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RESEARCH OF THE STRESS STATE OF AN ELEMENT OF A THICK-WALL PIPELINE UNDER CONDITIONS OF POWER AND CORROSION EFFECT

The stress state of an element of a thick-walled pipeline is studied under conditions of power and corrosion effect in the statement of plane deformation. The material of the element under the influence of external loads goes into an elastic-plastic state. The corrosive effect of the pumped medium leads to softening of the material in the plastic zone. This softening of the material is taken into account by a special inhomogeneity function in the Tresca-Saint-Venant plasticity condition. The elastic-plastic problem in the axisymmetric setting (uniform pressure) and non-axisymmetric setting (non-uniform external pressure along the contour) is considered. The problem is solved by the method of sharing static and physical equations for the considered elastoplastic material and the perturbation method in the theory of an elastoplastic body. An assessment of the strength and bearing capacity of a thick-walled element under corrosive force effect is given.

Key words: thick-wall element, elastoplastic state, corrosion damage to the material, plastic inhomogeneity, softening function.

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Қалың қабырғалы құбыр элементінің күштік және коррозиялық әсер ету жағдайларында кернеулі күйін зерттеу

Жазық деформация қойылымында күштік және коррозиялық әсер ету жағдайларында қалың қабырғалы құбыр элементінің кернеулі күйі зерттелді. Сыртқы жүктемелердің әсерінен элемент материалы серпімді-иілгіш күйге өтеді. Айдалатын ортаның коррозиялық әсері иілгіш аймақта материалдың жұмсаруына әкеледі. Материалдың жұмсаруы Треск-Сен-Венанның иілгіштік шартында біртексіздіктің арнайы функциясымен ескеріледі. Осесимметриялы қойылымда (бірқалыпты қысым) және симметриялы емес қойылымда (контұры бойынша бірқалыпты емес сыртқы қысым) серпімді иілгіш есеп қарастырылды. Есеп қарастырылып отырған серпімді иілгіш материал үшін статикалық және физикалық теңдеулерді бірлесіп пайдалану әдісімен және серпімді иілгіш дене теориясында ұйытқулар әдісімен шешілді. Коррозиялық-күштік әсер ету жағдайында қалың қабырғалы элементтің беріктігі мен көтергіштік қабілеттілігіне баға берілді.

Түйін сөздер: қалың қабырғалы элемент, серпімді иілгіш күй, материалдың коррозиялық зақымдануы, иілгіштік біртексіздік, беріктендіру функциясы.

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Исследование напряженного состояния элемента толстостенного трубопровода в условиях силового и коррозионного воздействия

Исследовано напряженное состояние элемента толстостенного трубопровода в условиях силового и коррозионного воздействия в постановке плоской деформации. Материал элемента под действием внешних нагрузок переходит в упругопластическое состояние. Коррозионное воздействие перекачиваемой среды приводит к разупрочнению материала в пластической зоне. Это разупрочнение материала учитывается специальной функцией неоднородности в условии пластичности Треска-Сен-Венана. Рассмотрена упругопластическая задача в осесимметричной постановке (равномерное давление) и неосесимметричной постановке (неравномерное по контуру наружное давление). Задача решена методом совместного использования статических и физических уравнений для рассматриваемого упругопластического материала и методом возмущений в теории упругопластического тела. Дана оценка прочности и несущей способности толстостенного элемента при коррозионно-силовом воздействии.

Ключевые слова: толстостенный элемент, упругопластическое состояние, коррозионные повреждения материала, пластическая неоднородность, функция разупрочнения.

1 Introduction

According to the normative documents [1, 2] in engineering practice, when calculating, as a rule, elastically deformable elements of pipelines are considered. At the same time, with an increase in external loads, the material of a thick-walled element passes into an elastoplastic state. Most pipelines are operated with prolonged exposure under increased loads in aggressive work environments. The corrosive effect of an aggressive pumped medium leads to damage and microcracking of the material during operation. In the plastic zone, as a result of corrosion-force action, a softening of the element occurs, which leads to a significant decrease in its strength and bearing capacity. The danger of exposure to aggressive media on the material working under load is due to the fact that in such cases the pipelines fail in a very short time, sometimes even with accidents. In this regard, this work is devoted to the study of the stress-strain state, strength and load-bearing capacity of the elastic-plastic element of a thick-walled pipeline under conditions of force and corrosion, which leads to the material softening in the plastic zone.

2 State of the problem

The effect of large loads and aggressive working environments (containing hydrogen sulfide, mercaptans), penetrating into the volume of structural elements, leads to their damage [3, 4]. The effect on the strength of the material of an individual crack or the final system of cracks in the framework of the linear theory of elasticity is the subject of linear fracture mechanics [5]. Nonlinear fracture mechanics [6] involves the analysis of the criteria and properties of material ductility in the vicinity of the tip of a single crack. Corrosion damage to the contact zone of the element material under high stress and corrosive environment leads to the appearance of many defects and microcracks, the account of the overall impact of which is not possible in the framework of fracture mechanics. Mathematical models and criteria describing the occurrence and development of scattered microdamages are formulated mainly within the framework of continuum mechanics [7]. Different types of corrosion affect on different parameters of metal structures. For example, the general mechanochemical continuous corrosion of metals leads to a loss of weight and volume of the material without significant loss of its strength [8]. The calculation of the decrease in the wall thickness of a pipeline subject to general

mechanochemical corrosion in the framework of the linear theory of elasticity is given in [9]. Moreover, the process of metal loss is significantly extended over time. In case of intergranular and transcrystalline corrosion, on the contrary, the mechanical properties of the metal change practically without loss of its weight [8]. The combined action of corrosion and static tensile stresses leads to a gradual decrease in the resistance to plastic deformation and a decrease in the plasticity limit in ductile metals, carbon and low alloy steels, and various alloys [10]. The decrease in the mechanical properties of the material during loading due to the accumulation of damage and defects can be taken into account in the framework of the plasticity theory of inhomogeneous bodies [11, 12, 13]. In [11, 12, 13], to take into account the accumulation of damage to the material, it was proposed to introduce a special softening function (radial inhomogeneity of strength characteristics) in the criteria used for plasticity of the material for axisymmetric and some plane problems. In [14], modified plasticity criteria were used, which can take into account the accumulation of material damage in the case of more complex boundary conditions under which plastic inhomogeneity should change in accordance with the change in the elastoplastic boundary. In this paper, this is exactly the approach taken.

3 Solution of the problem

3.1 Basic assumptions and relationships

The element of a thick-walled extended pipeline is in flat deformation conditions. Consider the cross section of a pipeline with an internal circuit $a_0 + f_1(r, \theta)$ and an external circuit $1 + f_2(r, \theta)$ in the polar coordinate system r, θ . The material of the pipeline is considered perfectly elastic-plastic.

The equations of equilibrium of the pipeline have the form:

$$\frac{\partial \sigma_r}{\partial r} + \frac{1}{r} \frac{\partial \tau_{r\theta}}{\partial \theta} + \frac{\sigma_r - \sigma_\theta}{r} = 0, \quad \frac{\partial \tau_{r\theta}}{\partial r} + \frac{1}{r} \frac{\partial \sigma_\theta}{\partial \theta} + 2 \frac{\tau_{r\theta}}{r} = 0. \quad (1)$$

Here are $\sigma_r, \sigma_\theta, \tau_{r\theta}$ the components of the stress tensor. In the elastic region, Hooke's law is valid for a homogeneous, isotropic linearly elastic material:

$$\varepsilon_{ij} = \frac{1}{E} ((1 + \mu) \sigma_{ij} - \mu \delta_{ij} \sigma_{kk}). \quad (2)$$

Where σ_{ij} and ε_{ij} are the components of the stress and strain tensors, E – is the elastic modulus, μ – is the Poisson's ratio, δ_{ij} – is the Kronecker symbol.

As a condition for the transition of the material into a plastic state, we accept the Tresca-Saint-Venant condition, which is widely used in the calculation of plastic deformable metal structures

$$(\sigma_\theta - \sigma_r)^2 + 4\tau_{r\theta}^2 = 4K_*^2 \quad (3)$$

where K_* is adhesion coefficient.

The material strength parameter K_* in condition (2) characterizes the plastic inhomogeneity formed as a result of varying degrees of damage to the material due to the force-corrosion effect and distributed over the thickness of the plastic zone, similar to the outline of its boundary. At the boundary of the plastic zone, the value K_* is constant: $K_* = K_1$. The value

K_* is a special softening function, depending on the coordinates r, θ and loading parameters r_0, δ [14]:

$$K_* = K_*(r, r_0, \theta, \delta) \quad (4)$$

Here r_0, δ – is the axisymmetric and nonaxisymmetric loading parameters: $r_0 = r_0(P_0, P_1 + P_2)$, $\delta = \delta(P_1 - P_2)$.

The problem is solved by the method of joint use of static and physical equations for the considered elastoplastic material. In the non-axisymmetric formulation, the perturbation method is applied [15]. The solution is searched for in the form of series by degrees of a small parameter, which is the parameter δ

$$\begin{aligned} \sigma_{ij} &= \sum_0^\nu \delta^\nu \sigma_{ij}^{(\nu)} = \sigma_{ij}^0 + \sum_1^\nu \delta^\nu \sigma_{ij}^{(\nu)}, & K_* &= \sum_0^\nu \delta^\nu K_*^{(\nu)} = K_*^0 + \sum_1^\nu \delta^\nu K_*^{(\nu)}, \\ r_s &= \sum_0^\nu \delta^\nu r_\nu = r_0 + \sum_1^\nu \delta^\nu r_\nu, \end{aligned} \quad (5)$$

where r_s is the desired elastoplastic boundary.

Linearizing equations (1), (3) and introducing, according to (1), the stress function $F = F(r, \theta)$

$$\sigma_r^{(\nu)} = \frac{1}{r} \frac{\partial F^{(\nu)}}{\partial r} + \frac{1}{r^2} \frac{\partial^2 F^{(\nu)}}{\partial \theta^2}, \quad \sigma_\theta^{(\nu)} = \frac{\partial^2 F^{(\nu)}}{\partial r^2}, \quad \tau_{r\theta}^{(\nu)} = -\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial F^{(\nu)}}{\partial \theta} \right), \quad \nu = 0, 1, 2, \dots \quad (6)$$

we obtain in the plastic zone an inhomogeneous partial differential equation for the function $F^{(\nu)}(r, \theta)$:

$$r^2 \frac{\partial^2 F^{(\nu)}}{\partial r^2} - r \frac{\partial F^{(\nu)}}{\partial r} - \frac{\partial^2 F^{(\nu)}}{\partial \theta^2} = r^2 f^{(\nu)}(r, \theta), \quad \nu \geq 0 \quad (7)$$

where $f^{(\nu)}$ is the right side of the corresponding linearized relation:

$$f^0 = 2K_*^0, \quad f^{(I)} = 2K_*^{(I)}, \quad f^{(II)} = -\frac{1}{K_0} (\tau_{r\theta}^{(I)})^2 + 2K_*^{(II)}.$$

The solution of equation (7) $F^{(\nu)}$ is determined taking into account static or geometric boundary conditions.

In the elastic region, the stress function $\Phi(r, \theta)$ satisfies the biharmonic equation (∇^2 is the Laplace operator)

$$\nabla^2 \nabla^2 \Phi = 0. \quad (8)$$

The solution of equation (8) with $r, m\theta$ can be represented as

$$\Phi_m = (C_1 r^m + C_2 r^{-m} + C_3 r^{m+2} + C_4 \varphi_m(r)) \cos m\theta, \quad m = 0, 1, 2, \dots, \quad (9)$$

where $\varphi_m(r) = r^m \ln r$ at $m = 0, 1$; $\varphi_m(r) = r^{-m+2}$ at $m \geq 2$. Constants $C_1 - C_4$ are found from the boundary conditions.

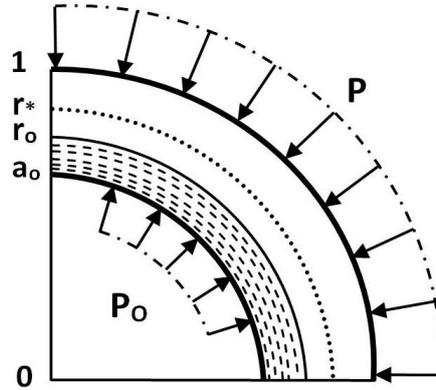


Figure 1: Design scheme for a thick-walled pipe element in an axisymmetric setting

3.2 Thick-walled element in an axisymmetric setting

First, find the axisymmetric state of the pipeline element. The axisymmetric problem for a thick-walled element is known in elasticity theory as the Lamé problem. Assuming in the initial equations $\tau_{r\theta}^0 = 0$, $\partial\sigma_{ij}^0/\partial\theta = 0$, $\partial K_*^0/\partial\theta = 0$, $\delta = 0$ we obtain the Lamé problem in the accepted formulation (Fig. 1).

The axisymmetric boundary conditions on the internal and external contour of a_0 and 1 and the conjugation conditions on the contour of r_0 have the form (square (round) brackets at the indices mean belonging to the plastic (elastic) zone

$$\begin{aligned} \sigma_{[r]}^0 &= P_0 \text{ at } r = a_0; \quad \sigma_{(r)}^0 = P \text{ at } r = 1; \\ [\sigma_r^0] &= [\sigma_\theta^0] = 0 \text{ at } r = r_0, \end{aligned} \quad (10)$$

Here big [...] brackets means a jump of a given value when crossing through the specific boundary.

In the axisymmetric (zero) state of a thick-walled element, the softening function in the plastic zone K_*^0 depends on the current radius r and the boundary radius r_0

$$K_*^0 = K_*(r, r_0) = (K_0 - K_1)\bar{f}(r, r_0) + K_1 \quad (11)$$

Where K_0 and K_1 is the value of the strength of the material on the inner contour of the element and on the elastic-plastic radius r_0 , $\bar{f}(r, r_0)$ – some core with properties $\bar{f}(a_0, 1) = 1$, $\bar{f}(r_0, r_0) = 0$. Such properties of the core allow us to describe the decrease in the value H_*^0 not only by the radius, but also depending on the position of the boundary radius r_0 . As a core $\bar{f}(r, r_0)$, one can take the core [12], which well describes the softening of the material during loading (Fig. 2). It has the form $\bar{f}(r, r_0) = \frac{a_0^n(r_0^n - r^n)}{r^n(1 - a_0^n)}$ (n – nonlinearity parameter).

Using equilibrium equations (1), plasticity condition (3), and also axisymmetric boundary conditions and conjugation conditions (10) we find all stress components and the radius of

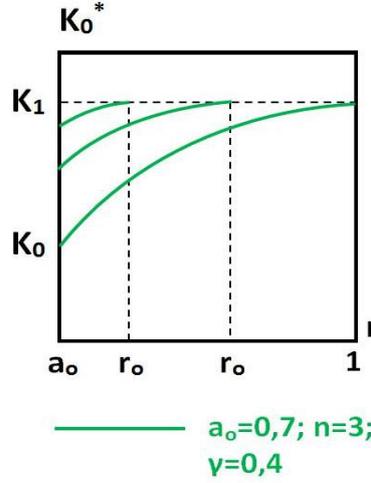


Figure 2: Decrease in the strength parameter K_*^0 depending on the position of the elastoplastic radius r_0 according to the presented softening function $K_*(r, r_0)$

the plastic zone r_0 .

$$\sigma_{[r]}^0 = P_0 + 2 \int_{a_0}^{r_0} r^{-1} K_*^0 dr, \quad \sigma_{[\theta]}^0 = \sigma_{[r]}^0 + 2K_*^0, \quad \tau_{[r\theta]}^0 = 0 \quad (12)$$

$$\left. \begin{array}{l} \sigma_{(r)}^0 \\ \sigma_{(\theta)}^0 \end{array} \right\} = P + K_1 r_0^2 \left(1 \mp \frac{1}{r^2} \right), \quad \tau_{(r\theta)}^0 = 0$$

The radius r_0 is implicitly determined from the equation

$$P_0 - P + 2 \int_{a_0}^{r_0} r^{-1} K_*^0 dr + K_1(1 - r_0^2) = 0 \quad (13)$$

In the absence of corrosion damage, the parameter $K_*^0 = K_1$ and radius of the plastic zone r_0 are found from the equation

$$P_0 - P + 2K_1 \left(\ln \left(\frac{r_0}{a_0} \right) + \frac{1}{2} (1 - r_0^2) \right) = 0 \quad (14)$$

Numerical calculations were performed using formulas (11) – (14) with the following data: $\mu = 0,34$; $E = 110GPa$; $\Delta P = P - P_0$; $E = 3667K_1$. According to numerical results, the presence of corrosion damage affects the elastic-plastic state of the thick-walled element. The greatest interest is the relationship $\Delta P = \Delta P(r_0)$ between value of uniform pressure ΔP and the radius of the plastic zone r_0 (Fig. 3). This dependence is determined by the thickness parameter a_0 and the softening parameters of the material n , $\gamma = K_0/K_1$. From

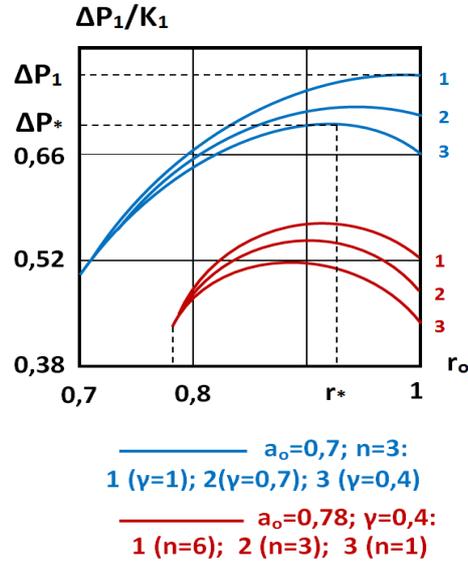


Figure 3: The relationship between the value of uniform pressure ΔP and the radius of the plastic zone r_0 for various thickness parameters and softening parameters of the material n , $\gamma = K_0/K_1$

the calculations it follows that the presence of corrosion damage ($\gamma < 1$) leads to an increase in radius r_0 . Moreover, all curves with $\gamma < 1$ have one maximum point with an abscissa $r_0 = r_*$, where $a_0 < r_* < 1$. In the absence of damage ($\gamma = 1$), the radius $r_* = 1$. The existence and uniqueness of maximum points follows analytically from the nonmonotonicity and convexity of the function $\Delta P = \Delta P(r_0)$.

The maximum point characterizes the moment of loss of the bearing capacity of a thick-walled element. Corresponding to the maximum point, pressure ΔP_* ($\Delta P_* \leq \Delta P$) and radius r_* ($r_* \leq 1$) are limiting for the destruction of a thick-walled element. The value r_* depends significantly on the thickness parameter a_0 : in a thinner element, it is located closer to the inner surface (Fig. 3). The obtained results can serve as an explanation of the phenomenon of premature failure of structural elements with corrosion damage.

We denote the relationship between external loads P , P_0 and radius r_0 as $g(P, P_0, r_0) = 0$. Then the existence of a maximum point on the interval $a_0 < r_0 < 1$ is expressed as an additional equation $\partial g(P, P_0, r_0)/\partial r_0 = 0$, which has the form: $K_*(a_0, r_0) - K_1 r_0^2 = 0$.

The bearing capacity of a thick-walled element is determined as follows. From the two transcendental equations $g = 0$, $\partial g/\partial r_0 = 0$, we first find the critical radius r_* , and then the critical loads at which the thick-walled element is destroyed.

With the known softening parameters and given external loads, it is possible to establish the optimal (minimum permissible) thickness parameter a_0 for a thick-walled element of two equations $g = 0$, $\partial g/\partial r_0 = 0$. Calculations show that the bearing capacity of the element is significantly reduced in the presence of corrosion damage, for example, when the thickness $d = 1 - a_0 = 0,4$ is reduced by 12-15%. The bearing capacity of an element that allows only elastic deformation is lower by 34-50% in comparison with elastoplastic elements. It follows

from the calculations that the bearing capacity of the elements increases most effectively when their thickness increases only to $d = 0.4 \div 0.5$.

3.3 Thick-walled element in a non-axisymmetric setting (outward pressure uneven in contour)

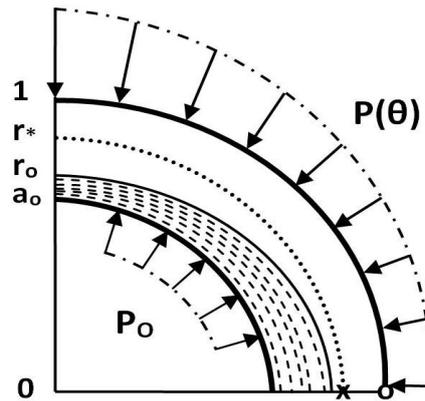


Figure 4: Design scheme for a thick-walled pipe element in a non-axisymmetric setting

Consider an extended thick-walled element loaded with an external pressure that is not uniform along the contour $P(\theta)$. The cross section of an element with an inner radius a_0 and an outer radius of 1 is in plane deformation (Fig. 4).

External pressure is elliptically distributed along the contour and can be written as

$$P(\theta) = P[1 - \delta \cos 2\theta] \quad (15)$$

where $P = (P_{\max} + P_{\min})/2$ is the averaged uniform pressure, $\delta = (P_{\max} - P_{\min})/(P_{\max} + P_{\min})$ – a parameter characterizing the deviation of external pressure from uniform.

The solution is sought in the form (5) at $\nu \geq 0$. The zero solution for $\nu = 0$ was found in the previous part of the paper. Find a solution at $\nu = 1$. The function $F^{(1)}$ in (7) is represented based on the static boundary conditions (15) on the outer contour of the element: $F^{(1)} = R(r) \cos 2\theta$.

Solving (7), we find the function $F^{(I)}$ in the plastic zone:

$$F^{(I)} = \left(A_i R_i + R_i \int_{a_0}^r \frac{\nu_i(r)}{\nu(r)} dr \right) \cos 2\theta, \quad i = \overline{1, 2}, \quad (16)$$

where $A_i R_i = A_1 R_1 + A_2 R_2 = r(A_1 \cos(\sqrt{3} \ln r) + A_2 \sin(\sqrt{3} \ln r))$, $V(r)$ – is the Wronskian of the solution system R_i , $V_i(r)$ is the determinant obtained from the determinant $V(r)$ by replacing the i -th column with a column with a single nonzero element $2K_*^{(I)} \cos^{-1} 2\theta$ located at its end.

Using (6), (16) we find the stress components in the plastic zone. They must satisfy the linearized boundary conditions on the inner contour

$$\sigma_{[r]}^{(I)} = 0, \quad \tau_{[r\theta]}^{(I)} = 0 \quad \text{at } r = a_0. \quad (17)$$

The components $\sigma_{[r]}^{(I)}$, $\sigma_{[\theta]}^{(I)}$, $\tau_{[r\theta]}^{(I)}$ of the stress tensor in the elastic region of the cylinder are determined from (6), (9). They satisfy the given static boundary conditions on the external contour (15) and two linearized conditions of stress conjugation σ_r and $\tau_{r\theta}$ on the boundary of the plastic zone:

$$[\sigma_{[r]}^{(I)}] = 0, \quad [\tau_{[r\theta]}^{(I)}] = 0 \quad \text{at } r = r_0. \quad (18)$$

As a result, we obtain the boundary-value problem for an elastic ring of radius r_0 and 1 under the following boundary conditions:

$$\begin{aligned} \sigma_{[r]}^{(I)} &= N_1(r) \cos 2\theta, \quad \tau_{[r\theta]}^{(I)} = N_2(r) \sin 2\theta \quad \text{at } r = r_0, \\ \sigma_{[r]}^{(I)} &= -P \cos 2\theta, \quad \tau_{[r\theta]}^{(I)} = 0 \quad \text{at } r = 1. \end{aligned}$$

Solving this problem, we find the stress components in the elastic region.

The equation of the boundary of the plastic zone r_s is sought in the form $r_s = r_0 + \delta r_1$. To determine the value of r_1 , we use the linearized conditions for the conjugation of the components σ_θ and K_* on r_0 :

$$\left[\sigma_{[\theta]}^{(I)} + \frac{d\sigma_\theta^0}{dr} r_1 \right] = 0, \quad \left[K_*^{(I)} + \frac{dK_*^0}{dr} r_1 \right] = 0 \quad \text{at } r = r_0, \quad (19)$$

where will we get it

$$r_1 = (\sigma_\theta^{(I)} - \sigma_{[\theta]}^{(I)}) / \left(\frac{d\sigma_{[\theta]}^0}{dr} - \frac{d\sigma_\theta^0}{dr} \right), \quad K_*^{(I)} = -\frac{dK_*^0}{dr} r_1 \quad \text{at } r = r_0.$$

Then for r_1 we will have $r_1 = \varphi(r_0)r_0 \cos 2\theta$, where $\varphi(r_0)$ is a function of r_0 :

$$\varphi(r_0) = \frac{X_0}{X_1 + X_2 + Z}, \quad X_0 = 4(1 - r_0^2 + r_0^{-2} - r_0^4) \frac{-P}{N},$$

$$N = 6 - 4(r_0^2 + r_0^{-2}) + r_0^4 + r_0^{-4}, \quad A = \frac{1}{2}(12 + 12\alpha_2 - \alpha_2^2)^{1/2},$$

$$X_1 = (10 - 4r_0^2 - 4r_0^{-2} - r_0^4 - r_0^{-4} + \alpha_1 N) \frac{\bar{K}_*}{N} \times$$

$$\times \left[\left(\sqrt{3} \cos(\sqrt{3} \ln r_0) + \sin(\sqrt{3} \ln r_0) \right) B_1 + \left(\cos(\sqrt{3} \ln r_0) - \sqrt{3} \sin(\sqrt{3} \ln r_0) \right) B_2 \right],$$

$$X_2 = -2(4 - 4r_0^2 + r_0^4 - r_0^{-4}) \frac{\bar{K}_*}{N} \times (\sin(\sqrt{3} \ln r_0) B_1 + \cos(\sqrt{3} \ln r_0) B_2),$$

$$Z = 4K_1, \quad \bar{K}_* = (K_0 - K_1) \frac{a_0^n}{1 - a_0^n} \frac{n}{r_0}, \quad B_1 = \frac{r_0}{2} \sin(\sqrt{3} \ln r_0 - \frac{\pi}{3}) - \frac{a_0}{2} \sin(\sqrt{3} \ln r_0 - \frac{\pi}{3}),$$

$$B_2 = \frac{r_0}{2} \cos(\sqrt{3} \ln r_0 - \frac{\pi}{3}) - \frac{a_0}{2} \cos(\sqrt{3} \ln r_0 - \frac{\pi}{3})$$

In the homogeneous case we have: X_0 – the same thing, $X_1 = X_2 = 0$, $H_*^0 = H_1$, $H_*^{(I)} = 0$

$$X_1 = X_2 = 0, \quad K_*^0 = K_1, \quad K_*^{(I)} = 0, \quad Z = 4K_1.$$

Moreover, the radius r_0 corresponds to the homogeneous case.

The equation of the boundary of the plastic zone r_s takes the form:

$$r_s = r_0(1 + \delta\varphi(r_0) \cos 2\theta).$$

The solution exists under the condition $r_0(1 - \delta\varphi(r_0)) \geq a_0$.

The bearing capacity of a thick-walled element can be determined from the transcendental equation

$$r_* = r_0(1 + \delta\varphi(r_0)) \tag{20}$$

First, from (20) we find the radius r_0 . Substituting the found radius r_0 into equation (13), we obtain critical loads at which the plastic zone reaches some "critical" points of the thick-walled element.

In the absence of corrosion damage to the element, $r_* = 1$ should be taken in equation (20) and equation (14) should be used. The critical points in this case (marked with zeros in Fig. 4) are on the external contour of the element in the directions of the minimum external pressure P_{\min} . In the presence of corrosion damage to the element, the critical points (marked with crosses in Fig. 4) are inside the element on circuit r_* in the same directions. Reaching these crosses with at least one point of the plastic zone will lead to the destruction of the element. As can be seen from Figure 4, the plastic zone is elongated in the directions of action of P_{\min} . Damage to the material leads to an increase in the size of the plastic zone. The degree of damage to the material depends on the size of this zone. The element has the greatest damage in the directions of action of P_{\min} . Therefore, the softening of the element depends both on the size of the plastic zone and on the orientation of its boundary.

4 Conclusion

The stress-strain state of an elastoplastic element of a thick-walled pipeline is studied under conditions of force and corrosion using a special softening function (plastic inhomogeneity) in the plasticity condition of Tresca-Saint-Venant.

The elastoplastic problem for a thick-walled element in an axisymmetric formulation (with uniform external and internal pressure) and non-axisymmetric formulation (with an external pressure uneven in contour) is considered. The problems are solved by the method of sharing static and physical equations and the perturbation method in the theory of an elastoplastic body.

An assessment of the strength and bearing capacity of a thick-walled element under corrosive force is given.

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