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KINEMATIC ANALYSIS AND SYNTHESIS OF THE LEVER MECHANISM OF CRANK PRESS STAMPING

Expanding technical and technological capabilities of forging and stamping machines and equipment can be carried out by introducing new designs of actuators with wide functionality. These features are provided by the crank lever mechanisms of the press. This article presents a kinematic analysis and synthesis of a six-lever mechanism for stamping a crank press with a forging feed mechanism. We propose an analytical method for kinematic analysis of the mechanism, which allowed us to implement a numerical calculation program in the integrated Maple environment. Methods of kinematic synthesis of the six-lever crank press mechanism based on the standard-square approximation, as well as the synthesis of the four-lever crank-slide forging feed mechanism have been developed. All the required constant geometric parameters of the stamping mechanism are determined; as a result, the mechanism implements the specified law of motion of the working slider with high accuracy. The comparative analysis was carried out in the ASIAN-2014 environment.

Key words: crank, press, linkage, the slider, the treatment of materials by pressure.

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Қосінді пресстің иінтіректі штампылау механизмін кинематикалық талдау және синтездеу

Ұсталық-штампылау машиналары мен жабдықтарының техникалық және технологиялық мүмкіндіктерін кеңейтуді кең функционалды мүмкіндіктері бар атқарушы механизмдердің жаңа құрылымдарын енгізу есебінен жүргізуге болады. Мұндай мүмкіндіктерге престің қосінді иінтіректі механизмдері ие. Бұл мақалада соғуды беру механизмі бар қосінді престі штампылаудың алтыбуынды иінтіректі механизмінің кинематикалық талдауы және синтезі берілген. Механизмді кинематикалық талдау үшін аналитикалық әдіс ұсынылған, бұл интегралдық Maple ортасында сандық есептеу бағдарламасын жүзеге асыруға мүмкіндік берді. Орташа квадраттық жуықтау, сондай-ақ соғуды берудің төрт буынды қосінді-жүгірткі механизмінің синтезі негізінде қосінді престің алтыбуынды механизмінің кинематикалық синтезі әдістері әзірленді. Штампылау механизмінің барлық іздестірілетін тұрақты геометриялық параметрлері анықталды, нәтижесінде механизм жұмыс жүгірткісі қозғалысының берілген заңын жоғары дәлдікпен іске асырады. Салыстырмалы талдау ASIAN-2014 аймағында жүргізілді.

Түйін сөздер: қосиін, пресс, иінтіректі механизм, жүгірткі, материалдарды қысыммен өңдеу.

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Кинематический анализ и синтез рычажного механизма штамповки кривошипного пресса

Расширение технических и технологических возможностей кузнечно-штамповочных машин и оборудования можно проводить за счет внедрения новых конструкций исполнительных механизмов с широкими функциональными возможностями. Такими возможностями обладает кривошипные рычажные механизмы пресса. В данной статье представлен кинематический анализ и синтез шестизвенного рычажного механизма штамповки кривошипного пресса с механизмом подачи поковки. Предлагается аналитический метод кинематического анализа механизма, который позволил реализовать программу численного расчета в интегрированной среде Maple. Разработаны методы кинематического синтеза шестизвенного механизма кривошипного пресса на основе среднеквадратического приближения, также синтеза четырехзвенного кривошипно-ползунного механизма подачи поковки. Определены все искомые постоянные геометрические параметры механизма штамповки, в результате механизм с высокой точностью реализовывает заданный закон движения рабочего ползуна. Сравнительный анализ проведен в среде ASIAN-2014.

Ключевые слова: кривошип, пресс, рычажный механизм, ползун, обработка материалов давлением.

1 Introduction

To increase the competitiveness of forging and stamping equipment, it is necessary to increase its operational characteristics (accuracy, durability, efficiency, high manufacturability) while reducing overall development and production costs [1, 2]. This encourages the transition to modern design methods based on mathematical modeling of ongoing processes throughout the technological cycle and rational use of modern CAD tools. Expanding the technical and technological capabilities of forging machines and equipment can be carried out by introducing new designs of actuators with wide functionality. These features are provided by the crank lever mechanisms of the press. The development begins with solving the problems of kinematic synthesis and analysis of mechanisms.

2 Literature review

The development of new machine mechanism designs, including crank presses [1], begins with solving problems of analysis and synthesis based on mathematical modeling. When implementing the technological process in crank presses, it is necessary to provide a specified cyclogram of the movement of the working slider: fast ascent, dwell, slow descent. Research on crank presses considers two ways to achieve this goal, the first is to synthesize a mechanism with a single degree of freedom [2, 3, 4, 5], where these properties are embedded in the properties of the kinematic chain, the second is the solution of this problem due to the additional freedom of the kinematic chain, which are called the hybrid press system [6].

M. Erkan Kyutyuk's work [6] provides a review of the scientific literature on the analysis and synthesis of hybrid mechanisms of crank presses. In this paper, we consider a seven-way

lever mechanism with two degrees of freedom (2 DOF), in which one degree operates on the basis of a DC power motor (for the implementation of the main technological process), the second – on the basis of a servomotor to provide a cyclogram of the technological process.

In many other studies, hybrid press systems are based on five-link and seven-link mechanisms with two degrees of freedom. The first study of this kind was performed by Dulger (originally Tokuz) and Jones in a hybrid configuration [7, 8, 9]. The constant-speed engine and servomotor were combined by a differential transmission, which further drives the crank mechanism [7].

Yuan and others explored the two combined machines having seven links, two DOF linkage [10]. Ouyang et al. proposed a five-link lever mechanism consisting of a five bar linkage, an AC CV motor and a frequency controller, an AC brushless servo motor and a servo amplifier with a gear transmission, a shift encoder, a flywheel and a belt [11]. Zhang proposed a hybrid five bar mechanism [12].

Connor et al. have presented a study on the synthesis of hybrid five bar path generating mechanisms using genetic algorithms [13]. Dulger et al. have presented a study on modeling and kinematic analysis of a hybrid actuator; a seven link mechanism with an adjustable crank [14]. Yu has offered a study with HM system using five bar mechanism [15]. Li and Zhang have applied a seven bar linkage configuration with kinematics analysis and optimum design of hybrid system [16]. Li and Tso have presented a seven bar mechanism [17]. Tso and Li have later used a seven bar mechanism to investigate the stamping capacity and energy distribution between the servomotor and the flywheel with different motion inputs [18]. Tso has again used a seven bar mechanism. A control system with iterative learning control and feedback control techniques was developed [19].

In all these mechanisms, the issue of providing the necessary cyclogram for moving the working slider is solved by controlling two or more engines and, accordingly, the problems of dynamic synthesis of drive control functions are solved.

The implementation of a technological cyclogram using a mechanism with a single degree of freedom requires a significant complication of the structure of the lever kinematic chain, the so-called Assur groups [20, 21]. In the works by A. Tuleshov [2, 3, 4, 5] a kinematic chain (structural group) of the fourth class is used for the synthesis of the crank press mechanism.

In [3], a vector method for kinematic analysis of crank two-rod presses has been developed on the basis of four-link groups [2]. As a follow-up to these studies, a vector model of the time diagram of the automaton was developed [2], which allows solving various dynamic problems by changing the parameters of the time cyclogram of its mechanisms, including the analysis of the mechanism using the Matlab / Simulink platforms [5]; based on this, it was possible to expand the motion scenarios for the slider with servo inputs. In [4], the authors used simulations to compare a conventional press with a power transmission using a crank mechanism and a press with an FEM yoke mechanism (Chval and Cechura 2014) [22].

3 Material and methods

3.1 Structural analysis

Figure 1 shows a kinematic diagram of the stamping mechanism under consideration with a feed-and-removal mechanism of the processed material. The structural formula of the mech-
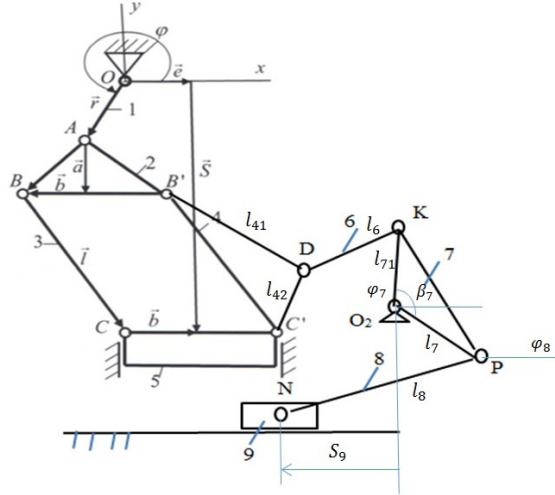
anism has the form [20].

Figure 1: Kinematic schemes of the stamping mechanism

$$I(1) \Rightarrow IV(2,3,4,5) \Rightarrow II(6,7) \Rightarrow II(8,9).$$

A special feature of the mechanism is that the modified contour $BB'C"C$ is a parallelogram and the ABB' triangle is equilateral. This imposes certain conditions on the movement of individual joints: joint 2 makes a forward movement on the plane and joints 3 and 4 occupy the same angular positions.

The following symbols for the coordinates and dimensions of joints were introduced: r – length of crank 1; a – height of ABB' triangle; l – length of parallel connecting rods $BC = B'C'$; φ – angular coordinate of crank 1; ψ – angular coordinate of two connecting rods 3 and 4; S – linear coordinate of slide 5; e – eccentricity of slide 5, i.e. the deviation of the trajectory of the center of gravity of the slider from Oy axis; b – distance between ball joint C and the center of the slide 5 along Ox axis; $l_{i,j}$ – the length of the leash triangular joints, where $i = 4.7$ takes the number value of the joint $j = 1, 2$ – number of sides on a i triangle; l_i – the length of the i-joint; φ_i – angular coordinate of the i-joint; S_9 – movement of slide 9 parallel to the axis O_2x.

3.2 Kinematic analysis

The kinematics equations of the Stephenson mechanism $I(1) \Rightarrow IV(2,3,4,5)$ in the crank press structure have the form [2]

$$\begin{cases} r\cos\varphi + l\cos\psi = e \\ r\sin\varphi - a + l\sin\psi = -S \end{cases} \tag{1}$$

Solutions of equations (1) with respect to $S = S(\varphi)$, $\psi = \psi(\varphi)$ are obtained explicitly

$$\begin{cases} S = a - r\sin\varphi \pm \sqrt{l^2 - (e - r\cos\varphi)^2} \\ \psi = \pm\arccos\left[\dfrac{1}{l}(e - r\cos\varphi)\right] \end{cases} \tag{2}$$

The signs $\pm$ correspond to different assemblies of the mechanism.
The first and second derivative (analogs of speed and acceleration) are written as

$$\begin{cases} S' = -r\cos\varphi - l\cos\psi \cdot \psi' \\ \psi'\sin\psi = -\dfrac{r}{l}\sin\varphi \end{cases} \tag{3}$$

$$\begin{cases} S'' = r\sin\varphi + l\sin\psi \cdot \psi'^2 - l\cos\psi \cdot \psi'' \\ \psi''\sin\psi + \cos\psi \cdot \psi'^2 = -\dfrac{r}{l}\cos\varphi \end{cases} \tag{4}$$

Solutions of equations (3) and (4) with respect to the first and second derivative are written as

$1°\quad T'_\psi(\varphi) = \psi' = -\dfrac{r}{l}\dfrac{\sin(\varphi)}{\sin(\psi)}, \quad \sin(\psi) \neq 0, \quad \psi \neq 0, \quad k\pi, \quad k = 1, 2, 3, \ldots$

$2°\quad T'_S(\varphi) = S' = -r\cos\varphi - l \cdot T'_\psi(\varphi)\cos\psi,$

$3°\quad T''_\psi(\varphi) = \psi'' = -\dfrac{1}{\sin\psi}\left[\dfrac{r}{l}\cos\varphi - (T'_\psi)^2\cos\psi\right], \quad \sin\psi \neq 0, \quad \psi \neq 0, \quad k\pi, \quad k = 1, 2, 3, \ldots$

$4°\quad T''_S(\varphi) = S'' = r\sin\varphi + l \cdot (T'_\psi)^2\sin\psi - l \cdot T''_\psi\cos\psi.$

$$\tag{5}$$

In real crank presses, the eccentricity $e = 0$, the above formulas are slightly simplified and the algorithm for kinematic analysis of the mechanism is recorded

$1°\quad \begin{cases} S = a - r\sin\varphi \pm \sqrt{l^2 - r^2\cos^2\varphi} \\ \psi = \pm\arccos\left[\dfrac{r}{l}\cos\varphi\right] \end{cases}$

$2°\quad \begin{cases} S' = -r\cos\varphi \pm \dfrac{r^2\sin\varphi}{2\sqrt{l^2 - r^2\cos^2\varphi}} \\ \psi' = \mp\dfrac{r\sin\varphi}{\sqrt{l^2 - r^2\cos^2\varphi}} \end{cases}$

$3°\quad \begin{cases} S'' = r\sin\varphi \pm r^2\left(\dfrac{\cos\varphi}{\sqrt{l^2 - r^2\cos^2\varphi}} - \dfrac{r^2\sin^2\varphi\cos\varphi}{2(l^2 - r^2\cos^2\varphi)^{3/2}}\right) \\ \psi'' = \pm r\left(\dfrac{\cos\varphi}{\sqrt{l^2 - r^2\cos^2\varphi}} - \dfrac{r^2\sin^2\varphi\cos\varphi}{2(l^2 - r^2\cos^2\varphi)^{3/2}}\right) \end{cases}$

$$\tag{6}$$

Next, we make the kinematics equations for the following mechanism structures $II(6,7) \Rightarrow II(8,9)$. To do this, write down the coordinates of the joints B' and C' :

$$x_{B'} = r\cos\varphi + b, \quad y_{B'} = r\sin\varphi - a, \quad x_{C'} = e + b, \quad y_{C'} = S. \tag{7}$$

Let us write the equations of the geometric connection of a $B'DC'$ triangle:

$$(x_{B'} - x_D)^2 + (y_{B'} - y_D)^2 = l_{41}^2,$$

$$(x_D - x_{C'})^2 + (y_D - y_{C'})^2 = l_{42}^2,$$

The solution of this system of equations with respect to two unknowns x_D and y_D can be represented as [1]

$$(x_D)_{1,2} = \frac{B \pm \sqrt{B^2 - AC}}{A}, \quad (y_D)_{1,2} = c(x_D)_{1,2} + d, \tag{8}$$

where $A = 1 + c^2$, $B = c(y_{B'} - d)$, $C = x_{B'}^2 + (y_{B'} - d)^2 - l_{41}^2$,

$$c = \frac{x_{C'} - x_{B'}}{y_{C'} - y_{B'}}, \quad d = \frac{l_{41}^2 - l_{42}^2 + x_{C'}^2 - x_{B'}^2 + y_{C'}^2 - y_{B'}^2}{2(y_{C'} - y_{B'})}, \quad y_{C'} \neq y_{B'}. \tag{9}$$

Let us write similar geometric connection equations for the group $II(6,7)$

$$(x_D - x_K)^2 + (y_D - y_K)^2 = l_6^2,$$

$$(x_K - x_{O_2})^2 + (y_K - y_{O_2})^2 = l_{71}^2,$$

The x_{O_2}, y_{O_2} coordinates are calculated, then the solution of this system of equations with respect to two unknowns x_K and y_K can be represented as

$$(x_K)_{1,2} = \frac{B \pm \sqrt{B^2 - AC}}{A}, \quad (y_K)_{1,2} = c(x_K)_{1,2} + d, \tag{10}$$

where $A = 1 + c^2$, $B = c(y_D - d)$, $C = x_D^2 + (y_D - d)^2 - l_6^2$,

$$c = \frac{x_{O_2} - x_D}{y_{O_2} - y_D}, \quad d = \frac{l_6^2 - l_{71}^2 + x_{O_2}^2 - x_D^2 + y_{O_2}^2 - y_D^2}{2(y_{O_2} - y_D)}, \quad y_{O_2} \neq y_D. \tag{11}$$

Let us determine φ_7 angle of the angular position of joint 7 (O_2P) using the formula

$$\varphi_7 = 2\pi - \beta_7 + \tan^{-1} \frac{y_K - y_{O_2}}{x_K - x_{O_2}}. \tag{12}$$

Now let us write the kinematics equations for the rocker-slider mechanism $I(7) \Rightarrow II(8,9)$ in the following form

$$x_N = x_{O_2} + l_7 \cos \varphi_7 + l_8 \cos \varphi_8, \quad y_N = y_{O_2} + l_7 \sin \varphi_7 + l_8 \sin \varphi_8.$$

Given that we have the kinematics equation $x_N - x_{O_2} = S_9$ and $y_N - y_{O_2} = h_9 = const$

$$S_9 = l_7 \cos \varphi_7 + l_8 \cos \varphi_8, \quad l_7 \sin \varphi_7 + l_8 \sin \varphi_8 = h_9. \tag{13}$$

Whence

$$\varphi_8 = \pm \sin^{-1} \frac{h_9 - l_7 \sin \varphi_7}{l_7} + k\pi, \quad k = 0, 1, 2, 3, \ldots, \quad S_9 = l_7 \cos \varphi_7 + l_8 \cos \varphi_8, \tag{14}$$

3.3 Choosing the law of motion

It is required to carry out the synthesis of the crank press reproducing the specified law of motion, $S_M = S_M(\varphi)$ on the site $0 \leq \varphi \leq 2\pi$.

Let us assume that it is necessary to implement an equidistant law of motion of the slider to the crank press. Then the analog of the slider accelerations has the form [1, 21]

$$T''_{SM}(\varphi) = \left\{ \begin{array}{ll} a_0 & \text{at } 0 \leq \varphi \leq 0,5\varphi_y, \\ -a_0 & \text{at } 0,5\varphi_y < \varphi \leq \varphi_y, \end{array} \right. \tag{15}$$

where $a_0 = const$ is the amplitude of the acceleration analog.

Using the unit function, equation (15) has the form

$$T''_{SM}(\varphi) = a_0 - 2a_0\delta(\varphi - 0,5\varphi_y), \tag{16}$$

where $\delta(\varphi - \varphi_0) = \left\{ \begin{array}{ll} 0 & at \ \varphi \leq \varphi_0 \\ 1 & at \ \varphi > \varphi_0 \end{array} \right.$ is the Dirac unit function.

By double integrating $T''_{SM}(\varphi)$ function we get the position functions (18) and the analog of the slider speed (17)

$$T'_{SM}(\varphi) = a_0\varphi - 2a_0(\varphi - 0,5\varphi_y)\delta(\varphi - 0,5\varphi_y) + C, \tag{17}$$

$$S_M = T_{SM}(\varphi) = 0,5a_0\varphi^2 - a_0(\varphi - 0,5\varphi_y)^2\delta(\varphi - 0,5\varphi_y) + C\varphi + D, \tag{18}$$

where C, D – are integration constants defined from initial conditions, if $\varphi = \varphi_0 = 0$, $T_{SM}(\varphi_0) = 0$, $T'_{SM}(\varphi_0) = 0$, then $C = 0$, $D = 0$.

From the boundary condition $T_{SM}(\varphi_0) = T_{\max}$, we determine the amplitude of the acceleration analog

$$a_0 = 4T_{\max}/\varphi_y^2. \tag{19}$$

3.4 Kinematic synthesis of the stamping mechanism

From the kinematic scheme of the six-lever crank press stamping mechanism, we have [5]

$$S(\varphi) = T_S(\varphi) = a - r\sin\varphi \pm \sqrt{l^2 - (e - r\cos\varphi)^2}. \tag{20}$$

The task of kinematic synthesis is to determine the constant parameters of the six-lever mechanism $a, l, r, , \varphi_0$ from the minimum functional conditions [5]

$$\|S(\varphi) - S_M(\varphi)\| \underset{l,a,r,e,\varphi_0}{\Longrightarrow} \min \tag{21}$$

The solution of the synthesis problem based on functional (21) has difficulties related to the nonlinearity of functions (2) (or (20)) and (18). Therefore, we apply another method related to the implicit representation of the kinematics equation of the mechanism and its transformations. To do this, we exclude the angle ψ from equation (1), then we get

$$l^2 = a^2 + S^2 + e^2 + r^2 - 2aS - 2ar \cdot \sin\varphi + 2Sr \cdot \sin\varphi - 2re \cdot \cos\varphi \tag{22}$$

Replace in equation (22) S with $S_i + S_0$ and φ to $\varphi_i + \varphi_0$ to account for the reference point S_i and φ_i, $= 1, 2, \ldots, N$

$$
\begin{aligned}
l^2 = a^2 + (S_i + S_0)^2 + e^2 + r^2 - 2a(S_i + S_0) - 2ar \cdot \sin(\varphi_i + \varphi_0) + \\
+ 2(S_i + S_0)r \cdot \sin(\varphi_i + \varphi_0) - 2re \cdot \cos(\varphi_i + \varphi_0)
\end{aligned}
\tag{23}
$$

We define the deviation function, which expresses the degree of proximity of the movement of the working joint and reproduced by the mechanism (21), in the form

$$
\Delta_i = \Delta_i(\varphi_i, S_i), \quad i = 1, 2, \ldots, N,
\tag{24}
$$

where

$$
\begin{aligned}
\Delta_i(\varphi_i, S_i) = a^2 + (S_i + S_0)^2 + e^2 + r^2 - l^2 - 2a(S_i + S_0) - 2ar \cdot \sin(\varphi_i + \varphi_0) + \\
+ 2(S_i + S_0)r \cdot \sin(\varphi_i + \varphi_0) - 2re \cdot \cos(\varphi_i + \varphi_0)
\end{aligned}
\tag{25}
$$

After transformations of the last expression, we get, assuming $S_0 = 0$

$$
\Delta_i = P_0 f_{0i} + P_1 f_{1i} + P_2 f_{2i} + P_3 f_{3i} + P_4 f_{4i} + P_5 f_{5i} - F_i
\tag{26}
$$

The following symbols are introduced here:

$$
\begin{aligned}
& f_{0i} = 1; \quad f_{1i} = S_i; \quad f_{2i} = \sin \varphi_i; \quad f_{3i} = \cos \varphi_i; \quad f_{4i} = S_i; \\
& f_{1i} = S_i \sin \varphi_i; \quad f_{5i} = S_i \cos \varphi_i; \quad F_i = S_i^2 \\
& P_0 = a^2 + e^2 + r^2 - l^2; \quad P_1 = -2a; \quad P_2 = 2re \cdot \sin \varphi_0 - 2ra \cdot \cos \varphi_0; \\
& P_3 = -2re \cdot \cos \varphi_0 - 2ra \cdot \sin \varphi_0; \quad P_4 = 2r \cdot \cos \varphi_0; \quad P_5 = 2r \cdot \sin \varphi_0; \quad F_i = S_i^2
\end{aligned}
\tag{27}
$$

Thus, the 5 required parameters of the lever mechanism of the crank press r, a, l, e, φ_0 are determined using 6 parameters $P_0, \ldots, P_5$. For the compatibility condition of the P_2 and P_3

$$
P_2 = P_5 e + \frac{P_1 P_4}{2}, \quad P_3 = -P_4 e + \frac{P_1 P_5}{2},
\tag{28}
$$

From here the value of the eccentricity can be found e

$$
e = \frac{P_2}{P_5} - \frac{P_1 P_4}{2P_5} \quad \text{or} \quad e = -\frac{P_3}{P_4} + \frac{P_1 P_5}{2P_4}
\tag{29}
$$

and the equation written down

$$
P_1(P_4^2 + P_5^2) = 2(P_2 P_4 + P_3 P_5)
\tag{30}
$$

The rest of the required parameters can be found from the ratios (27)

$$
a = -\frac{1}{2} P_1; \quad r = \frac{1}{2}\sqrt{P_4^2 + P_5^2}; \quad \text{tg}\, \varphi_0 = \frac{P_4}{P_5}; \quad l = \sqrt{a^2 + e^2 + r^2 - P_0}
\tag{31}
$$

To determine P_0, P_2, P_3, P_4, P_5, we apply the method of quadratic approximation [4], which consists in determining the minimum sum of square deviations for the positions of the mechanism N taking into account (26), i.e.

$$
C = \Delta_I^2 \underset{\min}{\Longrightarrow} 0.
\tag{32}
$$

The necessary conditions for the minimum (32) can be obtained by differentiating C by P_j

$$\frac{\partial C}{\partial P_j} = 2 \sum_{i=1}^{N} \Delta_i \frac{\partial \Delta_i}{\partial P_j} = 0, \quad j = 0, \ldots, 5 \tag{33}$$

Substitute in the equations (33) Δ_i and its derivatives $\dfrac{\partial \Delta_i}{\partial P_j}$, which are calculated according to (26). As a result we get a system of linear equations with respect to the desired parameters P_0, P_2, P_3, P_4, P_5

$$\begin{cases} P_0 \sum f_{0i}^2 + P_1 \sum f_{1i} f_{0i} + P_2 \sum f_{2i} f_{0i} + P_3 \sum f_{3i} f_{0i} + P_4 \sum f_{4i} f_{0i} + P_5 \sum f_{5i} f_{0i} = \sum F_i f_{0i} \\ P_0 \sum f_{01} f_{1i} + P_1 \sum f_{1i}^2 + P_2 \sum f_{2i} f_{1i} + P_3 \sum f_{3i} f_{1i} + P_4 \sum f_{4i} f_{1i} + P_5 \sum f_{5i} f_{1i} = \sum F_i f_{1i} \\ P_0 \sum f_{01} f_{2i} + P_1 \sum f_{1i} f_{2i} + P_2 \sum f_{2i}^2 + P_3 \sum f_{3i} f_{2i} + P_4 \sum f_{4i} f_{2i} + P_5 \sum f_{5i} f_{2i} = \sum F_i f_{2i} \\ P_0 \sum f_{01} f_{3i} + P_1 \sum f_{1i} f_{3i} + P_2 \sum f_{2i} f_{3i} + P_3 \sum f_{3i}^2 + P_4 \sum f_{4i} f_{3i} + P_5 \sum f_{5i} f_{3i} = \sum F_i f_{3i} \\ P_0 \sum f_{01} f_{4i} + P_1 \sum f_{1i} f_{4i} + P_2 \sum f_{2i} f_{4i} + P_3 \sum f_{3i} f_{4i} + P_4 \sum f_{4i}^2 + P_5 \sum f_{5i} f_{4i} = \sum F_i f_{4i} \\ P_0 \sum f_{01} f_{5i} + P_1 \sum f_{1i} f_{5i} + P_2 \sum f_{2i} f_{5i} + P_3 \sum f_{3i} f_{5i} + P_4 \sum f_{4i} f_{5i} + P_5 \sum f_{5i}^2 = \sum F_i f_{5i} \end{cases} \tag{34}$$

The system (34) must meet the minimum condition (32). Let's find from the first 5 equations of the system (34) coefficients P_i (i=0,2,3,4,5) expressed in P_1

$$P_i = c_i + d_i P_1, \quad i = 0, 2, \ldots, 5 \tag{35}$$

Substitute (31) in the coupling equations (28) with (22), then we get one equation with respect to P_1

$$P_1[(c_4 + d_4 P_1)^2 + (c_5 + d_5 P_1)^2] = 2[(c_2 + d_2 P_1)(c_4 + d_4 P_1) + (c_3 + d_3 P_1)(c_5 + d_5 P_1)], \tag{36}$$

which is converted to a third-degree equation

$$\begin{aligned} P_1^3(d_4^2 + d_5^2) + P_1^2(2c_4 d_4 + 2c_5 d_5 - 2d_2 d_4 - 2d_3 d_5) + \\ + P_1(c_4^2 + c_5^2 - 2c_2 d_4 - 2d_2 c_4 - 2c_3 d_5) = 2(c_2 c_4 + c_3 c_5) \end{aligned} \tag{37}$$

Solving this cubic equation, we find P_1, and then using the formula (35) we find P_0, P_2, P_3, P_4, P_5 coefficients. Then, based on the formulas (27), the desired parameters of r, a, l, e, φ_0 mechanism that implements the law (2) or (20) are determined. Thus, the problem of synthesis of this crank press mechanism by the quadratic approximation method is fundamentally solved.

3.5 Kinematic synthesis of the workpiece feed mechanism

The feed-and-remove mechanism works as follows (Figure 3): the slider 5 occupies three positions sequentially: 1 – upper; 2 – middle; 3 – lower position, when the workpiece is stamped, respectively, the slider 9 occupies three positions: 1 – right position where the workpiece is siezed to the feed table; 2 – average position, feed the workpiece into the matrix (table) and remove the finished stamping; 3 – left position, removal of the finished workpiece.

Kinematic synthesis of the rocker-slider mechanism $I(7) \Rightarrow II(8, 9)$ is performed using three specified positions of the slider and rocker [24]. The method is described in an analytical way.

So, x_{O_2} and y_{O_2} coordinates are known, the equation of the forward stroke of the slider relative to Oxy system has the form $y = -y_{O_2} - h_9$. On this line, $B_1(x_1, y_1)$, $B_2(x_2, y_2)$, $B_3(x_3, y_3)$ three slider positions are set, and $y_I = -y_{O_2} - h_9$, $I = 1, 2, 3$ are considered to be set $\varphi_{71}^{(1,2,3)} = \tan^{-1} \left(\dfrac{y_K - y_{O_2}}{x_K - x_{O_2}} \right)_{(1,2,3)}$ that correspond to the three positions of the slider 5 (see Figure 3). The algorithm for the synthesis of the rocker-slider mechanism is as follows:

1. Calculate the distance between $O_2(x_{O_2}, y_{O_2})$ point and the corresponding $B_1(x_1, y_1)$, $B_2(x_2, y_2)$, $B_3(x_3, y_3)$ points using the formulas

$$\sqrt{(x_I - x_{O_2})^2 + (y_I - y_{O_2})^2} = l_i, \quad i = 1, 2, 3.$$

2. Calculate $\varphi_{71}^{(2)} - \varphi_{71}^{(1)}$ and $\varphi_{71}^{(3)} - \varphi_{71}^{(1)}$ angle difference using the formula $\varphi_{71}^{(1,2,3)} = \tan^{-1} \left(\frac{y_K - y_{O_2}}{x_K - x_{O_2}} \right)_{1,2,3}$.

3. On a circle with a radius l_2 based on the equation $(x - x_{O_2})^2 + (y - y_{O_2})^2 = l_2^2$, find the coordinates $B_2'(x_2', y_2')$: find $d_2 = 2 l_2 \sin \frac{\varphi_{71}^{(2)} - \varphi_{71}^{(1)}}{2}$, then

$$x_2' = x_2 - d_2 \cos \frac{\pi - \varphi_{71}^{(2)} + \varphi_{71}^{(1)}}{2}, \quad y_2' = y_2 + d_2 \sin \frac{\pi - \varphi_{71}^{(2)} + \varphi_{71}^{(1)}}{2};$$

4. On a circle with a radius l_3 based on the equation $(x - x_{O_2})^2 + (y - y_{O_2})^2 = l_3^2$, find the coordinates $B_3'(x_3', y_3')$: find $d_3 = 2 l_3 \sin \frac{\varphi_{71}^{(3)} - \varphi_{71}^{(1)}}{2}$, then

$$x_3' = x_3 - d_3 \cos \frac{\pi - \varphi_{71}^{(3)} + \varphi_{71}^{(1)}}{2}, \quad y_3' = y_3 + d_3 \sin \frac{\pi - \varphi_{71}^{(3)} + \varphi_{71}^{(1)}}{2};$$

5. Let us write equations of lines that pass through points $B_1(x_1, y_1)$ and $B_2'(x_2', y_2')$ and $B_2'(x_2', y_2')$ and $B_3'(x_3', y_3')$: $y = k_2 x + q_2$ and $y = k_3 x + q_3$, respectively, where $k_2 = \frac{y_2' - y_1}{x_2' - x_1}$, $q_2 = y_1 - k_2 x_1$ and $k_3 = \frac{y_3' - y_2'}{x_3' - x_2'}$, $q_2 = y_1 - k_3 x_1$.

6. Let us create equations of perpendicular lines to lines that pass through points $B_1(x_1, y_1)$ and $B_2'(x_2', y_2')$, as well as $B_2'(x_2', y_2')$ and $B_3'(x_3', y_3')$:

   From the beginning we find the coordinates of C_2, C_3 points in the middle of the segments $(B_1(x_1, y_1),\ B_2'(x_2', y_2'))$ and $(B_2'(x_2', y_2'),\ B_3'(x_3', y_3'))$: $x_{C_2} = \frac{x_2' + x_1}{2}$, $y_{C_2} = \frac{y_2' + y_1}{2}$, and then the equations $x_{C_3} = \frac{x_2' + x_3'}{2}$, $y_{C_3} = \frac{y_2' + y_3'}{2}$, of perpendicular lines have the form

   $$y = -\frac{1}{k_2} x + q_4 \text{ and } y = -\frac{1}{k_3} x + q_5, \text{ where } q_4 = y_{C_2} + \frac{1}{k_2} x_{C_2}, \ q_5 = y_{C_3} + \frac{1}{k_3} x_{C_3}.$$

7. Next, we solve the equations of two lines $y = -\frac{1}{k_2}x + q_4$ and $y = -\frac{1}{k_3}x + q_5$, relative to $P(x_P, y_P)$ center coordinate:

$$x_P = \frac{k_2 k_3}{k_2 - k_3}(q_5 - q_4), \quad y_P = -\frac{1}{k_3}x_P + q_5,$$

8. As a result, we find the length of the rocker l_7 and the connecting rod l_9 using the formulas

$$l_7 = \sqrt{(x_P - x_{O_2})^2 + (y_P - y_{O_2})^2}, \quad l_9 = \sqrt{(x_P - x_{B_1})^2 + (y_P - y_{B_1})^2},$$

also the constant length of the base joint 7

$$l_{KR} = \sqrt{(x_P - x_K)^2 + (y_P - y_K)^2}.$$

3.6 Results and discussions

In order to solve the problem of analysis and synthesis of the six-lever mechanism (Figure 1), a program was developed in the integrated Maple environment.

To assess the quality of the sintered crank press stamping mechanism, the following criteria are used K_1, K_2

$$K_1 = T''_{\max} + \xi_1 |T''_{\min}|, \quad K_2 = (T'T'')_{\max} + \xi_2 |(T'T'')_{\min}|. \tag{38}$$

It is known [4] that geometric characteristics significantly affect the dynamics of mechanisms. Therefore, the criteria (38) can be used as preliminary dynamic criteria [4], which are used to compare different laws of motion, as well as to synthesize new laws that have optimal properties in a certain sense. This problem is relevant when studying the dynamics of the crank press and its further automation. Criterion (38) allows monitoring the pulsation of inertial loads on the slider and flywheel, from the external load on the workpiece side in the crank press [6].

Example. Consider an example of the synthesis of the Stephenson lever mechanism of a crank press. Initial data for synthesis: law of motion of slider 5: $S = S(\varphi)$, (see Figure 2) on the section of the crank rotation angle $0 \leq \varphi \leq 2\pi$. Slider stroke $S_{\max} = 120mm$. The angular velocity of the crank is constant $\omega = \dot{\varphi} = 10 \; rad/s$.
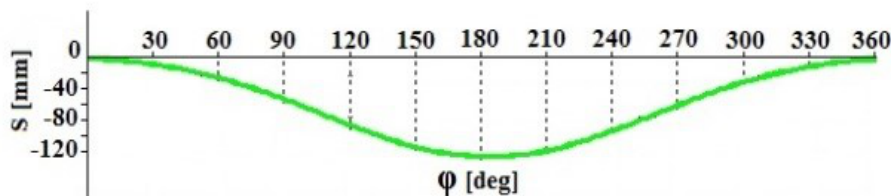

Figure 2: The law of motion of the slider of a six-lever mechanism

Note that since the slider moves in the opposite direction of the axis Oy, the axis Ox is directed to the right, then as a result of modeling the graphs relative to the specified graph in Figure 2 will be shifted to the left by $-\pi/2$.
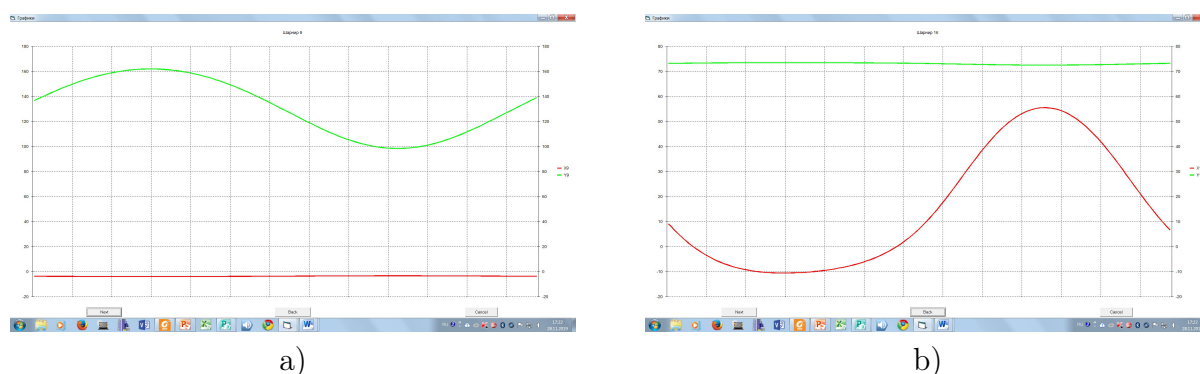
As a result of synthesis based on the above method in the Maple integrated environment, the following parameters of the Stephenson mechanism are obtained:

$$r = 60 \; mm, \; l = 160 \; mm, \; a = 41,85 \; mm, \; b = 43 \; mm, \; e = 0, \; \varphi_0 = -\pi/2.$$

According to the algorithms for the synthesis of the rocker-slider mechanism described in this paper, the mechanism with the following data is synthesized: $l_8 = 134.5mm$, $l_7 = 86 \; mm$. Other geometric parameters of the stamping mechanism are summarized in Table 1.

$B'D$	$C'D$	l_6	$l_{71} = O_2K$	KR	h_9	x_{O_2}	y_{O_2}
163 mm	36 mm	134.5 mm	103.5 mm	176.6 mm	56.5 mm	136,5 mm	-207.2 mm

The kinematic analysis of the mechanism in the ASIAN-2014 environment is carried out [25]. Figure 3 shows a kinematic diagram of the Stephenson synth mechanism with an analysis of the trajectory of the characteristic hinges.

Figure 3: Kinematic analysis of the stamping mechanism

Figures 4, 5, 6 show graphs of movement, speed, and acceleration of work joints: of the slider 5 and slider 9. Analysis of slider 5 movement graph (Figure 4,a) shows that the mechanism implements the specified law of movement of the working body (Figure 2). Slider 5 takes the upper position at $\varphi = \pi/2$, the lower position at $\varphi = 3\pi/2$. The forward and reverse moves of the slider occur at $\varphi \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ and $\varphi \in \left(\frac{3\pi}{2}, \frac{\pi}{2}\right)$. In the working lower position, slider 5 makes a uniform movement in the $\varphi \in (246°, 282°)$ interval.

The amplitude of the slider speed change at $\varphi^{(2)} = 186°$ and $\varphi^{(4)} = 339°$ remains the same (Figure 5, a). The amplitude of the change in the slider acceleration at $\varphi^{(1)} = 87°$ and $\varphi^{(3)} = 264°$ is 1.5 (Figure 6, a). At the interval $\varphi \in (53°, 124°)$, the acceleration of slider 5 is close to constant and the speed is linear.

Slider 9 makes a uniform movement in the interval $\varphi \in (66°, \; 180°)$. Acceleration is close to zero, speed is constant. The amplitude of forward slider movement is greater than the reverse by 4.7 times; speed – 2 times, acceleration –1.5 times (Figures 4,b, 5,b, 6,b). There are no abrupt changes in kinematic parameters that cause shock loads.

a)　　　　　　　　　　　　　　　　　　　b)

Figure 4: Analysis of the provisions of the stamping mechanism

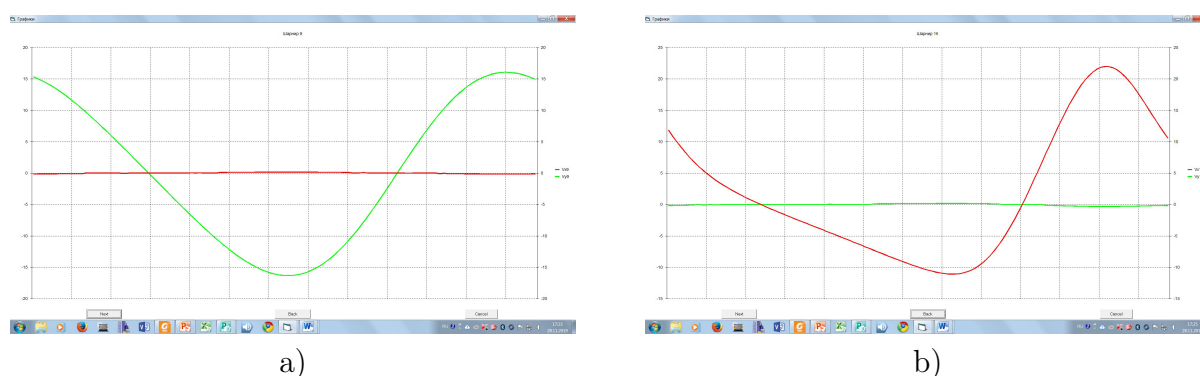a)　　　　　　　　　　　　　　　　　　　b)

Figure 5: Analysis of the stamping mechanism speeds
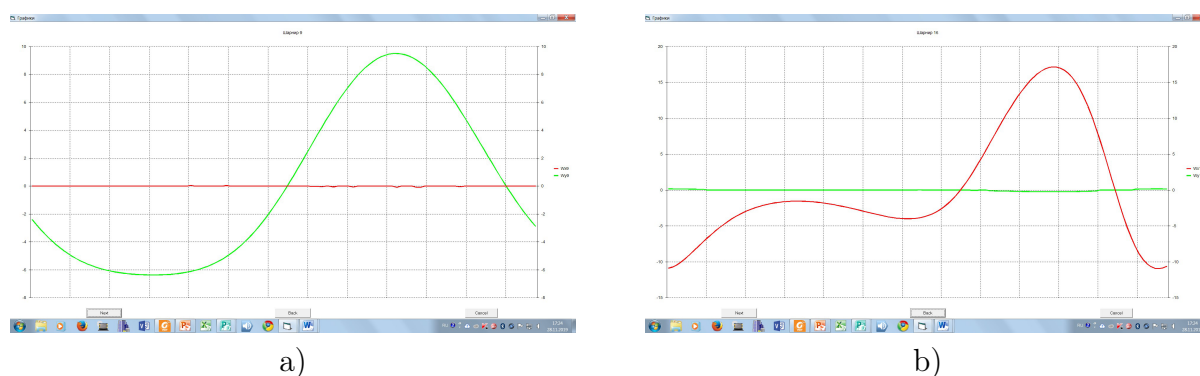

a)　　　　　　　　　　　　　　　　　　　b)

Figure 6: Analysis of the stamping mechanism acceleration

4 Conclusions

A method for kinematic synthesis of link mechanisms based on the mean-square minimization of the objective function has been developed, and a method for deriving this objective function is proposed taking into account the structural (constructive) features of the mechanism. Based on this method and the synthesis method for the three given positions of the slider and rocker of the four-link crank-slide mechanism, algorithms were constructed and Stephenson's stamping mechanism is synthesized with additional workpiece feeding mecha-

nism. Numerical modeling programs based on the Maple environment were developed, and verification calculations were performed to analyze the position, velocities, and accelerations in the ASIAN-2014 environment.

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