2-бөлім **Раздел 2** **Section 2**

Механика **Механика** **Mechanics**

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DIRECT KINEMATICS OF A 3-PRPS TYPE PARALLEL MANIPULATOR

Parallel manipulators with six degrees of freedom and three limbs have a large workspace and less complex singular configurations compared to the parallel manipulators with six degrees of freedom and six limbs. This paper is presented to solve the direct kinematics of a novel 3-PRPS type parallel manipulator with six-degrees-of-freedom, where P, R, and S are prismatic, revolute and spherical kinematic pairs respectively. The considered parallel manipulator is formed by connecting a moving platform with a fixed platform (base) through three closing kinematic chains of a PRPS type in which the prismatic kinematic pairs are active and they are located on a fixed platform and legs. The constant and variable parameters of the considered parallel manipulator characterizing its geometry and kinematics respectively are determined. In the direct kinematics, the positions of the moving platform are determined by the known constant parameters of the links and the given variable parameters of the active kinematic pairs. An analysis of the obtained equations of the direct kinematics showed that the variable parameters of the active prismatic kinematic pairs are set free, and these equations are reduced to a 16^{th} –order polynomial equation with passive kinematic pairs variables. Numerical examples of the considered parallel manipulator's direct kinematics are presented, and the results showed that the direct kinematics equations have four solutions corresponding to the four assemblies of the parallel manipulator.

Key words: parallel manipulator, moving and fixed platforms, direct kinematics.

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3-PRPS түрдегі параллел манипулятордың тура кинематикалық есебі

Алты еркіндік дәрежесі және үш аяғы бар параллель манипуляторлар алты еркіндік дәрежесі және алты аяғы бар пареллель манипуляторларға қарағанда үлкен жұмыс алаңы және қарапайым сингулярлық конфигурациясына ие. Бұл жұмыста P, R және S-ілгерілемелі, айналмалы және сфералық кинематикалық жұптар. Қарастырылған параллелді манипулятор ілгерілемелі кинематикалық белсенді болатын және олар қозғалмайтын платформа мен аяқтарда орналасқан PRPS типті үш тұйықтаушы кинематикалық тізбектер арқылы жылжымалы платформаны жылжымайтын платформамен (негізбен) біріктіру жолымен құрылған. Параллелді манипулятордың оның геометриясы мен кинематикасын сипаттайтын тұрақты және айнымалы параметрлері анықталды. Тура кинематикада қозғалатын платформаның орналасуы буындардың белгілі тұрақты параметрлерімен және белсенді кинематикалық жұптардың берілген айнымалы параметрлерімен анықталады. Тура кинематиканың алынған теңдеулерін талдау белсенді ілгерілемелі кинематикалық жұптардың айнымалы параметрлері босатылғанын көрсетті және бұл теңдеулер 16–ретті полиномиалдық теңдеуден пассивті кинематикалық жұптардың айнымалылар теңдеуінен құралады. Қарастырылған параллель манипулятордың тікелей кинематикасының сандық мысалдары келтірілген және нәтижелер тікелей кинематиканың теңдеулерінде параллель манипулятордың төрт түйініне сәйкес төрт шешім болатындығын көрсетеді.

Түйін сөздер: параллелді манипулятор, жылжымалы және қозғалмайтын платформалар, тура кинематика.

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Прямая задача кинематики параллельного манипулятора вида 3-PRPS

Параллельные манипуляторы с шестью степенями свободы и тремя ногами имеют большую рабочую зону и менее сложные сингулярные конфигурации по сравнению с параллельными манипуляторами с шестью степенями свободы и шестью ногами. Данная работа посвящена решению прямой кинематики нового параллельного манипулятора типа 3-PRPS с шестью степенями свободы, где P, R и S-призматические, вращательные и сферические кинематические пары соответственно. Рассматриваемый параллельный манипулятор образован путем соединения подвижной платформы с неподвижной платформой (основанием) через три замыкающие кинематические цепи типа PRPS, в которых активны призматические кинематические пары и они расположены на неподвижной платформе и ножках. Определены постоянные и переменные параметры рассматриваемого параллельного манипулятора, характеризующие соответственно его геометрию и кинематику. В прямой кинематике положения движущейся платформы определяются известными постоянными параметрами звеньев и заданными переменными параметрами активных кинематических пар. Анализ полученных уравнений прямой кинематики показал, что переменные параметры активных призматических кинематических пар освобождаются, и эти уравнения сводятся к полиномиальному уравнению 16 –го порядка с переменными пассивных кинематических пар. Представлены численные примеры прямой кинематики рассматриваемого параллельного манипулятора, и результаты показали, что уравнения прямой кинематики имеют четыре решения, соответствующие четырем узлам параллельного манипулятора.

Ключевые слова: параллельный манипулятор, подвижные и неподвижные платформы, прямая кинематика.

1 Introduction

Parallel manipulators with closed-loop kinematic chains, in comparison with serial manipulators with open-loop kinematic chains, have high stiffness, carrying capacity, positioning accuracy and dynamic characteristics [1–7]. Most parallel manipulators with six-degrees-of-freedom have six legs, i.e. six kinematic chains connecting a moving platform with a fixed platform (base). One of the problems of parallel mechanisms is a reduction in the number of their legs. With a decrease in the number of legs, the workspace of parallel manipulators increases and singular configurations are reduced.

Three-legged parallel manipulators (tripods) are currently being developed. The following types of tripods are known: 3-URS [8], 3-ESR [9], 3-RRPS [10], 3-RES [11–13], 3-PPSR [14], 3-CRS [16], 3-CCC [17, 18]. We have developed a modern 3-PRPS type parallel mechanism with six-degrees-of-freedom which in comparison with existing tripods, has a large load capacity. Figure 1 shows the 3D model of the developed parallel mechanism, where the moving platform 2 is connected to the fixed base 1 through three identical legs 4-5, 7-8, 10-11 of a PRPS type.

One of the legs 4-5 of this parallel manipulator (Fig. 1), where the hydraulic cylinder 3 moves the leg 3-4 on the fixed base 1, forms the prismatic kinematic pair A. Link 4 is connected by forming the revolute kinematic pair B. Link 4 is connected with link 3 by hydraulic cylinder and the moving platform 2 through the hydraulic cylinder C and spherical D kinematic pairs respectively.

In works [15] and [19], the geometry of this parallel mechanism was studied and the inverse kinematics was solved. According to the developed principle of forming of parallel manipulators, the considered parallel mechanism of a 3-PRPS type is formed by connecting the moving platform 2 with the fixed base 1 through three passive closing kinematic chains 4-5, 7-8, 10-11 of a PRPS type with zero degrees-of-freedom. To describe the geometry and study the kinematics of this

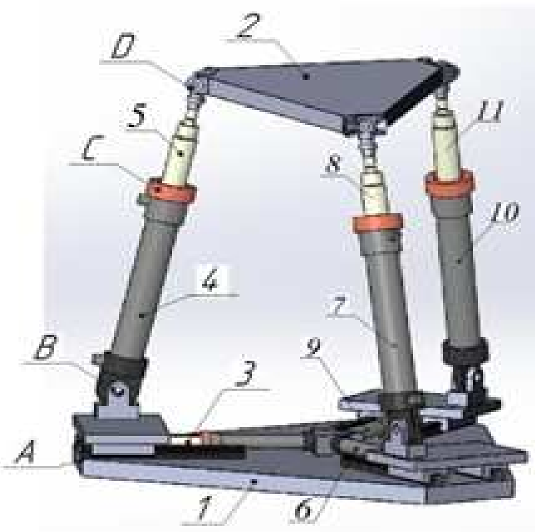

Figure 1 – 3D model of a 3-PRPS type parallel manipulator

parallel manipulator, two Cartesian coordinate systems UVW and XYZ are fixed at two elements of each kinematic pairs. The W and Z axes of these coordinate systems are directed along the axis of rotational and translational motions of the kinematic pairs elements, and the U and X axes are directed along the direction of the perpendicular t, drawn from the W axis to the Z-axis. The transformation matrix $\mathbf{T}_{jk} = \mathbf{T}_{jk}(a_{jk}, \alpha_{jk}, b_{jk}, \beta_{jk}, c_{jk}, \gamma_{jk})$ between the coordinate systems $U_jV_jW_j$ and $X_kY_kZ_k$, having six parameters has been formed in [19]. The transformation matrix $\mathbf{G}_{jk}$ of the coordinate systems fixed at the ends of binary links is called the matrix of binary link, and it has constant parameters characterizing the geometry of the links. The transformation matrix $\mathbf{P}_j$ of the coordinate systems fixed at the two elements of each kinematic pair is called the matrix of kinematic pair, and it has variable parameters characterizing the relative motions of the kinematic pair's elements.

2 Direct kinematics of a 3-PRPS type parallel manipulators

In Figure 2 the leg 3-4 with the chosen coordinate systems, where $U_1V_1W_1$ is an absolute coordinate system fixed at the base 1, and $PX_pY_pZ_p$ is a coordinate system fixed at the moving platform 2.

This leg has the following constant parameters $\alpha_{12} = 90^0, c_{13} = OO_3, a_{34} = O_3'O_4$, $\alpha_{12} = -90^0, \gamma_{13}$ and the following variable parameters $c_{33} = O_3O_3' = s_3$, $c_{42} = O_4O_2 = s_5, \gamma_{44} = \theta_4$ Other two legs have similar parameters. Let consider the direct kinematics of a 3-PRPS type parallel mechanism. Since all three legs 3-4, 7-8, 10-11 have similar constant and variable parameters, for the convenience of calculating, the parameters $a_{jk}, \alpha_{jk}, b_{jk}, \beta_{jk}, c_{jk}, \gamma_{jk}$ are denoted by $a_i, \alpha_i, b_i, \beta_i, c_i, \gamma_i$, and the lengths of the links 3,4,7,8,10,11 and the angles determining their positions, are denoted by s_{3i}, s_{5i} and θ_{4i} where i is the serial numbers of the legs, i.e. $i = 1, 2, 3$.

We, then, determine the coordinates of the centers of the spherical joints O_{2i} of the moving platform 2 in the absolute coordinate system $U_1V_1W_1$

$$[1, U_{O_{2i}}, V_{O_{2i}}, W_{O_{2i}}]^T = \mathbf{G}_{13i} \cdot \mathbf{P}_{3i}^P \cdot \mathbf{G}_{34i} \cdot \mathbf{P}_{5i}^R \cdot \mathbf{P}_{5i}^P [1, 0, 0, 0]^T, \quad (i = 1, 2, 3) \tag{1}$$

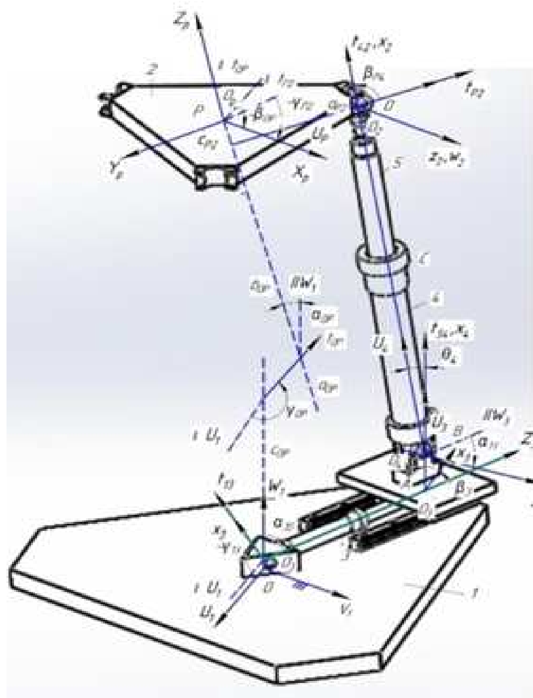

Figure 2 – Leg 3-4 with the coordinate systems

where $\mathbf{G}_{13i}, \mathbf{G}_{34i}$ are the matrices of links, and $\mathbf{P}^P_{3i}, \mathbf{P}^R_{5i}, \mathbf{P}^P_{5i}$ are the matrices of kinematic pairs.

From the systems of matrix equations (1) we obtain

$$\left.\begin{aligned} s_{3i} \cdot s\gamma_{13i} - s_{5i} \cdot s\gamma_{13i} \cdot s\theta_{4i} = U_{O_{2i}} \\ -s_{3i} \cdot c\gamma_{13i} - s_{5i} \cdot c\gamma_{13i} \cdot s\theta_{4i} = V_{O_{2i}} \\ c_{3i} + a_{34i} - s_{5i} \cdot s\theta_{4i} = W_{O_{2i}} \end{aligned}\right\} \tag{2}$$

where, s and c denote sin and cos.

Multiplying the first and the second equations of the system (2) on $c\gamma_{13i}$ and $s\gamma_{13i}$ respectively, and add them, we obtain, that all three variable parameters have disappeared. Therefore, these two equations are interdependent and the variable parameters s_{3i} should be setting.

Then, we set the parameters s_{5i} and for determine the angles θ_{4i} derive the following system of equations

$$\left.\begin{aligned} (U_{O_{21}} - U_{O_{22}})^2 + (V_{O_{21}} - V_{O_{22}})^2 + (W_{O_{21}} - W_{O_{22}})^2 = a^2 \\ (U_{O_{21}} - U_{O_{23}})^2 + (V_{O_{21}} - V_{O_{23}})^2 + (W_{O_{21}} - W_{O_{23}})^2 = a^2 \\ (U_{O_{22}} - U_{O_{23}})^2 + (V_{O_{22}} - V_{O_{23}})^2 + (W_{O_{22}} - W_{O_{23}})^2 = a^2 \end{aligned}\right\} \tag{3}$$

where a are the distances between the spherical joints O_{2i}. After some transformations, from the systems of equations (3) we obtain

$$\left.\begin{aligned} &-s_{51} \cdot s_{52} \cdot c(\gamma_{131} - \gamma_{132}) \cdot s\theta_{41} \cdot s\theta_{42} - s_{31} \cdot s_{51} \cdot s\theta_{41} + s_{31} \cdot s_{52} \cdot c(\gamma_{131} - \gamma_{132}) \cdot s\theta_{42} + \\ &\quad + s_{32} \cdot s_{51} \cdot c(\gamma_{131} - \gamma_{132}) \cdot s\theta_{41} - s_{32} \cdot s_{52} \cdot s\theta_{42} - s_{51} \cdot s_{52} \cdot c\theta_{42} \cdot c\theta_{41} = N_1, \\ &-s_{51} \cdot s_{53} \cdot c(\gamma_{131} - \gamma_{133}) \cdot s\theta_{41} \cdot s\theta_{43} - s_{31} \cdot s_{51} \cdot s\theta_{41} + s_{31} \cdot s_{53} \cdot c(\gamma_{131} - \gamma_{133}) \cdot s\theta_{43} + \\ &\quad + s_{31} \cdot s_{53} \cdot c(\gamma_{131} - \gamma_{133}) \cdot s\theta_{41} - s_{33} \cdot s_{53} \cdot s\theta_{43} - s_{51} \cdot s_{53} \cdot c\theta_{43} \cdot c\theta_{41} = N_2, \\ &-s_{52} \cdot s_{53} \cdot c(\gamma_{133} - \gamma_{132}) \cdot s\theta_{43} \cdot s\theta_{42} - s_{33} \cdot s_{53} \cdot s\theta_{43} + s_{52} \cdot s_{33} \cdot c(\gamma_{133} - \gamma_{132}) \cdot s\theta_{42} + \\ &\quad + s_{32} \cdot s_{53} \cdot c(\gamma_{133} - \gamma_{132}) \cdot s\theta_{43} - s_{32} \cdot s_{52} \cdot s\theta_{42} - s_{52} \cdot s_{53} \cdot c\theta_{42} \cdot c\theta_{43} = N_3, \end{aligned}\right\} \tag{4}$$

where

$$N_1 = \frac{a^2 - s_{31}^2 - s_{32}^2 - s_{51}^2 - s_{52}^2 + 2 \cdot s_{31} \cdot s_{32} \cdot c(\gamma_{131} - \gamma_{132})}{2},$$
$$N_2 = \frac{a^2 - s_{31}^2 - s_{51}^2 - s_{33}^2 - s_{53}^2 + 2 \cdot s_{31} \cdot s_{33} \cdot c(\gamma_{131} - \gamma_{133})}{2},$$
$$N_3 = \frac{a^2 - s_{32}^2 - s_{52}^2 - s_{33}^2 - s_{53}^2 + 2 \cdot s_{32} \cdot s_{33} \cdot c(\gamma_{133} - \gamma_{132})}{2}.$$

We use the following expressions $t_{4i} = tan(\theta_{4i}/2)$, $s\theta_{4i} = 2 \cdot t_{4i}/1 + t_{4i}^2$, $c\theta_{4i} = (1 - t_{4i}^2)/(1 + t_{4i}^2)$, $i = 1, 2, 3$ to the system of equations (4)

$$\left.\begin{array}{l} A \cdot t_{341}^2 + B \cdot t_{41} + C = 0, \\ D \cdot t_{41}^2 + E \cdot t_{41} + F = 0, \\ Q \cdot t_{41}^2 + R \cdot t_{41} + T = 0. \end{array}\right\} \tag{5}$$

where $A = A_{11} \cdot t_{41}^2 + A_{12} \cdot t_{41} + A_{13}$, $B = B_{11} \cdot t_{42}^2 + B_{12} \cdot t_{42} + B_{13}$, $C = C_{11} \cdot t_{42}^2 + C_{12} \cdot t_{41} + C_{13}$, $D = D_{11} \cdot t_{43}^2 + D_{12} \cdot t_{43} + D_{13}$, $E = E_{11} \cdot t_{43}^2 + E_{12} \cdot t_{43} + E_{13}$, $F = F_{11} \cdot t_{43}^2 + F_{12} \cdot t_{43} + F_{13}$, $Q = Q_{11} \cdot t_{42}^2 + Q_{12} \cdot t_{42} + Q_{13}$, $R = R_{11} \cdot t_{42}^2 + R_{12} \cdot t_{42} + R_{13}$, $T = T_{11} \cdot t_{42}^2 + T_{12} \cdot t_{42} + T_{13}$, $A_{11} = N_1 + s_{51} \cdot s_{52}$, $A_{12} = 2 \cdot s_{32} \cdot s_{52} - 2 \cdot s_{31} \cdot s_{52} \cdot c(\gamma_{131} - \gamma_{132})$, $A_{13} = N_1 - s_{51} \cdot s_{52}$, $B_{11} = 2 \cdot s_{31} \cdot s_{51} - 2 \cdot s_{32} \cdot s_{51} \cdot c(\gamma_{131} - \gamma_{132})$, $B_{12} = 4 \cdot s_{51} \cdot s_{52} \cdot c(\gamma_{131} - \gamma_{132})$, $B_{13} = 2 \cdot s_{31} \cdot s_{51} - s_{32} \cdot s_{51} \cdot c(\gamma_{131} - \gamma_{132})$, $C_{11} = N_1 - s_{51} \cdot s_{52}$, $C_{12} = 2 \cdot s_{32} \cdot s_{52} - 2 \cdot s_{31} \cdot s_{52} \cdot c(\gamma_{131} - \gamma_{132})$, $C_{13} = N_1 + s_{51} \cdot s_{52}$, $D_{11} = N_2 + s_{51} \cdot s_{53}$, $D_{12} = 2 \cdot s_{33} \cdot s_{53} - 2 \cdot s_{31} \cdot s_{53} \cdot c(\gamma_{131} - \gamma_{133})$, $D_{13} = N_2 - s_{51} \cdot s_{53}$, $E_{11} = 2 \cdot s_{31} \cdot s_{51} - 2 \cdot s_{51} \cdot s_{33} \cdot c(\gamma_{131} - \gamma_{133})$, $E_{12} = 4 \cdot s_{51} \cdot s_{53} \cdot c(\gamma_{131} - \gamma_{133})$, $E_{13} = 2 \cdot s_{31} \cdot s_{51} - s_{51} \cdot s_{33} \cdot c(\gamma_{131} - \gamma_{133})$, $F_{11} = N_2 - s_{31} \cdot s_{51}$, $F_{12} = 2 \cdot s_{33} \cdot s_{53} - 2 \cdot s_{31} \cdot s_{53} \cdot c(\gamma_{131} - \gamma_{133})$, $F_{13} = N_2 + s_{51} \cdot s_{53}$, $Q_{11} = N_3 + s_{52} \cdot s_{53}$, $Q_{12} = 2 \cdot s_{32} \cdot s_{52} - 2 \cdot s_{52} \cdot s_{33} \cdot c(\gamma_{133} - \gamma_{132})$, $Q_{13} = N_3 - s_{52} \cdot s_{53}$, $R_{11} = 2 \cdot s_{33} \cdot s_{53} - 2 \cdot s_{32} \cdot s_{53} \cdot c(\gamma_{133} - \gamma_{132})$, $R_{12} = 4 \cdot s_{52} \cdot s_{53} \cdot c(\gamma_{133} - \gamma_{132})$, $R_{13} = 2 \cdot s_{33} \cdot s_{53} - 2 \cdot s_{32} \cdot s_{53} \cdot c(\gamma_{133} - \gamma_{132})$, $T_{11} = N_3 - s_{52} \cdot s_{53}$, $T_{12} = 2 \cdot s_{32} \cdot s_{52} - 2 \cdot s_{52} \cdot s_{53} \cdot c(\gamma_{133} - \gamma_{132})$, $T_{13} = N_3 + s_{52} \cdot s_{53}$.

On the base of the Sylvester method [20] from the first two equations of the system (5) we derive the following matrix

$$\begin{vmatrix} A & B & C & 0 \\ 0 & A & B & C \\ D & E & F & 0 \\ 0 & D & E & F \end{vmatrix} = 0 \tag{6}$$

and get out of the variable t_{41}. From the determinant of the matrix (6), we obtain

$$\sum_{i=0}^{4} G_i \cdot t_{43}^i \tag{7}$$

where the coefficients G_i are the 4^{th}–order polynomial equation with the variable t_{42}. Continuing to use the Sylvester method from the equation (7) and the third equation of the system (4), we get out of the variable

$$\begin{vmatrix} Q & R & T & 0 & 0 & 0 \\ 0 & Q & R & T & 0 & 0 \\ 0 & 0 & Q & R & T & 0 \\ 0 & 0 & 0 & Q & R & T \\ G_{11} & G_{12} & G_{13} & G_{14} & G_{15} & 0 \\ 0 & G_{11} & G_{12} & G_{13} & G_{14} & G_{15} \end{vmatrix} = 0. \tag{8}$$

From the matrix (8) we obtain the 16$^{\text{th}}$–order polynomial equation with the variable t_{41}

$$\sum_{k=0}^{16} P_k \cdot t_{41}^k = 0 \tag{9}$$

The variable t_{41} is determined from the equation (9), then we can determine the variables t_{42} and t_{43}. The formation of coefficients G_i and P_k of the equations (7) and (9) and their calculation are based on Matlab. Of all the solutions of the equation (9), we choose those solutions for which the distances between the spherical joints are equal a.

3 Numerical examples

The following constant parameters $a_i = 15$, $c_i = 7$, $\gamma_1 = 90°$, $\gamma_2 = 210°$, $\gamma_3 = -30°$, $a = 74,48$, and for each case we set parameters of the legs 4-5, 7-8, 10-11.

Using the obtained values of the angles θ_{41}, θ_{42}, θ_{43}, the coordinates of the spherical joints centers of the moving platform 2 in the absolute coordinate system $OU_0V_0W_0$ are determined (Table - 1, 2, 3, 4).

Table - 1 – $s_{3i} = 60,\ (i = 1, 2, 3),\ s_{51} = 93,7618,\ s_{52} = 91,9882,\ s_{53} = 98,2112$

i	θ_{41}	θ_{42}	θ_{43}	U_{O4i}	V_{O4i}	W_{O4i}
1				37.004	-0.000	118.047
2	0.235	1.255	0.125	16.681	-28.892	52.479
3				-24.280	-42.053	113.274
1				-33.993	0.000	52.317
2	1.259	0.231	0.129	-18.737	32.454	117.593
3				-24.078	-41.704	113.222
1				42.970	-0.000	119.282
2	0.173	0.173	0.190	-21.514	37.263	118.733
3				-21.327	-36.940	112.338
1				42.563	-0.000	119.210
2	0.177	0.169	1.208	-21.717	37.615	118.804
3				12.996	22.510	54.665
1				42.563	-0.000	-75.210
2	2.964	2.972	1.934	-21.717	37.615	-74.804
3				12.996	22.511	-10.665
1				42.970	-0.000	-75.282
2	2.968	2.968	2.952	-21.513	37.262	-74.733
3				-21.327	-36.940	-68.338
1				-33.993	0.000	-8.317
2	1.883	2.910	3.012	-18.737	32.454	-73.593
3				-24.077	-41.703	-69.222

Table - 2 $-\ s_{3i} = 60,\ (i = 1, 2, 3),\ s_{51} = 95,\ s_{52} = 93,\ s_{53} = 96$

i	θ_{41}	θ_{42}	θ_{43}	U_{O4i}	V_{O4i}	W_{O4i}
1				-30.438	0.000	51.085
2	1.260	0.205	0.157	-20.245	35.065	115.997
3				-22.714	-39.341	113.851
1				41.317	-0.000	115.145
2	0.198	1.267	0.166	15.797	-27.362	50.747
3				-22.323	-38.665	113.724
1				43.016	-0.000	115.469
2	0.180	0.178	0.184	-21.488	37.218	116.478
3				-21.476	-37.197	113.424
1				43.799	-0.000	115.608
2	0.171	0.187	1.244	-21.096	36.539	116.334
3				14.046	24.328	51.813
1				43.799	-0.000	-71.608
2	2.970	2.955	1.897	-21.096	36.539	-72.334
3				14.046	24.328	-7.813
1				43.016	-0.000	-71.469
2	2.962	2.963	2.957	-21.096	37.218	-72.478
3				-21.476	-37.197	-69.424
1				41.316	-0.000	-71.145
2	2.943	1.874	2.976	15.797	-27.362	-6.747
3				-22.323	-38.665	-69.724

Table - 3 $-\ s_{3i} = 60,\ (i = 1, 2, 3),\ s_{51} = 95,\ s_{52} = 90,\ s_{53} = 75$

i	θ_{41}	θ_{42}	θ_{43}	U_{O4i}	V_{O4i}	W_{O4i}
1				39.671	-0.000	114.799
2	0.216	1.196	0.155	4.890	-8.470	49.488
3				-23.066	-39.950	110.925
1				42.199	-0.000	115.317
2	0.188	0.265	0.183	-20.161	34.920	94.373
3				-21.808	-37.773	110.496
1				-33.398	-0.000	39.372
2	1.387	0.076	0.343	-27.153	47.031	96.784
3				-14.864	-25.746	106.756
1				25.663	-0.000	110.578
2	0.370	0.040	1.357	-28.466	49.305	96.937
3				13.973	24.202	41.118
1				25.663	-0.000	-66.578
2	2.772	3.101	1.785	-28.466	49.305	-52.937
3				13.973	24.202	2.882
1				-33.3984	0.000	4.628
2	1.755	3.066	2.799	-27.153	47.031	-52.784
3				-14.864	-25.746	-62.756
1				42.199	-0.000	-71.317
2	2.953	2.876	2.959	-20.161	34.920	-50.372
3				-21.808	-37.773	-66.496

Table - 4 $-s_{31} = 60,\ s_{32} = 70,\ s_{33} = 50,\ s_{51} = 105,\ s_{52} = 90,\ s_{53} = 75$

i	θ_{41}	θ_{42}	θ_{43}	U_{O4i}	V_{O4i}	W_{O4i}
1	0.289	1.145	0.182	30.071	-0.000	122.644
2				9.159	-15.864	52.945
3				-26.839	-46.486	110.507
1	0.200	0.130	0.286	39.184	-0.000	124.916
2				-20.121	34.851	96.363
3				-22.300	-38.625	108.342
1	1.394	-0.031	0.430	-43.370	0.000	40.429
2				-26.174	45.334	96.963
3				-16.259	-28.162	103.824
1	0.418	-0.147	1.307	17.332	-0.000	117.940
2				-30.484	52.799	96.194
3				8.445	14.627	45.455
1	2.723	-2.995	1.834	17.3319	-0.000	-73.940
2				-30.484	52.799	-52.194
3				8.445	14.627	-1.455
1	1.747	-3.110	2.712	-43.370	0.000	3.571
2				-26.174	45.334	-52.963
3				-16.259	-28.162	-59.824
1	2.942	3.011	2.855	39.184	-0.000	-80.916
2				-20.121	34.851	-52.363
3				-22.300	-38.625	-64.342

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