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On a linear system of differential equations

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The linear systems of partial differential equations of the first order with the identical main parts is considered. Applying the well-known relation between a normal system of ordinary differential equations and a linear system of partial differential equations of the first order with the same main parts, the existence of integral basis of a linear inhomogeneous system of partial differential equations of the first order adjoining to some solution of the same linear inhomogeneous system of differential equations with partial derivatives of the first order is proved. A sign at which the nonlinear system of ordinary differential equations has a neighborhood such that any solution with initial values from it tends to zero is found. Using the equivalence of a linear system of partial differential equations of the first order with identical main parts to a linear differential equation with partial derivatives of the first order, the existence of integral basis of the adjoining to zero linear homogeneous system of partial differential equations of first order with nonlinear coefficients is shown.

Key words: equation, first order partial derivatives.

Дифференциалдық теңдеулердің сызықты жүйесі туралы

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Басты бөліктері бірдей бірінші ретті дербес туындылы дифференциалдық теңдеулердің сызықты жүйелері қарастырылады. Жай дифференциалдық теңдеулердің қалыпты жүйесі мен басты бөліктері бірдей бірінші ретті дербес туындылы дифференциалдық теңдеулердің сызықты жүйелері арасындағы айқын байланысты пайдалана отырып, сызықты біртекті емес бірінші ретті дербес туындылы дифференциалдық теңдеулердің және осы бірінші ретті дербес туындылы сызықты біртекті емес дифференциалдық теңдеулер жүйесінің қайсібір шешіміне жалғасатын интегралды базис бар екені көрсетілген. Жай дифференциалдық теңдеулердің сызықты емес жүйесінің сондай маңайы болуының сипаты табылған, сол маңайдағы бастапқы мәндері алынған кез келген шешім нөлге ұмтылады. Бірінші ретті дербес туындылы дифференциалдық теңдеулердің қалыпты жүйесі және басты бөліктері бірдей бірінші ретті дербес туындылы дифференциалдық теңдеулердің сызықты жүйелерінің эквиваленттігін пайдаланып, нөлге жалғасатын, коэффиценттері сызықты емес бірінші ретті дербес туындылы сызықты біртекті дифференциалдық теңдеулер жүйесінің итегралды базисының бар екені дәлеледенген

Түйін сөздер: теңдеу, бірінші ретті дербес туындылар.

Об одной линейной системе дифференциальных уравнений

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Рассматриваются линейные системы дифференциальных уравнений с частными производными первого порядка с одинаковыми главными частями. Применяя известную связь между нормальной системой обыкновенных дифференциальных уравнений и линейной системой дифференциальных уравнений с частными производными первого порядка с одинаковыми главными частями, доказано, что существует интегральный базис линейной неоднородной системы дифференциальных уравнений с частными производными первого порядка, примыкающий к некоторому решению этой же линейной неоднородной системы дифференциальных уравнений с частными производными первого порядка. Найден признак, по которому нелинейная система обыкновенных дифференциальных уравнений имеет такую окрестность, что любое решение с начальными значениями из этой окрестности стремится к нулю. Используя эквивалентность линейной системы дифференциальных уравнений с частными производными первого порядка с одинаковыми главными частями и линейного дифференциального уравнения с частными производными первого порядка, доказано, что существует интегральный базис, примыкающей к нулю линейной однородной системы дифференциальных уравнений с частными производными первого порядка с нелинейными коэффициентами.
Ключевые слова: уравнение, частные производные первого порядка.

1 Introduction

The article deals with linear systems of partial differential equations of the first order with the identical main parts.Applying the well-known relation between a normal system of ordinary differential equations and a linear system of partial differential equations of the first order with the same main parts, the existence of integral basis of a linear inhomogeneous system of partial differential equations of the first order adjacent to some solution of the same linear inhomogeneous system of differential equations with partial derivatives of the first order is proved. A sign at which the nonlinear system of ordinary differential equations has a neighborhood such that any solution with initial values from it tends to zero is found. Using the equivalence of a linear system of partial differential equations of the first order with identical main parts to a linear differential equation with partial derivatives of the first order, the existence of integral basis of the adjacent to zero linear homogeneous system of partial differential equations of first order with nonlinear coefficients is shown. In (see [3]), the equivalence of a linear system of partial differential equations with identical main parts to a certain system of ordinary differential equations is shown. In the article, the approach differs from said above and the well-known simple relations between solutions of a linear homogeneous and linear inhomogeneous system of partial differential equations of the first order is used.

2 Literature review

The general theory is presented in the books [1-10]. The domain of existence of solutions was investigated by Kamke and data is contained in the reference books [11-12]. The domain of existence of solutions was investigated in the work [13-15]. Non-analytic equations are considered in the papers[16-18]. In work of Kruzhkov, generalized solutions was considered [19]. Kovalevskaya's theorem was published in [20]. The problems of Lyapunovs second method were shown in [21, 22]. An example of nonexistence of a solution was constructed in [23-27]. The first-order partial differential equations were considered in [28]. Questions related to the nonlinear system of ordinary differential equations are in [29, 30].

3 Materials and research methods

Let us consider a linear inhomogeneous system of partial differential equations of the first order

$$\begin{cases} \frac{\partial u_1}{\partial x} + \sum\limits_{k=1}^{n} p_{1k}(x) y_k \frac{\partial u_1}{\partial y_1} + \ldots + \sum\limits_{k=1}^{n} p_{nk}(x) y_k \frac{\partial u_1}{\partial y_n} = b_1(x) u_1 \\ \frac{\partial u_2}{\partial x} + \sum\limits_{k=1}^{n} p_{1k}(x) y_k \frac{\partial u_2}{\partial y_1} + \ldots + \sum\limits_{k=1}^{n} p_{nk}(x) y_k \frac{\partial u_2}{\partial y_n} = b_2(x) u_2 \\ \\ \frac{\partial u_n}{\partial x} + \sum\limits_{k=1}^{n} p_{1k}(x) y_k \frac{\partial u_n}{\partial y_1} + \ldots + \sum\limits_{k=1}^{n} p_{nk}(x) y_k \frac{\partial u_n}{\partial y_n} = b_n(x) u_n; \end{cases} \tag{1}$$

where $u_1(x, y_1, \ldots, y_n), \ldots, u_n(x, y_1, \ldots, y_n)$ are unknown functions,$p_{ik}(x),\ i = 1, \ldots, n;\ k = 1, \ldots, n;$ and $b_s(x),\ s = 1, \ldots, n;$ have continuous first-order partial derivatives on the set $x_0 \le x < +\infty,\ -\infty < y_i < +\infty,\ i = 1, \ldots, n;$

Theorem 1. *If for some $\mu > 0$ and for some positive continuous function $\varphi(x)$, $\int\limits_{x_0}^{x} \varphi(s) ds \uparrow +\infty$, on $x \ge x_0$ next conditions are true:*
1) inequalities are fulfilled: $p_{k,k}(x) - p_{k+1,k+1}(x) \ge \mu\varphi(x),\ \mu > 0,\ k = 1, \ldots, n-1;$ при $x \ge x_0$;
2) the following limits exist: $\lim\limits_{x \to +\infty} \frac{|p_{ik}(x)|}{\varphi(x)} = 0,\ i \ne k,\ i = 1, 2, \ldots, n,\ k = 1, 2, \ldots, n;$
3) $\lim\limits_{x \to +\infty} \frac{1}{q(x)} \int\limits_{x_0}^{x} p_{kk}(s) ds = \beta_k,\ k = 1, 2, \ldots, n;$ и $\beta_1 < 0$; $q(x) = \int\limits_{x_0}^{x} \varphi(s) ds$. then a linear inhomogeneous system of first-order partial differential equations (1) *has an integral basis, which is adjacent to the solution of a linear inhomogeneous system.* Proof. System (1) corresponds to a linear homogeneous system of partial differential equations of the first order

$$\begin{cases} \frac{\partial u_1}{\partial x} + \sum\limits_{k=1}^{n} p_{1k}(x) y_k \frac{\partial u_1}{\partial y_1} + \ldots + \sum\limits_{k=1}^{n} p_{nk}(x) y_k \frac{\partial u_1}{\partial y_n} = 0 \\ \frac{\partial u_2}{\partial x} + \sum\limits_{k=1}^{n} p_{1k}(x) y_k \frac{\partial u_2}{\partial y_1} + \ldots + \sum\limits_{k=1}^{n} p_{nk}(x) y_k \frac{\partial u_2}{\partial y_n} = 0 \\ \\ \frac{\partial u_n}{\partial x} + \sum\limits_{k=1}^{n} p_{1k}(x) y_k \frac{\partial u_n}{\partial y_1} + \ldots + \sum\limits_{k=1}^{n} p_{nk}(x) y_k \frac{\partial u_n}{\partial y_n} = 0; \end{cases} \tag{2}$$

Obviously, the linear homogeneous system (2) is equivalent to a linear partial differential equation of the first order

$$\frac{\partial u}{\partial x} + \sum_{k=1}^{n} p_{1k}(x) y_k \frac{\partial u}{\partial y_1} + \ldots + \sum_{k=1}^{n} p_{nk}(x) y_k \frac{\partial u}{\partial y_n} = 0 \tag{3}$$

The characteristic system of equation (3) is the following linear system

$$\frac{dy_i}{dx} = \sum_{k=1}^{n} p_{1k}(t) y_k,\ i = 1, \ldots, n. \tag{4}$$

due to the condition, the linear system (4) has n linearly independent solutions

$$y_{1k}, y_{2k}, \ldots, y_{nk},\ k = 1, \ldots, n;$$

which satisfy equalities
a) $\lim\limits_{x\to+\infty}\left|\frac{y_{ik}}{y_{kk}}\right| = 0,\ i \neq k;$ b) $\lim\limits_{x\to+\infty}\left|\frac{1}{\varphi(x)}\frac{y'_{kk}}{y_{kk}} - \frac{p_{kk}(x)}{\varphi(x)}\right| = 0;$
This implies that for any $\varepsilon > 0$ there exists a $T \in I$ such that for any $x > T$, $k = 1, \ldots, n;$ there are inequalities

$$|y_{kk}(x_0)|e^{\int\limits_{x_0}^{x} p_{kk}(t)dt+\varepsilon q(x)} \leq \|\bar{y}_k(x)\| \leq n|y_{kk}(x_0)|e^{\int\limits_{x_0}^{x} p_{kk}(t)dt+\varepsilon q(x)}$$

$$\bar{y}_k = colon[y_{1k}, y_{2k}, \ldots, y_{nk}],\ k = 1, \ldots, n$$

it implies

$$\frac{1}{q(x)}\int\limits_{x_0}^{x} p_{kk}(t)dt - \varepsilon \leq \frac{1}{q(x)}\ln\|\bar{y}_k(x)\| \leq \frac{1}{q(x)}\int\limits_{x_0}^{x} p_{kk}(t)dt + \varepsilon;$$

Passing to the limit as $x \to +\infty$, by virtue of condition 3) we have

$$\lim_{x\to+\infty}\frac{1}{q(x)}\ln\|\bar{y}_k(x)\| = \beta_k,\ k = 1, 2, \ldots, n;$$

and besides

$$\beta_1 > \beta_2 > \ldots > \beta_n.$$

Therefore, $\bar{y}_k$, $k = 1, 2, \ldots, n;$ forms the normal basis of the linear system (4) and β_k, $k = 1, 2, \ldots, n;$ they are generalized exponents of the linear system (4) with respect to $q(x)$; moreover, they are exact generalized exponents of the linear system (4). Due to the condition $\beta_1 < 0$, we find that the generalized exponents of the linear system (4) are negative. Consequently, any solution of the linear system (4) tends to zero as $x \to +\infty$. Take the general solution of the linear system (4) of the Cauchy form

$$\begin{cases} y_1 = \varphi_1(x, x_0, y_1^0, \ldots, y_n^0) \\ \quad\ldots\ldots\ldots \\ y_n = \varphi_n(x, x_0, y_1^0, \ldots, y_n^0) \end{cases} \tag{5}$$

where the initial values$y_1^0, ..., y_n^0$ are arbitrary real numbers. There is equality

$$\lim_{x\to+\infty}\varphi_k(x, x_0, y_1^0, \ldots, y_n^0) = 0,\ k = 1, \ldots, n \tag{6}$$

solving (5) with respect to $y_1^0, ..., y_n^0$, we obtain an integral basis

$$\begin{cases} \psi_1(x, x_0, y_1, \ldots, y_n) \\ \quad\ldots\ldots\ldots \\ \psi_n(x, x_0, y_1, \ldots, y_n) \end{cases}$$

of linear equation (3). Consequently

$$\begin{pmatrix} \psi_1(x,x_0,y_1,\dots,y_n) \\ 0 \\ \dots \\ 0 \end{pmatrix}, \dots, \begin{pmatrix} 0 \\ \dots \\ 0 \\ \psi_n(x,x_0,y_1,\dots,y_n) \end{pmatrix}$$

the integral basis of the linear homogeneous system (2), which by virtue of (6) is adjacent to zero in the parameter x_0, as $x_0 \to +\infty$. It is easily verified that

$$\begin{pmatrix} u_1^0 e^{\int\limits_{x_0}^{x} b_1(\tau)d\tau} \\ 0 \\ \dots \\ 0 \end{pmatrix}, \dots, \begin{pmatrix} 0 \\ \dots \\ 0 \\ u_n^0 e^{\int\limits_{x_0}^{x} b_n(\tau)d\tau} \end{pmatrix} \tag{7}$$

is a special solution of the linear inhomogeneous system (1). Then

$$\begin{pmatrix} \psi_1(x,x_0,y_1,\dots,y_n) + u_1^0 e^{\int\limits_{x_0}^{x} b_1(\tau)d\tau} \\ 0 \\ \dots \\ 0 \end{pmatrix}, \dots, \begin{pmatrix} 0 \\ \dots \\ 0 \\ \psi_n(x,x_0,y_1,\dots,y_n) + u_n^0 e^{\int\limits_{x_0}^{x} b_n(\tau)d\tau} \end{pmatrix}$$

forms the integral basis of the linear inhomogeneous system (1), which is adjacent to the solution (7) of the linear inhomogeneous system.

Theorem 1 is proved.

A linear homogeneous system of first-order partial differential equations is considered.

$$\begin{cases} \frac{\partial u_1}{\partial x} + \left(\sum\limits_{k=1}^{n} p_{1k}(x)y_k + g_1(x,y_1,\dots,y_n)\right)\frac{\partial u_1}{\partial y_1} + \dots + \\ \quad + \left(\sum\limits_{k=1}^{n} p_{nk}(x)y_k + g_n(x,y_1,\dots,y_n)\right)\frac{\partial u_1}{\partial y_n} = 0 \\ \frac{\partial u_2}{\partial x} + \left(\sum\limits_{k=1}^{n} p_{1k}(x)y_k + g_1(x,y_1,\dots,y_n)\right)\frac{\partial u_2}{\partial y_1} + \dots + \\ \quad + \left(\sum\limits_{k=1}^{n} p_{nk}(x)y_k + g_n(x,y_1,\dots,y_n)\right)\frac{\partial u_2}{\partial y_n} = 0 \\ \\ \frac{\partial u_n}{\partial x} + \left(\sum\limits_{k=1}^{n} p_{1k}(x)y_k + g_1(x,y_1,\dots,y_n)\right)\frac{\partial u_n}{\partial y_1} + \dots + \\ \quad + \left(\sum\limits_{k=1}^{n} p_{nk}(x)y_k + g_n(x,y_1,\dots,y_n)\right)\frac{\partial u_n}{\partial y_n} = 0; \end{cases} \tag{8}$$

where $u_1(x,y_1,\dots,y_n),\dots,u_n(x,y_1,\dots,y_n)$ are unknown functions. $p_{ik}(x),\ i=1,\dots,n;\ k=1,\dots,n;$ are continuous in $I \equiv [x_0,+\infty)$, functions $g_i(x,\ y_1,\dots\ y_n),\ i=1,\dots,n;$ are continuous on x in the interval I, and have continuous partial derivatives y_s, $s=1,\dots,n;$ in the domain $\|y\| = \left(\sum\limits_{s=1}^{n} y_s^2\right)^{\frac{1}{2}} < h,\ y = colon\,[y_1, \ldots, y_n]\,,\ g_i(x, 0, \ldots, 0) = 0,\ i = 1, \ldots, n;$

Theorem 2. *If for some $\mu > 0$ and for some positive continuous function $\varphi(x)$, $\int\limits_{x_0}^{x} \varphi(s)ds \uparrow +\infty$, on $x \geq x_0$ next conditions are true:*
1) inequalities are fulfilled: $p_{k,k}(x) - p_{k+1,k+1}(x) \geq \mu\varphi(x),\ \mu > 0,\ k = 1, \ldots, n-1$; при $x \geq x_0$;
2) the following limits exist: $\lim\limits_{x\to+\infty} \frac{|p_{ik}(x)|}{\varphi(x)} = 0,\ i \neq k,\ i = 1, 2, \ldots, n,\ k = 1, 2, \ldots, n$;
3) $\lim\limits_{x\to+\infty} \frac{1}{q(x)} \int\limits_{x_0}^{x} p_{kk}(s)ds = \beta_k,\ k = 1, 2, \ldots, n$; and $\beta_1 < 0$; $q(x) = \int\limits_{x_0}^{x} \varphi(s)ds$.
4) $0 < \mu < |\beta_1|,\ m > 1$ и $0 < \varepsilon < (m-1)\mu \int\limits_{x_0}^{x} e^{[\varepsilon+\mu(1-m)][q(s)-q(x_0)]}ds < \infty$ are fulfilled;
5) for vector function $g(x,y) = colon[g_1(x,\, y_1, \ldots, y_n),\ \ldots, g_n(x,\, y_1, \ldots, y_n)]$ the inequality is true

$$\|g(x,y)\| \leq K\|y\|^m,\ K > 0,\ m > 1;$$

where

$$\|g(x,y)\| = \left(\sum_{i=1}^{n} g_i^2(x, y_1, \ldots, y_n)\right)^{\frac{1}{2}};$$

then a linear homogeneous system (8) *has an integral basis, which is adjacent to zero.*
Proof. Linear homogeneous system (8) is equivalent to a linear first-order partial differential equation

$$\frac{\partial u}{\partial x} + \Big(\sum_{k=1}^{n} p_{1k}(x)y_k + g_1(x, y_1, \ldots, y_n)\Big)\frac{\partial u}{\partial y_1} + \ldots + \Big(\sum_{k=1}^{n} p_{nk}(x)y_k + g_n(x, y_1, \ldots, y_n)\Big)\frac{\partial u}{\partial y_n} = 0 \quad (9)$$

The characteristic system of equation (9) is the following nonlinear system of differential equations.

$$\frac{dy_i}{dx} = \sum_{k=1}^{n} p_{ik}(x)y_k + g_i(x, y_1, \ldots, y_n),\ i = 1, \ldots, n. \tag{10}$$

The corresponding linear homogeneous system of differential equations is

$$\frac{dy_i}{dx} = \sum_{k=1}^{n} p_{ik}(x)y_k,\ i = 1, \ldots, n. \tag{11}$$

In the proof of Theorem 1 it was established that linear system (11) has exact generalized exponents β_k, $k = 1, 2, ..., n$; with respect to $q(x)$. There are equality

$$\sum_{k=1}^{n} \beta_k = \sum_{k=1}^{n} \lim_{x\to+\infty} \frac{1}{q(x)} \int_{x_0}^{x} p_{kk}(s)ds = \underline{\lim}_{x\to+\infty} \frac{1}{q(x)} \int_{x_0}^{x} \sum_{k=1}^{n} p_{kk}(s)ds$$

Consequently, linear system (11) is a generalized regular linear system with respect to $q(x)$. Let

$$\frac{dy}{dx} = P(x)y \tag{12}$$

is a vector matrix view of the linear system (11) and

$$\frac{dy}{dx} = P(x)y + g(x, y) \tag{13}$$

of nonlinear system (11). Further next lemma will be used: **Lemma 1.** *If the conditions of Theorem 2 are satisfied, then there exists a neighborhood of the point* y *and any solution of the nonlinear system of differential equations* (13) *with initial values from this neighborhood tends to zero as* $x \to +\infty$.
Proof. Lets take $\alpha > 0$ from 4th condition, i.e. $0 < \alpha < \beta_1$, where β_1 is senior generalized exponent with respect to $q(x)$ of the system (12) and in the system (13) irreplace

$$y = ue^{-\alpha[q(x)-q(x_0)]} \tag{14}$$

where $u(x)$ is new unknown function. Furthermore

$$\frac{du}{dx} = B(x)u + \nu(x, u) \tag{15}$$

where $B(x) = P(x) + \alpha\frac{dq}{dx}E$, $\nu(x, u) = e^{\alpha[q(x)-q(x_0)]}g\big(x, ue^{-\alpha[q(x)-q(x_0)]}\big)$. Vector function $\nu(x, u)$ is continuous on $x \in I$ and has continupus partials on u in the domain

$$\|u\| < he^{\alpha[q(x)-q(x_0)]}$$

Replacing in (14), due to choosing α, preserves the negativity of the senior generalized exponent and generalized correctness, therefore the linear system of differential equations

$$\frac{du}{dx} = B(x)u \tag{16}$$

is generalized regular and has negative generalized exponents with respect to $q(x)$. The nonlinear system of differential equations (15) with the initial condition $u(x_0) = y(x_0)$ is equivalent to an integral equation

$$u(x) = H(x)u(x_0) + \int_{x_0}^{x} K(x, s)\nu(s, u)ds \tag{17}$$

where $K(x, s) = H(x)H^{-1}(s)$ is a Cauchy matrix, $H(x)$ is a normalized fundamental matrix of the linear system (16).
Due to the negativeness of the generalized exponents of the linear system (16), there is an estimate

$$\|H(x)\| < C_1,\ x \in I,\ C_1 > 1 \tag{18}$$

and for $\varepsilon \in (0, (m-1)\alpha)$ (from condition 4), due to the negativity of the generalized exponents and regarding the generalized correctness of the linear system (16), the next estimate is true:

$$\|K(x,s)\| < C_2 e^{\alpha[q(s)-q(x_0)]}, \ x \geq s \geq x_0, \ C_2 > 1. \tag{19}$$

For $\nu(x,u)$ in the nonlinear system (15) the estimate holds:

$$\|\nu(x,u)\| \leq K e^{\alpha(1-m)[q(s)-q(x_0)]} \|u(x)\|^m \tag{20}$$

Estimating now by the norm in the interval $x_0 \leq x < x_0 + l$ of the existence of solutions of the integral equation (17), by virtue of (18), (19), (20), we will have

$$\|u(x)\| \leq C_1 \|u(x_0)\| \int_{x_0}^{x} C_2 K e^{\alpha(1-m)[q(s)-q(x_0)]} \|u(x)\|^m ds. \tag{21}$$

From inequality (21), using the Bihari Lemma ([29], p. 112), we find

$$\|u(x)\| \leq \frac{C_1 \|u(x_0)\|}{\left[1-(m-1)C_1^{m-1}\|u(x_0)\|^{m-1} C_2 K \int_{x_0}^{x} C_2 K e^{\alpha(1-m)[q(s)-q(x_0)]} ds\right]^{m-1}} \tag{22}$$

only if

$$(m-1)C_1^{m-1}\|u(x_0)\|^{m-1} C_2 K \int_{x_0}^{x} C_2 K e^{\alpha(1-m)[q(s)-q(x_0)]} ds < 1 \tag{23}$$

Since, due to condition 4), the inequality holds:

$$\int_{x_0}^{x} e^{\alpha(1-m)[q(s)-q(x_0)]} ds < \infty;$$

then inequality (23) can always be considered fulfilled by choosing a neighborhood of the initial values $u(x_0) = y(x_0)$. From formula (22) it follows that if $\|u(x_0)$ is sufficiently small, then for any $x \in [x_0, x_0 + l)$ the point $u(x)$ is the interior point of the domain

$$Z = \left\{ x_0 \leq x < \infty, \ \|u\| \leq \frac{h}{2} < h \right\}$$

and, therefore, the solution $u(x)$ is infinitely continued to the right, i.e. we can set $l = \infty$. Thus, in the infinite interval $x_0 \leq x < \infty$, the inequality

$$\|u\| \leq D\|u(x_0)\| < \frac{h}{2} \tag{24}$$

where D is some constant depending on the initial moment x_0. Returning to the variable y, by virtue of formula (14), when $x_0 \leq x < \infty$ and $\|y(x_0)\| < \delta < h$, we will have

$$\|y(x)\| \leq D\|y(x_0)\| e^{\alpha[q(x)-q(x_0)]} \tag{25}$$

where the constant δ is small enough. It follows that any solution of the nonlinear system of differential equations (13) with initial values from $\|y(x_0)\| < \delta$ neighborhoods tends to zero as $x \to +\infty$.
Lemma 1 is proved. Now, by Lemma 1, we take the general solution of the characteristic system (10) of the Cauchy form.

$$\begin{cases} y_1 = y_1(x, x_0, y_1^0, \ldots, y_n^0) \\ \qquad \ldots\ldots\ldots \\ y_n = y_n(x, x_0, y_1^0, \ldots, y_n^0) \end{cases} \tag{26}$$

where $y_i(x_0) = y_i^0$, $i = 1, \ldots, n$; $\sqrt{(y_1^0)^2 + \ldots + (y_n^0)^2} < \delta$. Solving (26) with respect to $y_1^0, \ldots, y_n^0$, we obtain an integral basis

$$\begin{cases} u_1(x, x_0, y_1, \ldots, y_n) \\ \qquad \ldots\ldots\ldots \\ u_n(x, x_0, y_1, \ldots, y_n) \end{cases}$$

of the equation (9). Consequently

$$\begin{pmatrix} u_1(x, x_0, y_1, \ldots, y_n) \\ 0 \\ \ldots \\ 0 \end{pmatrix}, \ldots, \begin{pmatrix} 0 \\ \ldots \\ 0 \\ u_n(x, x_0, y_1, \ldots, y_n) \end{pmatrix}$$

integral basis of the linear homogeneous system (8), which is adjacent to zero in the parameter x_0, as $x_0 \to +\infty$. Theorem 2 is proved.

4 Results and discussion

The paper considers linear systems of first-order partial differential equations. Existing of integral basis of a linear system of partial differential equations of the first order adjoining to some solution was proved. In the proof established a sighn, which became interesting, because a nonlinear system of ordinary differential equations has a solution tending to zero. The existence of an integral basis of an adjacent to zero linear homogeneous system of first-order partial differential equations with nonlinear coefficients is proved.

5 Conclusion

Exists of the integral basis of a linear system of partial differential equations of the first order adjoining to some solution was proved. A sign at which a nonlinear system of ordinary differential equations has a solution tending to zero was established. It is proved that there exists an integral basis of an adjacent to zero linear homogeneous system of first-order partial differential equations with nonlinear coefficients.

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References

[1] Courant R., *Uravneniya s chastnymi proizvodnymi.* [Partial equations]. (M: Mir, 1964): 845

[2] Mizohata S.S., *Teoriya uravnenii s chastnymi proizvodnymi.* [Partial equations theory]. (M.: Mir, 1977): 504

[3] Smirnov V.,*Kurs vysshei matematiki.* [Higher math course]. V.4, 2nd part, (M.: Nauka, 1981): 551

[4] Bers L., John D. and Shechter M., *Uravneniya s chastnymi proizvodnymi.* [Partial equations]. (M.: Mir, 1966): 352

[5] Trikomi F., *Lektsii po uravneniyam v chastnyh proizvodnyh.* [Lectures in partial equations]. (IL., 1957): 443

[6] Hartman P., *Obyknovennye differentsialnye uravneniya.* [Ordinary differential equations]. (M.:Mir, 1970): 719

[7] Petrovsky I.G., *Lektsii ob uravneniyah s chastnymi proizvodnymi.* [Lectures in partial equations]. 3rd ed. (M., 1961): 400

[8] Petrovsky I.G., *Lektsii po teorii obyknovennyh differentsialnyh uravnenii.* [Lectures in ordinary differential equations]. 6th ed., (M., 1970): 473

[9] Elsgoltc L., *Differentsialnye uravneniya.* [Differential equations]. (M., 2013): 312

[10] Yanenko N.N. and Rojdestvensky B.L., *Sistemy kvazilineinyh uravneenii i ih prilozhenie k gazovoi dinamike.* [Systems of quaziliniear differential equations and their application to gas dynamics]. VI.7-9, (M., 1978): 676

[11] Kamke E., *Spravochnik po differentsialnym uravneniyam v chastnyh proizvodnyh pervogo poryadka.* [Referense book in first-order partial differential equations]. (M.: Nauka, 1966): 260

[12] Massera H.L., *Lineinye differentsialnye i funktsionalnye prostranstva.* [Linear differential and functional spaces] (M.:Mir, 1970): 456

[13] Wazewski T., ”Sur l'appreciation du domain d'existence des integrals de l'equation aux derives partielles du premier ordre” [On the appreciation of the domain of existence of the integrals of the equation with partial derivatives of the first order], *Ann. Soc. Polon. Math.* VI.9, No.14, (1935): 149-177

[14] Wazewski T., ”Ueber die Bedingungen der Existenz der Integrale partieller Differentialgleichungen erster Ordnung” [On the conditions of existence of the integrals of partial differential equations of the first order], *Math. Zeit.*, VI.7-9 No.43. (1938): 522-532

[15] Gelfand I., *Nekotorye zadachi teorii kvazilineinyh uravnenii* [Some problems of quazilinear equations theory], No. 14(2). (UMN, 1959): 87-158.

[16] Gross W., "Bemerkung zum Existenzbeweise bei den partiellen Differentialgleichungen erster Ordnung"[Remark on proof of existence in the first order partial differential equations], *S.-B.K. Akad. Wiss. Wien, KI. Math. Nat.*, VI.7-9, (1914): 2233-2251

[17] Digel E., ”Uber die Bedingungen der Existenz der Integrale partieller Differentialgleichungen erster Ordnung” [On the Conditions of Existence of the Integrals of Partial Differential Equations of the First Order], *Math Z*, (1938): 445-451

[18] Caratheodory C., ”Variationsrechnung und partielle Differentialgleichungen erster Ordnung” [Variational calculus and partial differential equations of first order], VI 6, *Leipzig und Berlin:B. G. Teubner*, (1935): 7-9

[19] Kruzhkov S.N., ”Kvazilineinye uravneniya pervogo poryadka so mnogimi nezavisimymi peremennymi” [First order quazilin-iear equations with many independent variables], 81(2). *Mat. sbornik*, (1970): 228-255.

[20] Kovalevskaya S., "Zusatze und Bemerkungen zu Laplace's Untersuchung uber die Gestalt der Saturnsringe"[Additions and Remarks on Laplace's Investigation of the Shape of Saturn's Rings], *Astronomische Nachrichten*, CXI. (1885): 18-21

[21] Zubov V.I., *Voprosy teorii vtorogo metoda Lyapunova postroeniya obsh'ego v oblasti asimptoticheskoi ustoichivocti.* [General asimptotically stable domain building problems of the second method in lyapunov theory] Vol. XIX, 2nd edition (PMM., 1955): 25-31

[22] Frobenius G., "Ueber das Pfaffsche Problem", *Journal for die reine und angewandte Mathematik*, (1877): 230-315

[23] Perron O., "Ueber diejenigen Integrale linearer Differentialgleichungen, welche sich an einer Unbestimmtheitsstelle bestimmt verhalten"[On the integrals of linear differential equations which are determined at an uncertainty point], *Math. Ann.*, VI.13, No 70. (1911): 1-32

[24] Plis A.,"Characteristics of nonlinear partial differential equation", *Bull.Acad. Polon. Sci.*, No 2. (1954): 419-422

[25] Hartman P.,"On Jacobi bracket", *Amer. J. Math*, (1957): 187-189

[26] Hormander L., "On the uniqueness of the Cauchy problem I", *Math. Scand.*, No. 6. (1958): 213-225.

[27] Hormander L., "On the uniqueness of the Cauchy problem II", *Math. Scand.*, No. 7. (1959): 177-190.

[28] Aldibekov T.M., Aldazharova M.M., "On a first-order partial differential equation", *Veszprem Conference on Differential and Difference Equations and Applications. Program and Abstracts. Faculty of Information Technology University of Pannonia Veszprem, Hungary. July 2 - July 5*, (2018): 34

[29] Demidovich B.P., *Lektsii po matematicheskoi teorii ustoichivosti* [Lectures on math. stability theory]. (M: 1967): 472

[30] Aldibekov T.M., Aldazharova M.M., "On the Stability by the First Approximation of Lyapunov Characteristic Exponents in Critical Cases", *Differential Equations*, Vol. 50, No. 10. (2014): 1-5