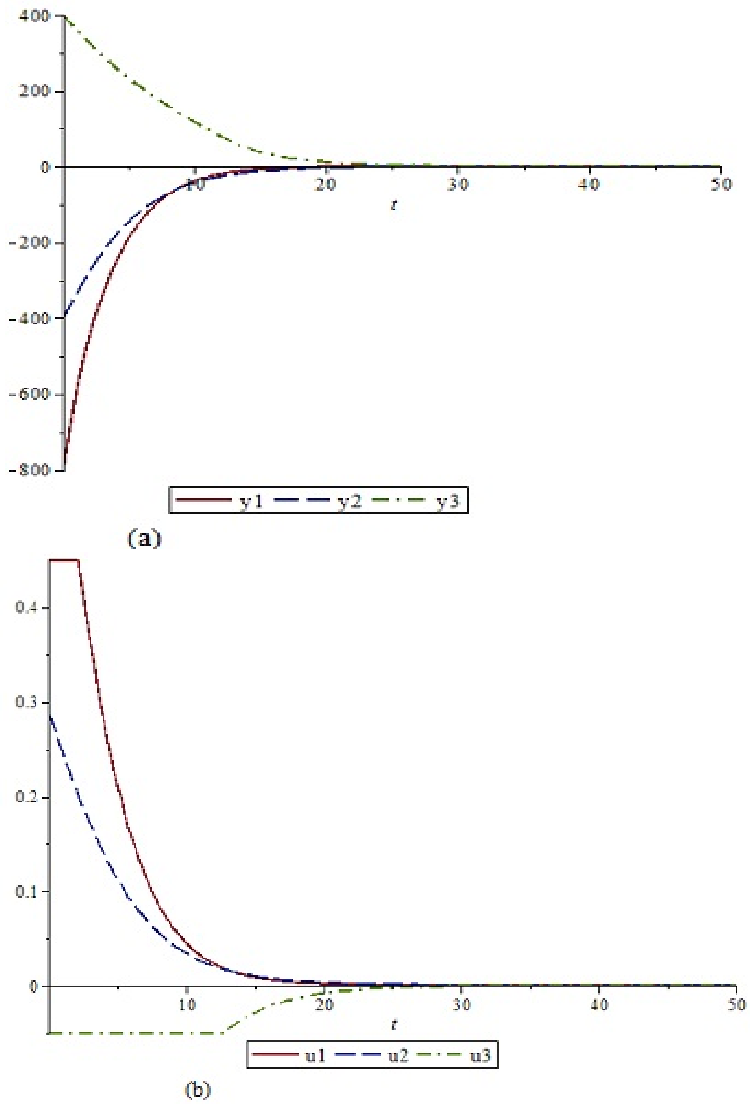


Figure 1: Graphs of trajectories $y(t)$ (a) and optimal control $u(t)$ (b).

For the considering example, these restrictions are of the form:

$$-0.45 \leq u_1 \leq 0.45, \quad -0.05 \leq u_2 \leq 0.5, \quad -0.05 \leq u_3 \leq 0.5 \tag{21}$$

Here, the components of control $u_1(t)$ and $u_3(t)$ lie on the boundary of the region $U$ in the time interval $[0,\, t_1]$ and $[0,\, t_2]$, accordingly, then at $t \in [t_1,\, \infty)$ $t \in [t_2,\, \infty)$ as they enter inside the region $U$. Switching controls occurs at the moment of time $t_1 = 2.193$ for the component$u_1(t)$ and for $u_3(t)$ at $t_2 = 12.762$. The optimal values of the system states at the finite moment of time at $T = 50$: $y_1(T) = -0.6739 \cdot 10^{-4}$; $y_2(T) = -0.2068 \cdot 10^{-2}$; $y_3(T) = 0.6282 \cdot 10^{-2}$, and the optimal values of the controls at the finite moment of time at $T = 50$: $u_1(T) = 0.3197 \cdot 10^{-5}$; $u_2(T) = 0.6089 \cdot 10^{-4}$; $u_3(T) = -0.1842 \cdot 10^{-2}$.

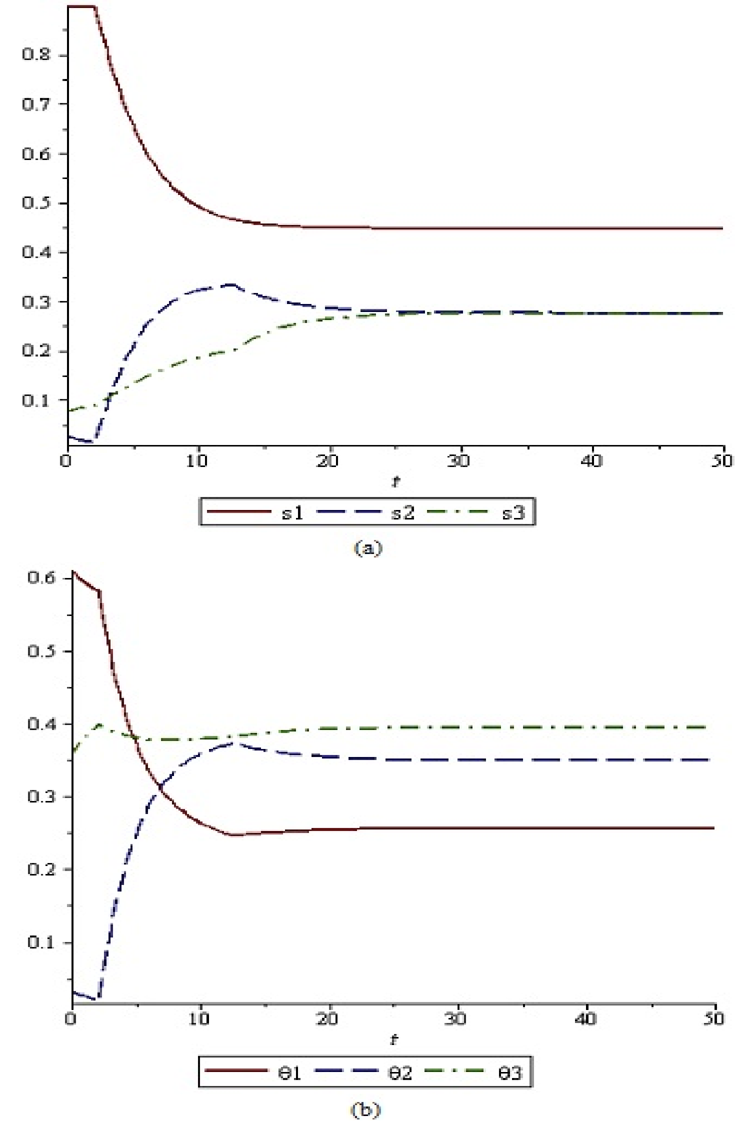


Figure 2: Graphs of the distribution of investment (a) and labor resources (b) for balance relations (3)-(5).

Using formulas (19)-(21), the optimal distribution of labor $(\theta_1(t),\, \theta_2(t),\, \theta_3(t))$ and investment resources $(s_1(t),\, s_2(t),\, s_3(t))$ was determined. Figure 2 shows the changes resources that satisfy the balance relations (3)–(5). The values of investment $(s_1(t),\, s_2(t),\, s_3(t))$ and workforce $(\theta_1(t),\, \theta_2(t),\, \theta_3(t))$ at the end point of time at $T = 50$ tend to a stationary state, with an approximation estimate: $|s_1(T) - s_1^s| = 0.3197 \cdot 10^{-5}$; $|s_2(T) - s_2^s| = 0.9437 \cdot 10^{-4}$; $|s_3(T) - s_3^s| = 0.9757 \cdot 10^{-4}$; $|\theta_1(T) - \theta_1^s| = 0.3058 \cdot 10^{-4}$; $|\theta_2(T) - \theta_1^s| = 0.2646 \cdot 10^{-5}$; $|\theta_3(T) - \theta_3^s| = 0.3323 \cdot 10^{-4}$.

## 5 Conclusion

An algorithm for solving the problem of optimal stabilization was developed and a nonlinear control was found, that based on the feedback principle using the matrix Riccati equation. A feature of the proposed approach is sufficient flexibility that determined by method of transforming the original nonlinear system to a linear form with respect to control and with coefficients depending on the state of the system. The use of a nonlinear quadratic quality functional allows at the synthesis of control to carry out the construction of the matrix Riccati equation with parameters independent of state of the control object.

The results obtained for nonlinear systems are used in the construction parameters of control for the mathematical model of a three-sector economic cluster. The equilibrium position in a closed system is asymptotically stable according to Lyapunov. Controlling parameters (19)-(21) are chosen in such a way that constraints on controls and balance relations (3)-(5) are satisfied. For the considered example, the optimal distribution of labor and investment resources has been determined, which satisfy the balance ratios. Figures 1 and 2 show the optimal trajectories and controls that satisfy the specified constraints.

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