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Processing, storage, and transmission of information are important processes in modern society. The practical application of cryptography has become an integral part of the life of modern society. In Kazakhstani, for the protection of the electronic information are mainly used foreign software and hardware-software tools. Therefore, the development of Kazakhstan cryptographic protection tools is certainly necessary. This article describes the new Qamal NPNS encryption algorithm, which is a modification of the previously developed Qamal encryption algorithm. The modification lies in the use of a transformation based on a non-positional polynomial notation (NPN). To build a new encryption algorithm, an SP-network is also used. The theoretical justification of the appropriateness of applying the NPN and the results of the analysis of the encryption algorithm are given. Algebraic cryptanalysis for multiplication in non-positional polynomial notations was considered separately. The study of the algorithm strength for separate procedures showed good results, which suggest the cryptographic strength of the developed algorithm.
Key words: cryptography, encryption, S-box, non-positional polynomial notation, SP-network.
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## «QAMAL NPNS» шифрлеу алгоритмінің позициялы емес полиномдық санау жүйесін пайдаланған модификациясы

Ақпаратты өңдеу, сақтау және алмасу қазіргі қоғамдағы маңызды процесс болып табылады. Криптографияны іс жүзінде қолдану қазіргі қоғам өмірінің ажырамас бөлігіне айналды. Қазақстанда электронды ақпаратты қорғау үшін негізінен шетелдіік бағдарламалық және аппараттық-бағдарламалық құралдар қолданылады. Сондықтан да қазақстандық криптографиялық қорғаудың құралдарын әзірлеу қажет. Бұл мақалада бұрын әзірленген Qamal шифрлеу алгоритмінің жаңа модификациясы сипатталады. Модификация позициялы емес полиномдық санау жүйелеріне (ПЕПСЖ) негізделген шифрлеу алгоритмін құру үшін SP желісі қолданылған. Сонымен қатар шифрлеуде ПЕПСЖ пайдалану мақсатының дұрыстығына теориялық түсініктеме берілді және шифрлеу алгоритмін талдау нәтижелері келтірілді. Позициялық емес полиномдық санау жүйелеріндегі көбейтуге арналған алгебралық криптоталдау нәтижелері бөлек көрсетілген. Сонымен қатар, алгоритмде пайдаланылған басқа да процедураларға арналған беріктілігін зерттеу жұмыстары жақсы нәтижелер көрсетті. Бұл өз кезегінде әзірленген алгоритмнің криптографиялық берік болатындығына болжам жасауға негіз болып табылады.
$\mathbf{T}^{\text {Үйін сөздер: криптография, шифрлеу, } S \text { блок, позициялы емес полиномдық санау жүйесі, }}$ SP жүйесі.

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Обработка, хранение и передача информации являются важными процессами в современном обществе. Практическое применение криптографии стало неотъемлемой частью жизни современного общества. В Казахстане для защиты электронной информации применяются в основном зарубежные программные и аппаратно-программные средства. Поэтому разработка казахстанских средств криптографической защиты безусловно является необходимой. В данной статье описывается новый алгоритма шифрования «Qamal NPNS», который является модификацией ранее разработанного алгоритма шифрования «Qamal». Модификация заключается в использовании преобразования, основанного на непозиционной полиномиальной системе счисления (НПСС). Для построения нового алгоритма шифрования также применяется SP-сеть. Приводятся теоретическое обоснование целесообразности применения НПСС и результаты анализа алгоритма шифрования. Отдельно приведены результаты алгебраического криптоанализ для умножения в непозиционных полиномиальных системах счислениях. Исследование стойкости алгоритма для отдельных процедур показало хорошие результаты, что предполагает криптостойкость разрабатываемого алгоритма.
Ключевые слова: криптография, шифрование, S-блок, непозиционная полиномиальная система счисления, SP-сеть.

## 1 Introduction

The science of secret transmission of information arose in ancient times. The development of writing and communications has greatly advanced its formation. The advent of affordable internet has taken cryptography to a new level. Due to the increasing dependence of society on information technology and the need to ensure information security, the use of cryptographic methods has become relevant for almost everyone. However, secrecy can be inferior in importance to ensuring integrity, authenticity and other aspects of security. The invention of new principles of cryptography and the emergence of the so-called public key cryptography gave a powerful impetus to the widespread use of this science for the needs of civil society, business, banking and other fields of activity [1].

Cryptographic information protection is one of the main subsystems of any information protection system. The processes of handling, storage, transmission and use of information become dominant in the life of modern society [1-3]. All specific tasks of cryptography substantially depend on the level of development of engineering and technology, on the means of communication used, and the methods of transmitting information $[4,5]$.

## 2 Literature review

The security of sensitive information has begun to be governed primarily by the key. The encryption algorithm itself is considered to be known to the enemy and available for study,
but the algorithm provides for the use of an unknown to the adversary key, on which the applied information transformations substantially depend [3-6].

Claude Shannon was the first who with mathematical rigor formulated questions about the absolute and theoretical strength of ciphers. Namely, to what extent a cryptosystem is resistant to an attacker with unlimited resources [7]. Requirements for perfect secrecy: 1) the key is truly random (equally probable); 2) the key is exactly as long as the message that is encrypted; 3) the key is used one time only. In case of violation of at least one of these conditions, the cipher ceases to be completely unbreakable, and there appear possibilities in principle to break it. But these conditions make a completely unbreakable cipher very expensive and impractical. Before using such a cipher, it is necessary to provide all subscribers with a sufficient supply of random keys and exclude the possibility of their repeated use. And this is extremely difficult and expensive to do [5, 6]. Therefore, completely unbreakable ciphers are used only in communication networks with a small amount of transmitted information, and these are usually networks for transmitting sensitive or critical information.

Most typically, legitimate users are forced to use not completely unbreakable ciphers to protect their information. Symmetric block encryption algorithms have gained wide use, and now they are the main cryptographic means to ensure confidentiality in the processing of information in modern information and telecommunication systems [5-7].

The main types of block ciphers are a Feistel network and a substitution-permutation network (SP-network). An SP-network is a block cipher in which the transformation of each round is a combination of substitutions (S-boxes) and permutations. Two fundamental principles for constructing cryptographic transformations, confusion and diffusion, proposed by Claude Shannon in 1949, can clearly be implemented in the structure of SP-network[7-9]. Recall that confusion means complicating all kinds of connections between the plaintext and the ciphertext. Examples of SP-networks are the ciphers IDEA, AES (Rijndael), Serpent, Kuznyechik [10-13]. Every day there are more and more such examples. It is the new practical applications of cryptography that are one of the sources of its development.

For Kazakhstan, information and communication technologies play a big part in the development of the young state. In 2017, the Cybersecurity Concept was adopted. The objectives of the Concept are to achieve and maintain the level of security of electronic information resources, information systems, and the information and communication infrastructure from external and internal threats, ensuring sustainable development of the Republic of Kazakhstan in the context of global competition [14].

In recent years, the Institute of Information and Computational Technologies of the Science Committee of the Ministry of Education and Science of the Republic of Kazakhstan has carried out research on the study of symmetrical block encryption algorithms for electronic messages and has developed various modifications, including those based on non-positional polynomial notations (NPNs) [15-21]. These works, in turn, will contribute to the creation of domestic cryptographic information protection facilities.

## 3 Materials and methods

The paper [19] introduced the new symmetric block encryption algorithm Qamal. The Qamal algorithm scheme is shown in Figure 1 (a). The encryption algorithm includes pairing a plaintext with a key using the bitwise addition (XOR) operation, a substitution S-box, and
mixing procedures Mixer1 and Mixer2.
The considered algorithm is a modification of the above one, where a non-positional polynomial notation is used (Figure - $1(\mathrm{~b})$ ). Instead of the operation of pairing (addition) a key modulo 2 (XOR operation) to a plaintext block, multiplication by the NPN is performed. For this reason, the algorithm was named Qamal NPNS. The developed algorithm supports a fixed block and a key length of 128 bits. This is yet another difference from the basic algorithm.

Building an NPN is the selection of its bases designated as working bases. Let some irreducible polynomials be chosen as such bases:

$$
\begin{equation*}
p_{1}(x), p_{2}(x), \ldots, p_{S}(x) \tag{1}
\end{equation*}
$$

Let us denote their degrees by $m_{1}, m_{2}, \ldots, m_{S}$ respectively. The polynomials (1), considering their arrangement, form a single system of bases. The main working range of the NPN is the polynomial $P^{m}(x)=p_{1}(x) p_{2}(x) \ldots p_{S}(x)$ of degree $m=\Sigma_{i=1}^{S} m_{i}$. In the NPN, any polynomial $F(x)$ whose degree is fewer than m has a unique non-positional representation in the form of a sequence of residues of its division by the bases(1):

$$
\begin{equation*}
F(x)=\left(\alpha_{1}(x), \alpha_{2}(x), \ldots, \alpha_{S}(x)\right) \tag{2}
\end{equation*}
$$

where $\alpha_{i} \equiv F(x)(\operatorname{modp}(x)), i=1, \ldots, S$. The positional notation of $F(x)$ is restored by its nonpositional form (2) [22-25]:

$$
F(x)=\sum_{i=1}^{S} \alpha_{i}(x) B_{i}(x)
$$

where

$$
\begin{equation*}
B_{i}(x)=\frac{p^{m}(x)}{p^{i}(x)} M_{i}(x) \equiv 1\left(\bmod p_{i}(x)\right) \tag{3}
\end{equation*}
$$

The polynomials $M_{i}(x)$ are selected in such a way as to satisfy the congruence in (3). In the case of only the transmission and storage of information, the positional form of the polynomial $F(x)$ according to the formula:

$$
F(x)=\sum_{i=1}^{S} \alpha_{i}(x) P_{i}(x)
$$

where

$$
\begin{equation*}
P_{i}(x)=\frac{p^{m}(x)}{p^{i}(x)} \tag{4}
\end{equation*}
$$

Each working base must have a degree not higher than the value of L (in our case, 128). The bases (1) are selected from among all irreducible polynomials of degree $m_{1} t o m_{S}$ with the condition that equation (4) holds:

$$
\begin{equation*}
k_{1} m_{1}+k_{2} m_{2}+\ldots+k_{S} m_{S}=L \tag{5}
\end{equation*}
$$

In the equation (5), $0<k_{i}<n_{i}, i=1, \ldots, S$ are unknown coefficients and the number of selected irreducible polynomials of degree $m_{i}$. One specific set of these coefficients is a solution of (5) and defines one system of working bases, $n_{i}$ is the number of all irreducible polynomials of degree $m_{i}, 1 \leq m_{i} \leq L, S=k_{1}+k_{2}+\ldots+k_{S}$ is the number of selected working

a)

b)

Figure 1: Qamal encryption algorithm scheme, b) Qamal NPNS encryption algorithm scheme
bases. Complete residue systems modulo polynomials of degree $m_{i}$ include all polynomials of degree at most $m_{i-1}$, for the notation of which $m_{i}$ bits are required.

Encryption. The used key sequence of $L$ bits long is also interpreted as the sequence of remainders $k_{1}(x), k_{2}(x), \ldots, k_{S}(x)$, but from dividing some other polynomial $\mathrm{K}(\mathrm{x})$ by the same working bases of the system:

$$
\begin{equation*}
K(x)=k_{1}(x), k_{2}(x), \ldots, k_{S}(x) . \tag{6}
\end{equation*}
$$

where $K(x) \equiv k(x)\left(\bmod p_{i}(x)\right), i=1, \ldots, S$. Then, as a cryptogram $\omega_{1}(x), \omega_{2}(x), \ldots, \omega_{S}(x)$, some encryption function $H(F(x), K(x))$ can be considered:

$$
\begin{equation*}
H(x)=\omega_{1}(x), \omega_{2}(x), \ldots, \omega_{S}(x) \tag{7}
\end{equation*}
$$

where $H(x) \equiv \omega_{i}(x)\left(\operatorname{modp}_{i}(x)\right), i=1, \ldots, S$.
In accordance with the operations in the NPN, the operations in the functions $\mathrm{F}(\mathrm{x}), \mathrm{K}(\mathrm{x})$, $\mathrm{H}(\mathrm{x})$ are performed in parallel modulo the polynomials (1) selected as the working bases of the NPN.

For encryption, elements of the residue sequence $\omega_{1}(x), \omega_{2}(x), \ldots, \omega_{S}(x)$ in the cryptogram are used, which are the least remainders on dividing the products $\alpha_{i}(x) k_{i}(x)$ by the corresponding bases $p_{i}(x)$, if the multiplication operation is used as the function $H(F(x), K(x))$ [22-25]:

$$
\begin{equation*}
\alpha_{i}(x) k_{i}(x) \equiv \omega_{i}(x)\left(\operatorname{modp} p_{i}(x)\right), i=1, \ldots S . \tag{8}
\end{equation*}
$$

Decryption. When decrypting the cryptogram $\mathrm{H}(\mathrm{x})$ using the known key $K(x)$, for each value $k_{i}(x)$, we calculate, as follows from (8), the reciprocal (inverse) polynomial $k_{i}^{(-1)}(x)$ from the following congruence:

$$
\begin{equation*}
k(x) k_{i}^{(-1)}(x)=1\left(\operatorname{modp}_{i}(x)\right), i=1, \ldots, S \tag{9}
\end{equation*}
$$

The result is the polynomial

$$
K^{(-1)}(x)=\left(k_{1}^{(-1)}(x), k_{2}^{(-1)}(x), \ldots, k_{S}^{(-1)}(x)\right)
$$

which is inverse to the polynomial $\mathrm{K}(\mathrm{x})$. Then the elements of the residue sequence (2) in accordance with (8) and (9) are restored by the congruence:

$$
\alpha_{i}(x)=k_{i}^{(-1)}(x) \omega_{i}(x)\left(\bmod _{i}(x)\right), i=1, \ldots, S
$$

Thus, in the considered model of the encryption algorithm for an electronic message of a given length L bits in the NPN, the complete key is the selected system of the polynomial working bases $p_{1}(x), p_{2}(x), \ldots, p_{S}(x)$ and the inverse key $K^{(-1)}(x)=$ $\left(k(x), k_{2}^{(-1)}(x), \ldots, k_{S}^{(-1)}(x)\right)$ to decrypt the message.

Round keys. The round-key generation algorithm remains the same as in the basic algorithm [19]. The round keys $K_{i}$ are generated from the cipher key $K_{0}$ using the key extension procedure. As a result, an array of round keys is formed, from which the required round key is then directly selected.

The complete key in the developed encryption algorithm modification is comprised of the chosen system of polynomial bases $p_{1}(x), p_{2}(x), \ldots, p_{S}(x)$, the key $K(x)=$ $\left(k_{1}(x), k_{2}(x), \ldots, k_{S}(x)\right)$ obtained while generating a pseudo-random sequence, and the inverse key $K^{(-1)}(x)=\left(k_{1}^{(-1)}(x), k_{2}^{(-1)}(x), \ldots, k_{S}^{(-1)}(x)\right)$ calculated according to expression (9).

## 4 Results and discussions

### 4.1 Encryption algorithm analysis

The main methods for analyzing the strength of such algorithms include brute force attacks, statistical and algebraic methods. Brute force attacks are to check all possible keys by using them to decrypt the ciphertext and then to verify whether the result obtained represents a plaintext.

Statistical methods for purposes of analysis use some statistical dependence of the algorithm, which is performed for the correct key with a greater frequency than for a false key.

The basis of algebraic methods is the building of a system of linear equations in which the elements of plaintext and key are selected as variables. When solving the system of equations using the linearization method, the possibility of finding key elements in parts is considered.

Keyspace calculation. In the algorithm, the key consists of two parts that are generated independently of each other. The length of each key is 128 bits. One part of the key is a pseudo-random sequence generated for the bitwise addition operation and for the nonpositional encryption system. In an NPN, the second part of the key is the selected set of polynomial bases $p_{1}(x), p_{2}(x), \ldots, p_{S}(x)$. It is known that the number of operations to
enumerate all candidate keys with a length of 128 bits is equal to $2^{128}$. The cryptographic strength of the encryption algorithm based on an NPN is determined by the number of all possible and different options for choosing complete keys. The cryptostrength of encryption of a message of a given length L is calculated by the formula [26]:

$$
\begin{equation*}
Q_{k}=2^{L} \cdot \Sigma_{k_{1}, k_{2}, \ldots, k_{S}}\left(k_{1}+\ldots+k_{S}\right)!C_{n_{1} \ldots}^{k_{1}} \ldots C_{n_{S}}^{k_{S}} \tag{10}
\end{equation*}
$$

To find the exact value of $Q_{k}$ for each L, it is necessary to calculate the number of irreducible polynomials of degrees up to L and the compositions of $L$.

The number of irreducible binary polynomials of degree $L$ is calculated by the following formula [26]:

$$
I_{L}=\frac{1}{L} \sum_{d \backslash L} \mu(d) 2^{L / d}=\frac{1}{n} \sum_{d \backslash L} \mu(L / d) 2^{d}
$$

where d are divisors of $L, \mu(x)$ is the Mobius function defined as follows:

$$
\begin{cases}0, & \text { if } x \text { has a squared prime factor } \\ (-1)^{k}, & \text { if } x \text { is the product of } k \text { different numbers } \\ +1, & \text { if } \mathrm{x}=1\end{cases}
$$

Table 1 shows the values of $I_{L}$ from 1 to 32. If $L=128$, then $I_{L} \approx 2^{122}$.

Table 1: Values of $I_{L}$ from 1 to 32

| $I$ | $I_{L}$ | $I$ | $I_{L}$ | $I$ | $I_{L}$ | $I$ | $I_{L}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 9 | 56 | 17 | 7,710 | 25 | $1,342,176$ |
| 2 | 1 | 10 | 99 | 18 | 14,532 | 26 | $2,580,795$ |
| 3 | 2 | 11 | 186 | 19 | 27,594 | 27 | $4,971,008$ |
| 4 | 3 | 12 | 335 | 20 | 52,377 | 28 | $5,586,395$ |
| 5 | 6 | 13 | 630 | 21 | 99,858 | 29 | $18,512,790$ |
| 6 | 9 | 14 | 1,161 | 22 | 190,557 | 30 | $35,790,267$ |
| 7 | 18 | 15 | 2,182 | 23 | 364,722 | 31 | $69,273,666$ |
| 8 | 30 | 16 | 4,080 | 24 | 698,870 | 32 | $134,215,680$ |

It is known from the theory of numbers that in the general case for the number $L$ there are $2^{L-1}$ compositions, of which exactly $C_{L-1}^{k-1}$ ones have the length of $k$.

Based on this fact, the total number of complete keys was calculated for different values of $L$. For $L$ equal to 16,32 , and 64 , the number of enumeration operations is $2^{34}, 2^{69}$, and $2^{138}$, respectively. Taking into account these calculations, it is suggested that when $L$ takes the value of 128 , the number of enumeration operations is close to $2^{276}$.

### 4.2 Algebraic analysis results

Algebraic methods are based on the algebraic properties of an information transformation algorithm. The strength of algorithms against statistical methods depends on the amount of
accumulated information about plaintexts and the corresponding converted texts. Algebraic methods usually do not require a lot of statistics when using the same key.

Algebraic cryptanalysis for multiplication in non-positional polynomial notations was considered separately. For multiplication in an NPN, a partial attack was used. Earlier studies had been conducted in this direction [27]. The system of equations binding the key, plaintext, and ciphertext in the encryption scheme based on an NPN for one irreducible polynomial is given below:

$$
\left\{\begin{array}{l}
c_{n-1} d_{n-1} \bigoplus k_{n} s_{n-2}=0 \\
c_{n-1} d_{n-2} \bigoplus c_{n-2} d_{n-1} \bigoplus k_{n} s_{n-3} \bigoplus k_{n-1} s_{n-2}=0 \\
\ldots \\
c_{n-1} d_{1} \bigoplus c_{n-2} d_{2} \bigoplus \ldots \bigoplus c_{1} d_{n-1} \bigoplus k_{n} s_{0} \bigoplus k_{n-1} s_{1} \bigoplus \ldots \bigoplus k_{2} s_{n-2}=0 \\
c_{n-1} d_{0} \bigoplus c_{n-2} d_{1} \bigoplus \ldots \bigoplus c_{0} d_{n-1} \bigoplus k_{n-1} s_{0} \bigoplus \ldots \bigoplus k_{1} s_{n-2}=a_{n-1} \\
c_{n-2} d_{0} \bigoplus c_{n-3} d_{1} \ldots \bigoplus c_{0} d_{n-2} \bigoplus k_{n-2} s_{0} \bigoplus k_{n-3} s_{1} \bigoplus \ldots \bigoplus k_{0} s_{n-2}=a_{n-2} \\
\ldots \\
c_{2} d_{0} \bigoplus c_{1} d_{1} \bigoplus c_{0} d_{2} \bigoplus k_{2} s_{0} \bigoplus k_{1} s_{1} \bigoplus k_{0} s_{2}=a_{2} \\
c_{1} d_{0} \bigoplus c_{0} d_{1} \bigoplus k_{1} s_{0} \bigoplus k_{0} s_{1}=a_{1} \\
c_{0} d_{0} \bigoplus k_{0} s_{0}=a_{0}
\end{array}\right.
$$

Here $c=\left(c_{n-1}, c_{n-2}, \ldots, c_{2}, c_{1}, c_{0}\right)$ is a numerical sequence of the given ciphertext, $a=\left(a_{n-1}, a_{n-2}, \ldots, a_{2}, a_{1}, a_{0}\right)$ is a sequence of characters of the unknown plaintext, $k=$ $\left(k_{n}, k_{n-1}, \ldots, k_{2}, k_{1}, k_{0}\right), d=\left(d_{n-1}, d_{n-2}, \ldots, d_{2}, d, d_{0}\right)$, and $s=\left(s_{n-2}, s_{n-3}, \ldots, s_{2}, s_{1}, s_{0}\right)$ are the sequences of unknown variables.

In this context, the input data are random sequences resulting from other transformations. It was shown in [25] that after one cycle, each bit of the intermediate result depends on each bit of the plaintext and on the key. Minimal changes in the plaintext or in the key lead to changes of about $50 \%$ of the bits (an avalanche effect). In view of the above, an attack in parts is impractical.

In the case of an algebraic attack, provided that the ciphertext and plaintext are known, the number of search operations for finding the key lies within the following interval [27]:

$$
\sum_{i=1}^{s} I\left(m_{i}\right) \leqslant J(m)<\prod_{i=1}^{s} I\left(m_{i}\right)
$$

where $I\left(m_{i}\right)$ is the number of irreducible polynomials of degree fewer than $m_{i}, J(m)$ is the number of search operations for complete keys of length $m$.

## 5 Conclusion

The study of the cryptostrength of the algorithm begins with the cryptanalysis of each transformation separately. Then, depending on the results obtained, an analysis of the entire algorithm, i.e. for the whole round transformation, is conducted.

The basis of algebraic methods is combining a set of equations describing the internal transformations in the cipher system, and solving the simultaneous equations. Typically,
these internal transformations include linear and non-linear parts. Developers of modern encryption systems often use S-boxes, which due to their non-linearity significantly increase the level of strength of such encryption systems against algebraic cryptographic attacks. In addition, in order to complicate the use of analytical approaches, iterative (round) schemes are widely used, when the transformation output is again fed to the input a certain number of times.

The study of the algorithm strength for separate procedures showed good results, which suggest the cryptographic strength of the developed algorithm and the possibility to study the algorithm comprehensively, i.e. considering all transformation procedures and rounds. Work in this direction is ongoing. The results will be presented in the following publications.

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