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STUDY OF FORCED VIBRATIONS TRANSITION PROCESSES OF VIBRATION PROTECTION DEVICES WITH ROLLING-CONTACT BEARINGS

Many seismic isolation and vibration protection devices use asan essential element the various types of rolling-contact bearings. The rolling-contact bearing is used for creation of moving base of body protected against vibration. The most dynamic disturbances acting in the constructions and structures have highly complex and irregular nature.
This article considers the oscillation of a solid body on kinematic foundations, the main elements of which are rolling bearers bounded by the high order surfaces of rotation at horizontal displacement of the foundation. It is ascertained that the equations of motion are highly nonlinear differential equations. Stationary and transitional modes of the oscillatory process of the system have been investigated. It is determined that several stationary regimes of the oscillatory process exist. Equations of motion have been investigated also by quantitative methods.
In this paper the cumulative curves in the phase plane are plotted, a qualitative analysis for singular points and study of them for stability is performed. In the Hayashi plane a cumulative curve of body protected against vibration forms a closed path which does not tend to the stability of singular point. This means that the vibration amplitude of body protected against vibration is not remain constant in steady-state, but changes periodically.
Key words: protection against vibration,rolling-contact bearing, nonlinear vibrations, cumulative curves, singular point.

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Теңселмелі тірекке орналастырылған дірілденқорғау қондырғысының мәжбүр тербелісінің өтпелі процесстерін зерттеу

Көптеген діріденқорғау және сейсмоқорғау қондырғыларында негізгі элемент ретінде әртүрлі түрдегі теңселмелі тіректер қолданады.Теңселмелі тірек дірілден қорғалатын денеге қозғалмалы табан жасау үшін қолданылады. Ғиаратарға және құрылғыларға әсер ететін динамикалық ұйытқытулардың көпшілігі өте күрделі және жүйесіз сипаттарда болады.
Берілген мақалада, табаны горизонталь бағытта орын ауыстырған жағыдайдағы, негізгі элементі жоғары дәрежелі айналу беттерімен шектелген теңселмелі тірек болатын кинематикалық табанға орнатылған қатты дененің тербелісі қарастырылады.Қозғалыс теңдеуі айтарлықтай сызықты емес дифференциальдық теңдеу болады. Жүйенің тебелмелі процессінің стационарлы және өтпелі режимдері зерттелді. Тербелмелі процесстердің бірнеше режимдері бар екендігі тағайындалды. Қозғалыс теңдеуі сандық әдіс арқылы да зерттелді.

Бұл жұмыста, фазалық жазықтықта интегральдық қисықтар ұрғызылған, ерекше нүктелерге сапалы талдаулар жасалынып және оларды орнықтылыққа зерттеген. Хаяси жазықтығында дірілден қорғалатын дененің интегральдық қисығы тұйық траектория жасайды және ол орнықты ерекше нүктеге ұмтылмайды. Бұл дірілденқорғалатын дененің тербелісінің, орныққан режимдегі амплитудасының тұрақты болмайтындығының,оның периодты өзгеретіндігін білдіреді.
Түйін сөздер: дірілденқарғайтын қондырғы, теңселмелі тірек, сызықты емес тербелістер, интегральдық қисықтар, ерекше нүктелер.

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Исследование переходных процессов вынужденных колебаний виброзащитных устройств на опорах качения

Во многих виброзащитных и сейсмозащитных устройствах, в качестве основного элемента используется опора качения различного вида. Опора качения применяется для создания подвижного основания виброзащищаемого тела. Большинство данимических возмушений, действующих в сооружениях и конструкциях, носит весьма сложный и нерегулярный характер.
В данной статье рассматриваются колебания твердого тела на кинематических основаниях, основными элементами которых являются подвижные опоры, ограниченные поверхностями вращения высокого порядка при горизонтальном смещении основания. Установлено, что уравнения движения являются сильно нелинейными дифференциальными уравнениями. Исследованы стационарные и переходные режимы колебательного процесса системы. Установлено, что существует несколько стационарных режимов колебательного процесса. Уравнения движения были исследованы также количественными методами.
В данной работе, построено интегральные кривые на фазовой плоскости, проведено качественное анализ на особые точки и исследовано их на устойчивость. На плоскости Хаяси интегральная кривая виброзащищаемого тела образует замкнутую траекторию, которая не стремится к устойчивой особой точке. Это означает, что амплитуда колебания виброзащищаемого тела в установившемся режиме не остается постоянной, а периодически меняются.

Ключевые слова: виброзащитные устройства, опора качения, нелинейные колбания, интегральные кривые, особая точка.

1 Introduction

The tasks considered in this work have arisen from the problems of earthquake-resistant constructing.

The essence of the matter is that the protection of building structures from the destructive forces of nature, appeared in earthquakes, is carried out almost exclusively by strengthening the structures nowadays. The taken measures, although they provide the seismic resistance of the facilities under construction to a certain extent, lead to a rise in the cost of construction in seismically active areas, depending on the seismic zone score. Therefore, along with further improvement of measures to increase the seismic resistance of building structures, clarifying the parameters of seismic effects and the values of the calculated seismic loads, the search for new effective methods of seismic protection is of great relevance.

First of all, these searches are carried out by developing new structures and their elements that ensure reliability and high economic efficiency of construction in seismic areas.

The use of devices called seismic insulating foundations involves the counteraction of building structures to seismic forces not by improving the strength properties of structures, but, as it is done in a wide variety of vibration protection systems, by reducing the seismic load on the protected objects. This is quite new for the earthquake-resistant constructing.

In work [1] we gave a review, a classification and the comparison of the devices designed to reduce the seismic load on buildings and which are the integral part of their foundations. Two classes of seismic isolating devices have been identified, which are the example of the direct transfer of vibro-isolation principles to constructing.

These are foundations with elastic support elements and dynamic dampers of seismic vibrations. Two classes of shock absorbers of a different kind have been established:

1. Foundations with servomechanisms, which include rigid supports with an indifferent or even unstable equilibrium position (balls, rollers, vertically arranged spars, etc.) and servomechanisms that return the building to its equilibrium position; at the same time, a compromise solution is often given, combining rolling or sliding bearings and elastic shock absorbers that replace servomechanisms;

2. Kinematic foundations, in which, as in foundations with servomechanisms, the seismic isolation is carried out not due to the elasticity of the shock absorber, but using supports of a special geometric shape; a building, a structure installed on such supports, has a stable equilibrium position, when removed from that position it oscillates with a frequency that depends [1,2] mainly on the geometric dimensions of the supports and the acceleration of gravity (for this reason, such devices are called kinematic [2] and [3] gravitational seismic isolation systems);

3. The most acceptable and promising from an engineering point of view, as noted in [1, 3], is the newest class of seismic isolating devices — a class of supporting kinematic foundations that favorably differ from other types of seismic shock absorbers in cost-effectiveness and simplicity of technical solution.

The kinematic supports developed in connection with the requests of earthquake-resistant constructions can be used as shock absorbers in vibro-isolation systems of various machines and equipment, and as elements of devicesas well.

This article [19, 20] considers the oscillation of a solid body on kinematic foundations, the main elements of which are rolling bearers bounded by high-order surfaces of rotation at horizontal displacement of the foundation. Equations of motion of the vibro-protected body have been obtained. Stationary and transitional modes of the oscillatory process of the system have been investigated.

The work contains geometrical analysis of non-linear vibrations of vibro-protective systems on rolling bearers bearing elements of which are restricted by high order spherical surfaces in transition regime.

2 Literature review

Rolling bodies of various types are applied as the main element in many vibro-protective and seismo-protective devices.

The work [4] contains systematic depiction of non-linear systems analysis method, described by differential equations of second rate. This work contains also topological and graphical methods, applicable for calculation of autonomic and, especially, non-autonomic systems.

In the work [5] the author focuses attention on decision of tasks on determination of orders of initial conditions, leading to various stable stationary decisions. In the work [6] the author considers problems of self-oscillations of various mechanical systems, particularly, examines in detail self-oscillations of rotors.

The work [7] studies the features of vibrational motion of an orthogonal mechanism with disturbances, such as restricted power in the presence of a fixed load on the horizontal link. Dynamic and mathematical models were prepared, and the operating conditions' fields of existence for the vibration mechanism in terms of the driving power were defined.

This paper [8] presents results of modelling of vibrations of rigid rotor caused by the degradation of hydrodynamic bearings. Model is composed applying equations of nonlinear hydrodynamic forces and measured parameters of a real rotary machine.

In order to study the resonance of a rotating circular plate under static loads in magnetic field, in the work [9] the nonlinear vibration equation about the spinning circular plate is derived according to Hamilton principle. The algebraic expression of the initial deflection and the magneto elastic forced disturbance differential equation are obtained through the application of Galerkin integral method.

This paper [10] presents a new semi analytical approach for geometrically nonlinear vibration analysis of Euler-Bernoulli beams with different boundary conditions. The method makes use of Linstedt-Poincar'e perturbation technique to transform the nonlinear governing equations into a linear differential equation system, whose solutions are then sought through the use of differential quadrature approximation in space domain and an analytical series expansion in time domain.

In the work [11] a systematic method is developed for the dynamic analysis of the structures with sliding isolation which is a highly non-linear dynamic problem. According to the proposed method, a unified motion equation can be adapted for both stick and slip modes of the system. Unlike the traditional methods by which the integration interval has to be chopped into infinitesimal pieces during the transition of sliding and non-sliding modes, the integration interval remains constant throughout the whole process of the dynamic analysis by the proposed method so that accuracy and efficiency in the analysis of the non-linear system can be enhanced to a large extent.

The paper [12] features a survey of some recent developments in asymptotic techniques, which are valid not only for weakly nonlinear equations, but also for strongly ones. Further, the obtained approximate analytical solutions are valid for the whole solution domain. The limitations of traditional perturbation methods are illustrated, various modified perturbation techniques are proposed, and some mathematical tools such as variational theory, homotopy technology, and iteration technique are introduced to over-come the shortcomings.

The effects of neglecting small harmonic terms on estimation of dynamical stability of the steady state solution determined in the frequency domain are considered in the paper[13]. For that purpose, a simple single-degree- of-freedom piecewise linear system excited by a harmonic excitation is analyzed. In the time domain, steady state solutions are obtained by using the method of piecing the exact solutions (MPES) and in the frequency domain, by the incremental harmonic balance method(IHBM). The stability of the solutions obtained in the frequency domain by IHBM is determined by using Floquet-Liapounov theorem and by digital simulation of the corresponding perturbed motion.

In the paper[14]the nonlinear response of a base-excited slender beam carrying an attached mass is investigated with 1:3:9 internal resonances for principal and combination parametric resonances.

3 Material and methods

3.1 Equations of the motion

Let us consider the principle of work of the kinematic foundation of moving supporting elements, which is a rolling bearing with bounded surfaces of rotation of a high (n) order (Fig.1).

On the Figure 1, the object I is a rolling bearing with bounded (top and bottom) surfaces of rotation, expressed by formulas

$$y_1 = a_1 x_1^n, \quad y_2 = a_2 x_1^m \tag{1}$$

and having a common axis of symmetry; but objects 2 and 3 are stationary base (foundation) and inner coat of the vibro-protected body.

Equations (1) are referred to the coordinate system associated with the rolling bearings (See Fig.1). The curvature radius of the vertices of these surfaces at $n, m > 2$ tends to infinity, i.e. there is straightening of the bearing surfaces. Let us denote the horizontal offset of the bases as $\tilde{x}_0(t)$. As $\tilde{x}(t)$ we denote a displacement of the upper body, supporting on the rolling bearing.

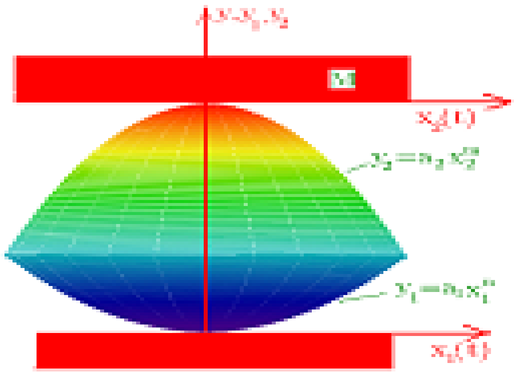


Figure 1: Scheme of rolling bearingswith higher ordersurfaces

The equation (2) can be reduced to an equation in dimensionless form [19]:

$$\ddot{x} + \Phi(x - x_0) - x = -x_0(t), \tag{2}$$

where

$$\Phi(x - x_0) = N_n (x - x_0)^{\dfrac{1}{n-1}}, \tag{3}$$

$$N_n = \frac{1}{\sqrt[n-1]{nH}}\left(\frac{1}{\sqrt[n-1]{a_1}} + \frac{1}{\sqrt[n-1]{a_2}}\right). \tag{4}$$

3.2 Periodic solutions and their stability

Let us study the vibrations of a body at harmonic horizontal displacement of the lower base

$$x_0(t) = Q \sin pt, \tag{5}$$

where Q and p – dimensionless amplitude and frequency of perturbations.

Assuming that in the case of harmonic oscillations, a component of the fundamental frequency, having period $2\pi/p$, dominates over the higher harmonics. Periodic solution and first derivative of the equation (5) can be approximately represented as,

$$x = a \sin pt + b \cos pt,\ \dot{x} = ap \cos pt - bp \sin pt, \tag{6}$$

Let us suppose that the amplitudes a and b are functions of time and slowly vary depending on t.

For the nonlinear term of the equation (2), Fourier series expansion looks as:

$$\Phi(x - x_0) = N_n C^{\overline{\frac{1}{n-1}}} \sin^{\overline{\frac{1}{n-1}}}(pt+\gamma) = \sum_{k=1}^{\infty} B_{2k-1} \sin(2k-1)pt + D_{2k-1}\cos(2k-1)pt, \tag{7}$$

where

$$\begin{gathered} C = \sqrt{(a-Q)^2 + b^2},\quad tg\gamma = \frac{b}{a-Q},\quad B_{2k-1} = N_n K_{2k-1} \frac{(a-Q)}{[(a-Q)^2+b^2]}^{\overline{\frac{n-2}{2(n-1)}}}, \\ D_{2k-1} = N_n K_{2k-1} \frac{b}{[(a-Q)^2+b^2]^{\overline{\frac{n-2}{2(n-1)}}}},\quad K_{2k-1} = \sqrt{L_{2k-1}^2 + M_{2k-1}^2}, \end{gathered} \tag{8}$$

$$L_{2k-1} = \frac{1}{\pi}\int_0^{2\pi} \sin^{\overline{\frac{1}{n-1}}} \psi \sin(2k-1)\psi d\psi,\ M_{2k-1} = \frac{1}{\pi}\int_0^{2\pi} \sin^{\overline{\frac{1}{n-1}}} \psi \cos(2k-1)\psi d\psi,\ \psi = pt+\gamma.$$

Substituting (6), (7) to (2) and equating to zero the individual coefficients of the terms, containing $\sin pt$ and $\cos pt$, we have

$$\frac{da}{dt} = \frac{1}{p}\left\{(p^2+1) - N_n K_1 \frac{1}{[(a-Q)^2+b^2]^{\overline{\frac{n-2}{2(n-1)}}}}\right\} b = X(a,b),$$

$$\frac{db}{dt} = -\frac{1}{p}\left\{\left[(p^2+1) - N_n K_1 \frac{1}{[(a-Q)^2+b^2]^{\frac{n-2}{2(n-1)}}}\right](a-Q) + p^2 Q\right\} = Y(a,b). \quad (9)$$

Let us consider the steady state, when amplitudes $a(t)$ and $b(t)$ in (6) are constant, i.e.

$$\frac{da}{dt} = X(a,b) = 0, \quad \frac{db}{dt} = Y(a,b) = 0. \quad (10)$$

In light of these conditions, from equations (9) we can get that the set amplitude $a_0 = A$, $b_0 = 0$ of the periodic solution $x(t)$ is determined by the formula

$$A = \frac{1}{p^2+1}\left[N_n K_1 (A-Q)^{\frac{1}{n-1}} + Q\right]. \quad (11)$$

Let us derive the conditions for the stability of periodic solutions. We will consider small deviations ξ and η from the amplitudes a_0 and b_0 and will find out, when these deviations (with increasing time) are close to zero.

From equation (9) we get

$$\begin{aligned} \frac{d\xi}{dt} &= \alpha_1 \xi + \alpha_2 \eta, \\ \frac{d\eta}{dt} &= \beta_1 \xi + \beta_2 \eta, \end{aligned} \quad (12)$$

Where

$$\begin{aligned} \alpha_1 &= \frac{(n-2)}{(n-1)}\frac{1}{p}\frac{W_0}{C_0^2}(a_0-Q)b_0, \\ \alpha_2 &= \frac{1}{p}\left\{(p^2+1) - W_0 + \frac{(n-2)}{(n-1)}\frac{W_0}{C_0^2}b_0^2\right\}, \\ \beta_1 &= \frac{1}{p}\left\{-(p^2+1) + W_0 - (\frac{n-2}{n-1})\frac{W_0}{C_0^2}(a_0-Q)^2\right\}, \\ \beta_2 &= -\frac{1}{p}\left\{(\frac{n-2}{n-1})\frac{W_0}{C_0^2}(a_0-Q)b_0\right\}, \end{aligned} \quad (13)$$

where

$$W_0 = \frac{N_n K_1}{C_0^{\frac{n-2}{n-1}}}, \quad C_0 = A - Q.$$

The characteristic equation of the system has the form:

$$\lambda^2 - (\alpha_1 + \beta_2)\lambda + \alpha_1\beta_2 - \alpha_2\beta_1 = 0. \tag{14}$$

The stability condition is given by Routh-Hurwitz criteria, i.e.

$$\alpha_1 + \beta_2 = 0, \quad (\alpha_1 = 0, \beta_2 = 0).$$
$$\alpha_1\beta_2 - \alpha_2\beta_1 > 0$$

to

$$\left[(p^2 + 1) - W_0\right]\left[(p^2 + 1) - \frac{W_0}{n-1}\right] > 0. \tag{15}$$

The singular point, i.e. steady system state, is a center.

The boundary of unstable periodic solutions of equations (9) is determined by the curves.

$$p^2 = W_0 - 1, \quad p^2 = \frac{W_0}{n-1} - 1 \tag{16}$$

and stability areas are determined by the following inequalities [19]

$$\begin{gathered} p^2 - (W_0 - 1) > 0, p^2 - (\frac{W_0}{n-1} - 1) > 0, \\ p^2 - (W_0 - 1) < 0, \quad p^2 - (\frac{W_0}{n-1} - 1) < 0. \end{gathered} \tag{17}$$

4 Simulation Results:Geometric analysis of the integral curves

From equation (9), we have

$$Y(a, b)da - X(a, b)db = 0. \tag{18}$$

As due to equations (9) $\dfrac{\partial X}{\partial a} + \dfrac{\partial Y}{\partial b} = 0$, the equation (9) becomes integrable, and its complete integral has the form

$$-(p^2 + 1)\frac{C^2}{2} + \frac{2(n-1)}{n} N_n K_1 C^{\overline{n-1}^{\,n}} - p^2 Qa = E, \tag{19}$$

where E – constant of integration. In order to examine the integral curves in the neighborhood of a singular point, we move the origin of coordinates to this particular point a_0, b_0 introducing new variables ξ and η, namely:

$$a = a_0 + \xi, \quad b = b_0 + \eta.$$

Then the basic system of equations (9) takes the form,

$$
\begin{aligned}
\frac{d\xi}{dt} &= \alpha_1\xi + \alpha_2\eta + (\frac{n-2}{n-1})\frac{1}{p}\left[\frac{1}{2}b_0\xi^2 + (a_0-Q)\xi\eta + \frac{3}{2}b_0\eta^2 + \frac{1}{2}\xi^2\eta + \frac{1}{2}\eta^3\right]\frac{W_0}{C_0^2},\\
\frac{d\eta}{dt} &= \beta_1\xi + \beta_2\eta + (\frac{n-2}{n-1})\frac{1}{p}\left[\frac{3}{2}(a_0-Q)\xi^2 + b_0\xi\eta + \frac{1}{2}(a_0-Q)\eta^2 + \frac{1}{2}\xi\eta^2 + \frac{1}{2}\xi^3\right]\frac{W_0}{C_0^2},
\end{aligned}
\tag{20}
$$

where the following relations are used

$$
W = W_0 - (\frac{n-2}{n-1})\frac{W_0}{C_0^2}\left[(a_0-Q)\xi + \frac{1}{2}\xi^2 + b_0\eta + \frac{1}{2}\eta^2\right], \quad W = \frac{N_nK_1}{C^{\frac{n-2}{n-1}}}, \quad W_0 = \frac{N_nK_1}{C_0^{\frac{n-2}{n-1}}}
$$

Taking into account that $b_0 = 0$due to the equation (9), we get

$$
\begin{aligned}
\frac{d\xi}{dt} &= \bar{\alpha}_2\eta + (\frac{n-2}{n-1})\frac{1}{p}\frac{N_nK_1}{(a_0-Q)^{\frac{3n-4}{n-1}}}\left[(a_0-Q)\xi\eta + \frac{1}{2}\xi^2\eta + \frac{1}{2}\eta^3\right],\\
\frac{d\eta}{dt} &= \bar{\beta}_1\xi + (\frac{n-2}{n-1})\frac{1}{p}\frac{N_nK_1}{(a_0-Q)^{\frac{3n-4}{n-1}}}\left[\frac{3}{2}(a_0-Q)\xi^2 + \frac{1}{2}(a_0-Q)\eta^2 + \frac{1}{2}\xi\eta^2 + \frac{1}{2}\xi^3\right],
\end{aligned}
\tag{21}
$$

where

$$
\bar{\alpha}_2 = \frac{1}{p}\left[(p^2+1) - \frac{N_nK_1}{(a_0-Q)^{\frac{n-2}{n-1}}}\right], \quad \bar{\beta}_1 = \frac{1}{p}\left[-(p^2+1) - \frac{N_nK_1}{(n-1)(a_0-Q)^{\frac{n-2}{n-1}}}\right].
$$

Equations (21) are integrated. As a result of integration we obtain

$$
\bar{\beta}_1\xi^2 - \bar{\alpha}_2\eta^2 + (\frac{n-2}{n-1})\frac{1}{p}\frac{N_nK_1}{(a_0-Q)^{\frac{3n-4}{n-1}}}\left[(a_0-Q)\xi^3 + \frac{1}{4}(\xi^4 - \eta^4)\right] = F, \tag{22}
$$

where F – constant of integration.

In order to classify the type of singular points, we calculate the roots of the characteristic equation (14):

$$
\lambda_{1,2} = \frac{\alpha_1 + \beta_2 \pm \sqrt{(\alpha_1-\beta_2)^2 + 4\alpha_2\beta_1}}{2},
$$

where from

$$\lambda_1, \quad \lambda_2 = \pm\sqrt{\alpha_2\beta_1}, \quad \alpha_1 + \beta_2 = 0.$$

The dependence curve between Q and A_0 is presented on Fig. 2. Let us divide the curve into three parts 1, 2, 3 (as shown in Fig. 2): the boundaries of each part are determined by the points D and B, for which,

$$A = \frac{N_n K_1}{p^2+1} \sqrt[n-2]{\frac{N_n K_1}{p^2+1}},$$

$$A = \frac{[(n-1)(p^2+1)-1]}{(n-1)(p^2+1)p^2} \cdot \sqrt[n-2]{\frac{N_n K_1}{(n-1)(p^2+1)}}.$$

These areas correspond to the following special terms:
1. Center ($\alpha_2\beta_1 < 0$)
2. Saddle ($\alpha_2\beta_1 > 0$)
3. Center ($\alpha_2\beta_1 < 0$)

We shall consider the case when $Q = 0.014081$. Here there are three possible states of equilibrium (Fig. 2); data for the respective singular points are shownin Table 1.

Table 1. – Singular points (Fig. 3)

Singular point	A_0	λ_1, λ_2	μ_1, μ_2	Classification
1	-0.055919	$\pm 1.209i$		Center
2	0.045107	1.552	∓ 1.139	Saddle
3	0.015344	$\pm 18.965i$		Center

Directions of the integral curves at the singular points (node and saddle) can be found by the following expression

$$\mu_{1,2} = \frac{-(\alpha_1 - \beta_2) \pm \sqrt{(\alpha_1-\beta_2)^2 + 4\alpha_2\beta_1}}{2\alpha_2}.$$

Integral curves of equation (9) can be easily obtained by using equation (19) for different values of E. The results are shown in Fig. 3. We see that in a conservative system, each integral curve forms a closed trajectory, which does not tend to a stable singular point.

This means that the amplitude and phase fluctuations angle in a steady state do not remain constant, but vary periodically. Thus the phase fluctuations can outperform an external force and be behind it. If a closed trajectory does not cover the origin of coordinates, so the angle of advance and the angle of retard are mutually compensated after passing the representation point over a closed trajectory, and an oscillation will be synchronized with an external force. On the other hand, if the origin of coordinates is located inside a closed trajectory, so as a result of each cycle, there will be a phase difference of 2π radian, and oscillation will not be synchronized with the external force.

a. The integral curves of the system, corresponding to the point D (Fig. 2).

Assuming that $Q = 0$ in equations (9), we have

$$\frac{db}{da} = -\frac{a}{b}$$

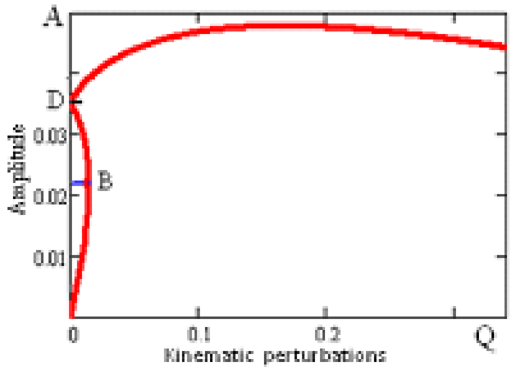


Figure 2: Amplitude response curve for harmonic vibration

or after integration

$$a^2 + b^2 = const.$$

Consequently, the integral curves form a family of concentric circles with a center at the coordinate origin, so that the singular point (in this case - the origin of coordinates) is a center.

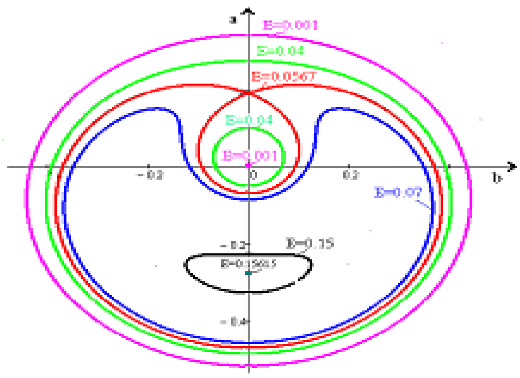


Figure 3: Cumulative curves for harmonic vibration

Period T, necessary in order to the representation points $a(t)$ and $b(t)$ make one revolution along a closed trajectory, is defined by the expression.

$$T = \oint \frac{ds}{\sqrt{X^2(a,b) + Y^2(a,b)}} = \oint \frac{pds}{\left[p^2 - (W-1)\right] A} = \frac{2\pi p}{p^2 - (W-1)},$$
$$ds = \sqrt{(da)^2 + (db)^2}, \quad W = \frac{N_n K_1}{A^{\frac{n-2}{n-1}}}. \tag{23}$$

Now let us suppose, that the initial condition is given by the point $a(0)$, $b(0)$ located on a circle of radius $A = (\frac{N_n K_1}{p^2+1})^{\frac{n-1}{n-2}}$; then the period T will be equal to infinity. As can be seen from the equations (9), the representation point $a(t)$, $b(t)$ in this case remains in its initial position. This means that the oscillation frequency coincides with the frequency of the external forces. Then from equations (9) we can see that the representation point $a(t)$, $b(t)$ is moving circumferentially in the counterclockwise direction when $A > (\frac{N_n K_1}{p^2+1})^{\frac{n-1}{n-2}}$, and in the clockwise direction – when $A < (\frac{N_n K_1}{p^2+1})^{\frac{n-1}{n-2}}$.

In the first case, the oscillation frequency is higher than the external force; in the second case, the pattern will be reversed. So we can conclude, that the oscillation frequency varies depending on A and coincides with the frequency of an external force only in the case where

$$A = (\frac{N_n K_1}{p^2+1})^{\frac{n-1}{n-2}}.$$

b. The integral curves of the system, corresponding to the point B ***(Fig. 2)***

In this case, from equations (11), we obtain

$$\begin{aligned} &Q = \frac{(n-1)(p^2+1)}{p^2}\left[\frac{N_n K_1}{(n-1)(p^2+1)}\right]^{\frac{n-1}{n-2}}, \quad b_0 = 0, \\ &a_0 = A = \frac{n(p^2+1)-1}{p^2}\cdot\left[\frac{N_n K_1}{(n-1)(p^2+1)}\right]^{\frac{n-1}{n-2}}. \end{aligned} \tag{24}$$

Let us investigate nature of the singular point B. From (21) we have

$$\bar{\alpha}_2 = -(n-2)\frac{p^2+1}{p}, \quad \bar{\beta}_1 = 0,$$

Where from $\lambda_1 = \lambda_2 = 0$. Then equation (21) takes the form

$$\begin{aligned} &\frac{d\xi}{dt} = -\gamma\eta + C_1(C_0\xi\eta + \frac{1}{2}\xi^2\eta + \frac{1}{2}\eta^3), \\ &\frac{d\eta}{dt} = \frac{1}{2}C_1(3C_0\xi^2 + C_0\eta^2 + \xi\eta^2 + \xi^3), \end{aligned} \tag{25}$$

Where

$$\begin{aligned} &\gamma = (n-2)\frac{p^2+1}{p}, \quad C_1 = \frac{1}{p}(\frac{n-2}{n-1})\frac{[(n-1)(p^2+1)]^{\frac{3n-4}{n-2}}}{(N_n K_1)^2}, \\ &C_0 = \left[\frac{N_n K_1}{(n-1)(p^2+1)}\right]. \end{aligned} \tag{26}$$

Substituting $dz = -\gamma dt$, we get

$$\begin{aligned} \frac{d\xi}{dz} &= \eta - \frac{C_1}{\gamma}(C_0\xi\eta + \frac{1}{2}\xi^2\eta + \frac{1}{2}\eta^3), \\ \frac{d\eta}{dz} &= -\frac{C_1}{2\gamma}(3C_0\xi^2 + C_0\eta^2 + \xi\eta^2 + \xi^3). \end{aligned} \tag{27}$$

Integral curves in the plane ξ, η approach to the origin of coordinates, touching the straight line $\eta = 0$. Applying the substitution $\eta = x_1\xi$, we have

$$\begin{aligned} \frac{d\xi}{dz} &= x_1\xi - \frac{C_1C_0}{\gamma}x_1\xi^2 - \frac{C_1}{2\gamma}x_1\xi^3 - \frac{C_1}{2\gamma}x_1^3\xi^3 \\ \frac{dx_1}{dz} &= -\frac{3C_1C_0}{2\gamma}\xi - x_1^2 - \frac{C_1}{2\gamma}\xi^2 + \frac{C_1C_0}{2\gamma}x_1^2\xi + \frac{C_1}{2\gamma}x_1^4\xi^2 \end{aligned} \tag{28}$$

Now the integral curves in the plane ξx approach to the origin of coordinates, touching the straight line $\xi = 0$. Next, using the substitution $\xi = x_1y_1$, equation (28) is reduced to the form:

$$\begin{aligned} \frac{dy_1}{dz} &= \frac{3C_1C_0}{2\gamma}y_1^2 + 2x_1y_1 + \left\{-\frac{C_1}{2\gamma}x_1y_1^3 - \frac{3C_1C_0}{2\gamma}x_1^2y_1^2 - \frac{C_1}{2\gamma}x_1^3y_1^3 - \frac{C_1}{\gamma}x_1^5y_1^3\right\} \\ \frac{dx_1}{dz} &= -\frac{3C_1C_0}{2\gamma}x_1y_1 - x_1^2 + \frac{C_1}{2\gamma}\left\{-x_1^2y_1^2 + C_0x_1^3y_1 + x_1^6y_1^2\right\} \end{aligned} \tag{29}$$

Tangents to the integral curves at the coordinate origin on the plane x_1, y_1 are determined by the Theorem of Bendixson [4]

$$x_1y_1(x_1 + \frac{C_1C_0}{\gamma}y_1) = 0 \tag{30}$$

However, in the equation tangents $x_1 = 0$, $y_1 = 0$ degenerate at the coordinate origin of the plane ξ, η, therefore, we consider only the integral curves, having at the coordinate origin a tangent $x_1 + \frac{C_1C_0}{\gamma}y_1 = 0$. For this purpose, we apply the transformation

$$x_1 = (x_2 - \frac{C_1C_0}{\gamma})y_1$$

Then equation (29) takes the form

$$y_1\frac{dx_2}{dy_2} = \frac{-\frac{3C_1C_0}{\gamma}x_2 + 3x_2^2 + y_1^2\varphi(x_2, y_2)}{\frac{C_1C_0}{\gamma} - 2x_2 + y_1^2\psi(x_2, y_1)} \tag{31}$$

where $\varphi(x_2, y_1)$ and $\psi(x_2, y_1)$ are polynomials relatively x_2 and y_1. Equation (31) can be presented as:

$$y_1 \frac{dx_2}{dy_1} = -6x_2 + B(x_2, y_1) \tag{32}$$

where $B(x_2, y_1)$ consists of the terms of higher degree relatively x_2 and y_1. Bendixson investigated the differential equation of the form

$$x''' \frac{dy}{dx} = ay + bx + B(x, y) \tag{33}$$

and determined, that if $a < 0$, m – an odd number, then the origin of coordinates is a saddle point.

For equation (32) we have $m = 1$ (odd number) and $a = -b < 0$ so, the singular point $(x_2 = 0, y_1 = 0)$ is a saddle; and the integral curves tend to it, having tangents $x_2 = 0, y_1 = 0$. Thus, as a result of all the transformations we have

$$\xi = x_1 y_1 = (x_2 - \frac{C_1 C_0}{\gamma}) y_1^2, \quad \eta = x_1 \xi = x_1^2 y_1 = (\frac{C_1 C_0}{\gamma})^2 y_1^3$$

As it was mentioned previously, tangent $y_1 = 0$ in the plane ξ, η reduces to the origin of coordinates; tangent $x_2 = 0$ enters the curve

$$\xi = -\frac{C_1 C_0}{\gamma} y_1^2, \quad \eta = (\frac{C_1 C_0}{\gamma})^2 y_1^3 \tag{34}$$

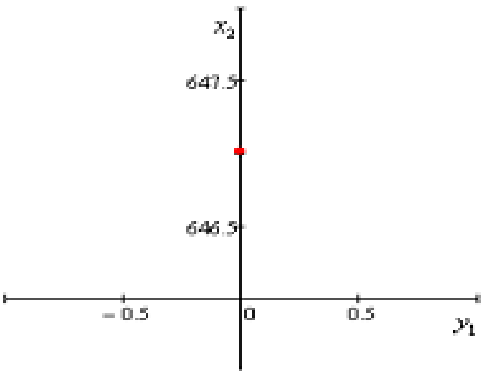
and we can assume, that it represents the integral curves in the neighborhood of the origin of the plane ξ, η.

Fig. 4 (in the corresponding coordinates) represents the tangent $x_2 = 0$.

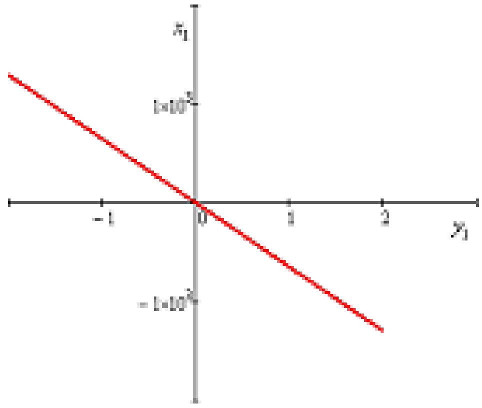
In conclusion we shall note, that this singular point is a saddle-node: as it can be seen from the equations (25), the representation point $\xi(t), \eta(t)$ with increasing time is moving along the integral curve along the direction, indicated by arrows.

5 Conclusions

Peculiarities of integral curves of vibro-protective systems on rolling bearings in absence of rolling friction are investigated. Special points of integral curves ar defined and it is ascertained that special points are centre, saddle and centre. The special point D (in this case the point of reference) is centre. Oscillation frequency (fig.8) changes depending on A and coincides with frequency of external force frequency only in case, when $A = (N_n K_1/p^2 + 1)^{\frac{n-1}{n-2}}$. The special point B is regarded as sadle-knot.



a)



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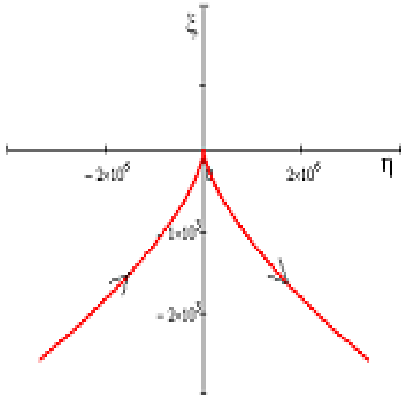

c)

Figure 4: Integral curves in respective coordinates: a special point suits the point B in Fig. 2

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