4-бөлім | Раздел 4 | Section 4

Қолданылмалы математика | Прикладная математика | Applied Mathematics

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Research of controllability of dynamical systems with constraints on control using interval mathematics

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The article is devoted to the actual problem of the mathematical theory of controllability. It investigated the mathematical model of control, described by ordinary differential equations, taking into account the restrictions on the control. As is known, the problem of finding controllability of dynamic systems with phase and control constraints is still relevant. There are many approaches to solving the determined problem. The classical control theory is being modified today and it finds new methods for solving problems of controllability, optimal control and stability, the solutions obtained. In the course of studying the controllability of a dynamic system, the authors applied interval mathematics, which made it possible to obtain an effective controllability criterion for dynamic systems with phase and control constraints. This method is applicable for a certain class of problems in which the data are described by the normal distribution law.
The constructiveness of the proposed criterion is demonstrated in two examples. The first is a model problem described by 2-nd order equations. The second is an electromechanical tracking system of an automatic manipulator, described by equations of the 3rd order. Thus, for dynamic systems, we obtained a sufficient condition for controllability.
Key words: criterion, controllability, control, interval mathematics, dynamical systems, interval, interval vector, differential equation.

Интервалды математиканы қолдана отырып, басқаруға шектеу қойылған кездегі динамикалық жүйлердің басқарылуын зерттеу

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Мақала басқарудың математикалық теориясының өзекті мәселесіне арналған. Және мақалада қарапайым дифференциалдық теңдеулермен сипатталған басқарудың математикалық моделі, басқаруға қатысты шектеулерді ескере отырып зерттеледі. Белгілі болғандай, фазалық шектеулермен және басқаруға қатысты шектеулері бар динамикалық жүйелердің басқарылуын табу проблемасы қазіргі уақыттада маңызды. Аталған мәселені шешудің түрлі жолдары бар. Классикалық басқару теориясы бүгінгі таңда өзгертілуде және басқарудың, оңтайлы басқарудың және тұрақтылықтың, алынған шешімдердің мәселелерін шешудің жаңа әдістері табылуды. Динамикалық жүйенің басқарылуын зерттеу барысында авторлар фазалық және басқаруға қойылған шектеулері бар динамикалық жүйелер үшін тиімді басқарылатын талапқа қол жеткізуге мүмкіндік беретін интервалды математиканы қолданды. Бұл әдіс қарапайым таралу заңымен сипатталатын арнайы есептер тобын шешуге арналған.
Ұсынылған критерий екі мысалда көрсетілген. Біріншісі – 2-ші реттік теңдеулермен сипатталған модельдік мәселе. Екіншісі - 3-ші ретті теңдеулермен сипатталатын автоматты манипулятордың электромеханикалық қадағалау жүйесі. Осылайша, динамикалық жүйелер үшін басқарудың жеткілікті шарты алынды.
Түйін сөздер: критерий, бақылау, интервалды математика, динамикалық жүйелер, интервал, интервалды вектор, дифференциалды теңдеу.

Исследование управляемости динамических систем при наличии ограничения на управления с применением интервальной математики

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Статья посвящена актуальной проблеме математической теории управляемости. В ней исследована математическая модель управления, описываемая обыкновенными дифференциальными уравнениями, учитывающая ограничения на управление. Как известно, проблема нахождения управляемости динамических систем с фазовыми ограничениями и ограничениями на управление до сих пор остается актуальной. Существует множество подходов к решению названной задачи. Классическая теория управления сегодня модифицируется и находит новые методы решения задач управляемости, оптимального управления и устойчивости, полученных решений. В ходе исследования управляемости динамической системы авторы применили интервальную математику, которая позволила получить эффективный критерий управляемости динамический систем с фазовыми ограничениями и ограничениями на управления. Данный метод применим для определенного класса задач, в которых данные описываются нормальным законом распределения.
Конструктивность предложенного критерия демонстрируется на двух примерах. Первый – модельная задача, описываемая уравнениями 2-го порядка. Второй – электромеханическая следящая система автоматического манипулятора, описываемая уравнениями 3-го порядка. Таким образом для динамических систем получили достаточное условие управляемости.
Ключевые слова: критерий, управляемость, управление, интервальная математика, динамические системы, интервал, интервальный вектор, дифференциальное уравнение.

1 Introduction

The need to obtain effective criteria for ensuring the transfer of a dynamic system from a certain initial state to the desired final state for a certain period of time is an urgent task. For example, the inability to transfer electric power system after the electric circuit (or other emergency) the operating mode for a particular time interval leads to large economic losses.

The above technical problem in mathematical control theory is called the problem of controllability [1].
The main approaches used in the mathematical theory of controllability imply the accuracy of the presentation of the initial data. However, in real dynamic systems physical parameters are measured with some error related to wear and operating conditions (temperature, etc.). To take into account these features, you can use the new direction of computational mathematics - interval analysis, the main idea of which is to replace arithmetic operations and real functions on real numbers with interval operations and functions that convert the intervals containing these numbers [2]. The use of the "classical" interval analysis in solving the problem of controllability of dynamic systems makes it possible to obtain a criterion of guaranteed stability. However, these solutions are obtained "super-sufficient", which in practice is a strict limitation. In work [3] "practical" interval mathematics was introduced, which allows one to expand the field of application (with appropriate restrictions on the systems under study).
Interval analysis is currently actively developing in many countries. Initially, interval methods appeared as a means of automatic control of rounding errors on a computer and later turned into one of the sections of modern applied mathematics. Interval methods have long gone beyond a purely theoretical study and are widely used in practice with the help of appropriate software. As a result, there were interval arithmetic, interval algebra, interval topology, interval methods for solving problems of computational mathematics, optimal control, stability, etc. [2].
In scientific research, technology and mass production, it is often necessary to measure any values (length, mass, current strength, etc.). When repeating measurements of the same object, performed with the same measuring device with the same care due to the influence of various factors, the same data is never obtained. Such factors include random vibrations of individual parts of the device, physiological changes in the sensory organs of the contractor, various unaccounted changes in the environment (temperature, optical, electrical and magnetic properties, etc.). Although it is not possible to predict the result of each individual measurement in the presence of random dispersion, it corresponds to the "normal distribution curve". In this case, the bulk of the results obtained will be grouped around some central or average value of a, which is answered by an unknown "true value" of the object being measured. Deviations in one direction or another will occur even less often, the greater the absolute value of such deviations, and are characterized by the value of σ - the standard deviation. On the section from $a-\sigma$ to $a+\sigma$ the share on average is equal to 0,6287 (68,27%) of the total mass of repeated measurements. Within the limits $(a-2\sigma,\ a+2\sigma\ )$ an average of 0,9545 (95,45%) of all measurements is placed, and in the area $(a-3\sigma,\ a+3\sigma\ )$ - already 0,9973 (99,73%), so, only 0,0027 (0,27%) of the total number of measurements goes beyond the "three lung" limits, i.e. insignificant share of them.
"Classical" interval arithmetic assumes that all values of the interval are equally probable. Therefore, all the results obtained with its help encompass all possible values and are "super-sufficient".
We introduce the formal concept of the interval a in the following form:

$$a = [[\bar{a} - \varepsilon_a, \bar{a} + \varepsilon_a]] = (\bar{a}, \varepsilon_a)$$

where $\bar{a}$ - the middle of the interval (or the mathematical expectation), ε_a - interval width (or variance). Denote the set of all such intervals as $I_{\text{вер}}(R)$.

Let a, b, c are intervals from $I_{\text{Вер}}(R)$. We introduce the following interval arithmetic operations (assuming that the intervals are independent normally distributed quantities):

1. Addition of two intervals $a, b \in I_{\text{Вер}}(R) : c = a + b$,

$$\bar{c} = \bar{a} + \bar{b}, \qquad \varepsilon_c = \sqrt{\varepsilon_a^2 + \varepsilon_b^2}.$$

2. Subtraction of two intervals $a, b \in I_{\text{Вер}}(R) : c = a - b$,

$$\bar{c} = \bar{a} - \bar{b}, \qquad \varepsilon_c = \sqrt{\varepsilon_a^2 + \varepsilon_b^2}.$$

3. Multiplication of two intervals $a, b \in I_{\text{Вер}}(R) : c = a * b$,

$$\bar{c} = \bar{a} \cdot \bar{b}, \qquad \varepsilon_c = \sqrt{\bar{a}^2\varepsilon_b^2 + \bar{b}^2\varepsilon_a^2}.$$

   reverse interval $a, b \in I_{\text{Вер}}(R) : c = \frac{1}{a}$;

$$\bar{c} = \frac{1}{\bar{a}}, \qquad \varepsilon_c = \frac{\varepsilon_a}{\bar{a}^2}.$$

4. division of two intervals $a, b \in I_{\text{Вер}}(R) : c = \frac{1}{a}$

$$\bar{c} = \frac{\bar{a}}{\bar{b}}, \qquad \varepsilon_c = \sqrt{\frac{\bar{a}^2\varepsilon_b^2}{\bar{b}^4} + \frac{\varepsilon_a^2}{\bar{b}^2}}.$$

For the numerical computation of interval expressions developed software that allows you to operate as classical interval arithmetic [2], and interval arithmetic, introduced in the paper [3, 4] (taking into account the uneven distribution of values within the interval).

2 Literature review

Management problems in various areas of scientific research have been relevant for a long time, today we are exploring various objects for which it is necessary to find the required management.

The classical theory of control of multidimensional systems [5, 6], which was actively developed in the 20th century, is being modified and strengthened in new, expanded problems. One of the current trends is the construction of models of controlled systems with a priori taking into account the uncertainties arising in the input data and parameters.

In this article, we consider one of the classical problems of control, the problem of controllability of dynamic systems [7], based on the ideas of interval analysis actively developing from the second half of the 20th century [8, 9].

The complexity of real objects often does not allow us to give an exact model description of the object. Thus, the uncertainty was originally incorporated into the model under study. In such systems, controls are used to satisfy a certain, often extreme, criterion that specifies the characteristics (properties) of the system that are desirable for the user. The construction of managements adjusted for accounting for uncertainties is a current, actively developing area of research [10–13]. Recently popular interval analysis methods, which suppose known changes to the boundaries of segments (intervals) of parameter changes, are actively used in control problems. In [14], interval methods in control theory were divided into the following groups:

- Methods based on the use of the apparatus of sensitivity functions, the frequency representation of the object [12, 15, 16].
- Methods with infinite gains [17]
- Adaptive methods [18].
- Methods of modal control [19–23].
- Optimal control [24, 25].

One of the disadvantages of using interval analysis: the operation of interval multiplication greatly expands the interval [9], which overestimates the degree of uncertainty of the initial data in the calculation process when solving a problem. The second significant disadvantage is that in many cases, building control requires the condition of complete controllability for all independent realizations of the system's interval parameters.
To address these shortcomings, various approaches are used [26–32]. The control constructed for the deterministic system is applied to the interval system. Based on the analysis of the beam [13] of the trajectories of a closed system, estimates are made for the size of the intervals, which preserve the required properties of the constructed control as a whole for the interval system.
Pros of the variational approach. First, conventional deterministic methods of solution are used that do not involve the apparatus of interval arithmetic, which leads to simpler algorithms from a computational point of view. Secondly, the requirements of controllability are usually put forward only with respect to the chosen deterministic system, which is a much milder condition compared with the requirement of controllability for all systems from the allowable intervals.
The disadvantage of this approach is the overestimation of estimates when constructing the external approximation of the beam of trajectories of the interval system, and, as a result, the narrowing of guarantees for the fulfillment of the criterion. Such an approach is suitable only for fairly small initial uncertainty intervals. Our approach to the use of interval analysis allows us to obtain an effective controllability criterion for dynamic systems with a control constraint.

3 Materials and Methods

3.1 Formulation of the problem

The control system described by the following linear ordinary differential equations

$$\dot{x} = Ax + Bu, \tag{1}$$

where $A - n \times n$ - constant matrix, $B - n$ - dimensional constant vector, $x - n$ - dimensional vector of system state, u - scalar control.
The following restriction is imposed on management

$$l_1 \le u(t) \le l_2, \quad t \in [0, T]\,. \tag{2}$$

The task is to determine whether there is a control that satisfies the restriction (2) and transfers the system (1) from the initial state

$$x(0) = x_0 \tag{3}$$

to the final specified state

$$x(T) = x_1 \tag{4}$$

for a fixed time T.
The study of the problem in the presence of restrictions on the management of the forms (2) is of some interest, since there are still no effective criteria [1]. In addition, the results can be used in solving practical problems of optimal control of systems described by ordinary differential equations with fixed ends and restrictions on the control actions. In particular, equations of the form can describe robotic or electric power systems, where the coefficients of the matrix and vector are determined through parameters (such as weight, metric characteristics, inertia, etc.), which are usually calculated with some error.

3.2 Application of interval analysis to obtain the criterion of controllability

In recent years, such a direction of computational mathematics as interval has been developed, that operate not with numbers, and intervals (which allow to take into account the error of the initial data) [2].
Next, we apply the results of interval mathematics to the problem of controllability. If $\Phi(t, \tau) = \theta(t) \cdot \theta^{-1}(\tau)$, where $\theta(t) = e^{At}$ - the fundamental matrix of solutions of the system described by a homogeneous vector differential equation

$$\dot{x} = Ax. \tag{5}$$

Introducing the notation:

$$u = v + \frac{l_1 + l_2}{2}, \qquad L = \frac{l_2 - l_1}{2}.$$

Then the system (1) can be represented as

$$\dot{x} = Ax + B\frac{l_1 + l_2}{2} + Bv, \tag{6}$$

Where

$$-L \le v(t) \le L, \forall t \in [t_0, t_1]\,. \tag{7}$$

The solution of equation (6) can be represented as

$$x(t) = \Phi(t,t_0)x(t_0) + \frac{l_1+l_2}{2}\int_{t_0}^{t}\Phi(t,\tau)Bd\tau + \int_{t_0}^{t}\Phi(t,\tau)Bv(\tau)d\tau. \tag{8}$$

Introducing the notation $y_1 = x_1 + \Phi(T,0)x_0 - \frac{l_1+l_2}{2}\int_0^T \Phi(t,\tau)Bd\tau, \quad f(\tau) = \Phi(T,\tau)B.$
Then the problem of controllability is reduced to the existence of the solution of the integral equation

$$y_1 = \int_0^T f_1(\tau)\mathrm{v}(\tau)d\tau, \tag{9}$$

satisfying the condition (7).
To solve this problem, we apply the results of interval analysis [2].
Replace the integral in the right part (9) with the next

$$h = \sum_{i=1}^{n} f_i \mathrm{v}_i \text{ where } n = \tfrac{T}{h},\ h \geq 0, \quad -L \leq \mathrm{v}_i \leq L, \quad i = \overline{1,n}.$$

Denote by $\bar{f}_l = (f_i, 0)$ – the interval centered in f_i and radius of 0, $\bar{\mathrm{v}}_l = (0, L)$ - the interval from $-L$ to L [2].
If $i = 1$. Calculating $\bar{f}_l\bar{\mathrm{v}}_l = (0, |f_1, L|)$ - the interval with the center at point 0 and radius of $|f_i * L|$, here all arithmetic operations are performed according to the rules defined for interval calculations [2].
Obviously multitude

$$\{hf_1\mathrm{v}_1 | \forall \mathrm{v}_1 \in (-L, L)\}$$

same as interval

$$h(0, |f_1 L|) \quad \forall h \geq 0.$$

By the method of mathematical induction it can be shown that the set of

$$\left\{ h\sum_{i=1}^{n} f_i \mathrm{v}_i \mid \forall \mathrm{v}_i \in (-L, L), i = \overline{1,n} \right\}$$

same as interval

$$h(0, \sum_{i=1}^{n} |f_i L|) \quad \forall h \geq 0.$$

It can be seen that the set

$$\left\{ \int_0^T f(\tau)\mathrm{v}(\tau)d\tau \mid \mathrm{v}(t) \in (-L, L), \forall t \in [0, T] \right\}$$

same as interval $y_2 = \int_0^T f(\tau)\bar{v}d\tau$, where all arithmetic operations are performed using interval calculations [2].

Thus, the following theorem is proved.

Theorem. In order for the system (6)-(7) to be controllable, it is necessary and sufficient that the vector belongs to the interval vector y_2.

For numerical modeling in Pascal language the software is developed, which implements calculations of the proposed criterion and uses the library of interval calculation [4].

Lemma (Gronwall-Belman) [5]. If a scalar continuous function $x(t)$ and $g(t) \geq 0$ satisfy the inequality

$$x(t) \leq \alpha(t) + \int_0^t g(s)x(s)ds, \quad t \geq 0,$$

where $\alpha(t)$ – some non-decreasing function. Then

$$x(t) \leq \alpha(t) exp\left(\int_0^t g(s)ds\right).$$

Applying the Gronwall-Belman Lemma to the problem (1) and (4) we obtain the following inequality

$$\|x(t_1)\| \leq \|x(t_0)\| + \int_0^{t_1} \|B(\tau)\| u(\tau)d\tau exp\left(\int_0^{t_1} \|A(\tau)\| d\tau\right). \tag{10}$$

Choose as the norm of the vector $\|x\| = \sum_{i=1}^{n} |x_i|$ and the norm of the matrix $\|A\| = \max_{1\leq j\leq n}\left(\sum_{i=1}^{n} |a_{ij}|\right)$.

3.3 Verification of the obtained controllability criterion with examples

Example 1. The second order system is considered as an example

$$\begin{aligned} \dot{x_1} &= 3x_1 + 2x_2 + u \\ \dot{x_2} &= x_1 - x_2 - u, \end{aligned} \tag{11}$$

when partial conditions

$$x_0 = (1, 1) \qquad t_0 = 0,\ t_1 = 1. \tag{12}$$

The conditions for management and endpoint will vary.

$$\text{a)if} - 1.5 \leq u(t) \leq 1.0, \quad t \in [0, 1]. \tag{13}$$

Calculate the value of the interval vector $y_2 = \begin{pmatrix} (39.97, 17.99) \\ (9.33, 7.50) \end{pmatrix}$.

Substituting the values of the example parameters in (10) we get $\|x(t_1)\| \leq 4e^4 \approx 218,3$. Consequently, when $x_1 = (109, 110)^*$ according to the Gronwall-Belman Lemma, the system (11)-(13) is not controllable, i.e. there is no control satisfying the restriction $-1.5 \leq u(t) \leq 1/0$ and translating system for time 1 of point $x_0 = (1, 1)^*$ into the point $x_1 = (109, 110)^*$.
Applying the proposed criterion, we obtain that the vector $x_1 = (109, 110)^*$ does not belong to the interval vector y_2, since $109 > 39.97 + 17.99$ and $110 > 9.33 + 7.5$, that means there is no controllability for both variables.
b) at the point x_1 we take the solution of the Cauchy problem (11)-(12) at time t_1 at management $u \equiv 0$, which satisfies the restriction (13): $x_1 = (41.13, 9.43)$.
Applying the proposed criterion, we obtain that the vector belongs to the interval vector y_2, since $39.97 - 17.99 < 41.13 < 39.97 + 17.99$ and $9.33 - 7.50 < 9.43 < 9.33 + 7.50$, i.e. the system is controllable.
Example 2. A system of equations of the third order of the form (1) describing the state of the circuits of an Electromechanical tracking system of an automatic manipulator is considered [5], where $x = x(t) = (i_я(t), \Omega(t), \theta(t))^*$ – the state vector of the system, $u = u(t) = (\Omega_0(t), \theta_0(t))^*$ – control input vector-system signal, with restrictions

$$l_i^1 \leq u_i \leq l_i^2, \quad i = \overline{1,2}; \quad t \in [t_0, t_1], \tag{14}$$

$$A = \begin{pmatrix} -\left(\frac{1}{T_я} + \frac{k_{oc}k_{ум}R_ш}{L_я}\right) & -\left(\frac{k_e}{L_я} + \frac{k_1k_{ум}k_m}{L_я}\right) & -\frac{k_1k_{ум}k_n}{L_я} \\ \frac{k_М}{J} & -\frac{1}{T_М} & 0 \\ 0 & 1 & 1 \end{pmatrix}, \qquad B = \begin{pmatrix} -\frac{k_1k_{ум}k_Г}{L_я} & \frac{k_1k_{ум}k_n}{L_я} \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Numerical values of matrix coefficients A and B depend on parameters and structure of the tracking system.
If

$$x_0 = (1, 1, 1) \tag{15}$$

$T = 1$,
$T_я = 2,\ L_я = 3,\ k_{oc} = 1,\ k_{ум} = 1.5,\ R_ш = 1.1,\ k_e = 2.1,$
$k_1 = 0.1,\ k_m = 2,\ k_n = 4,\ k_Г = 6,\ J = 5,\ T_М = 4.$

Then the system of equations (1) is represented as

$$\begin{aligned} \dot{i_я} &= -1.05 i_я - 0.8\Omega - 3.0\theta - 3.0\Omega_0 + 2.0\theta_0, \\ \dot{\Omega} &= 0.4 i_я - 0.25\Omega, \\ \dot{\theta} &= \Omega + \theta. \end{aligned}$$

Setting a constraint on the control vector $u = (\Omega_0(t), \theta_0(t))^*$ in form

$$\begin{aligned} -0.4 \leq \Omega_0 \leq 0.6, &\quad t \in [0, 1]. \\ -0.25 \leq \theta_0 \leq 1.25, &\quad t \in [0, 1]. \end{aligned} \tag{16}$$

Calculate the value of the interval vector $y_2 = \begin{pmatrix} (4.94\ 12.29) \\ (0.14\ 1.62) \\ (4.33\ 5.87) \end{pmatrix}$.

Substituting the values of the example parameters in (10), we obtain

$$\|x(1)\| \le (3 + 3 * 1.85)e^4 \approx 466.6$$

Then at $x(1) = (160, 160, 150)^*$ the system is not manageable, i.e. there is no management of the translating system in time $T = 1$ from the point $(1, 1, 1)^*$ to the point $x(1) = (160, 160, 150)^*$.

4 Results

Applying the proposed criterion, we get that the vector $x(1) = (160, 160, 150)^*$ does not belong to the interval vector y_2, as $160 > 4.94+12.29, 160 > 0.14+1.62$ and $150 > 4.33+5.87$, that means there is no controllability for the three variables.
As the point $x(1)$ we'll take the solution of the Cauchy problem (1) at a time T at management $u \equiv 0$, which satisfies the restriction (16), then $x(1) = (-4.87, 0.12, 4.1)^*$.
Applying the proposed criterion, we obtain that the vector belongs to the interval vector y_2, as $4.94 - 12.29 < -4.87 < 4.94 + 12.29, 0.14 - 1.62 < 0.12 < 0.14 + 1.62$ and $4.33 - 5.87 < 4.1 < 4.33 + 5.87$, that means that the system is manageable.

5 Conclusion

For a linear control system with limited control on the basis of interval mathematics, a controllability criterion is obtained. A feature of the obtained criterion is its algorithmic constructiveness.
Results of numerical calculations show the effectiveness of the proposed controllability criterion and the possibility of their application in practical applications.

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