2-бөлім **Раздел 2** **Section 2**

Механика **Механика** **Mechanics**

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RESEARCH OF THE STRESS STATE OF A PIPELINE ELEMENT WITH OVALIZATION UNDER CORROSION-POWER EFFECT

The stress state of an element of a thick-walled pipeline when ovalizing a cross section is studied under conditions of power and corrosion effect in the statement of plane deformation. The material of the element under the influence of external loads goes into an elastic-plastic state. The corrosive effect of a pumped medium leads to softening of the material in the plastic zone. This softening of the material is taken into account by a special inhomogeneity function in the Tresca-Saint-Venant plasticity condition. The elastic-plastic problem for an thick-walled elliptical element under uniform external and internal pressure is considered in non-axisymmetric setting. The problem is solved by the method of sharing static and physical equations for the considered elastoplastic material and the perturbation method in the theory of an elastoplastic body. An assessment of the strength and bearing capacity of a thick-walled pipeline element with ovalization under presence and absence of corrosion damage is given.
Key words: thick-walled pipeline element with ovalization, elastoplastic state, corrosion damage to the material, plastic inhomogeneity, softening function.

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Коррозиялық-күштік әсер ету жағдайында қалың қабырғалы құбыр элементінің көлденең қимасының сопақталу кезіндегі кернеулі күйін зерттеу

Жазық деформация қойылымында коррозиялық-күштік әсер ету жағдайында қалың қабырғалы құбыр элементінің оның көлденең қимасының сопақталу кезіндегі кернеулі күйі зерттелді. Сыртқы қысымның әсерінен элемент материалы серпімді пластикалық күйге өтеді. Айдалатын ортаның коррозиялық әсері иілгіш аймақта материалдың жұмсаруына әкеледі. Материалдың жұмсаруы Треск-Сен-Венанның иілгіштік шартында біртексіздіктің арнайы функциясымен ескеріледі. Асимметриялық есеп қойылымында бірқалыпты сыртқы және ішкі қысымның әсерінен қалың қабырғалы эллиптикалық элемент үшін серпімді пластикалық есеп қарастырылды. Есеп қарастырылып отырған серпімді иілгіш материал үшін статикалық және физикалық теңдеулерді бірлесіп пайдалану әдісімен және серпімді иілгіш дене теориясында ұйытқулар әдісімен шешілді. Коррозиялық-күштік әсер ету жағдайында қалың қабырғалы құбыр элементінің көлденең қимасының сопақталуы кезіндегі беріктігі мен көтергіштік қабілеттілігіне баға берілді.
Түйін сөздер: қалың қабырғалы эллиптикалық элемент, серпімді иілгіш күй, материалдың коррозиялық зақымдануы, беріктендіру функциясы. анықтамасы.

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Исследование напряженного состояния элемента трубопровода с овализацией при коррозионно-силовом воздействии

Исследовано НС элемента толстостенного трубопровода при овализации его поперечного сечения в условиях коррозионно-силового воздействия в постановке плоской деформации. Материал элемента под действием внешнего давления переходит в упругопластическое состояние. Коррозионное воздействие перекачиваемой среды приводит к разупрочнению материала в пластической зоне. Это разупрочнение материала учитывается специальной функцией неоднородности в условии пластичности Треска-Сен-Венана. Рассмотрена упругопластическая задача для толстостенного эллиптического элемента под действием равномерного наружного и внутреннего давления в неосесимметричной постановке. Задача решена методом совместного использования статических и физических уравнений для рассматриваемого упругопластического материала элемента и методом возмущений в теории упругопластического тела. Дана оценка прочности и несущей способности толстостенного элемента трубопровода с овализацией при коррозионно-силовом воздействии.

Ключевые слова: толстостенный элемент трубопровода с овализацией, упругопластическое состояние, коррозионные повреждения материала, функция разупрочнения.

1 Introduction

We studied the stress state, strength and carrying capacity of an element of a thick-walled pipeline with the circular cross-section under the action of uniform and non-uniform external pressure along the contour [1] with accounting of softening of material in the plastic zone due to corrosion effect of a pumping medium.

During operation of the elements of pipeline structures are subjected to various exorbitant loads, leading to ovalization of pipes. Ovalization of pipes significantly affects their strength and bearing capacity [2,3]. Moreover, during operation, corrosive wear of the inner surface of a thick-walled element occurs in aggressive working environments, which further reduces its bearing capacity.

In this regard, this work is devoted to the study of the stress state, strength and bearing capacity of an elastoplastic element of a thick-walled pipeline with ovalization under conditions of corrosion-force action, leading to weakening of the material in the plastic zone.

2 State of the problem

Ovalization is a factor that is taken into account in regulatory documents at the stage of delivery and installation of pipes, during design and construction [2,3], however there are no standards for the limiting value of ovality of pipelines in operation, despite the large amount of diagnostic data on their ovalization [4]. Meanwhile, the coincidence of the zone of increased stresses caused by ovalization of pipes with places of rupture and corrosion damage indicates that ovalization should be taken into account and be able to evaluate from the point of view of the operability of the pipeline. The ovalization of the cross section of an infinitely long elastic pipe in pure bending was studied by Brazier [5]. Elastoplastic ovalization of the profile of an initially straight cylindrical pipe bent by external moments along the mandrel is considered in [6]. Since in [6] only a part of the pipe is subject to bending, the study of the deformation of the pipe section is carried out in the framework of the plane stress state. In [7], the reliability and residual life of gas pipeline sections with defects such as ovalization and wall thinning due to corrosion and erosion processes are considered on the basis of a probabilistic approach. The strength characteristics of steel pipelines with geometric defects of the "dint" type were studied in [8]. In this case, a dint leads to a stress concentration in the defect zone under the action of internal pressure and can cause the appearance of other surface defects, including corrosion.

Ovality of the section is a geometric defect in the section of the pipe resulting from the transformation of the initial annular section of the pipe into an elliptical. Ovalization of the section is considered by us as a result of significant external transverse (radial) loads on the pipeline. This allows us to research the elastoplastic stress state, strength and bearing capacity of a pipeline element under conditions of corrosion-force action in the formulation of plane deformation. At the same time, the decrease in the strength properties of the pipe material during loading due to the accumulation of damage and defects can be taken into account by introducing a special softening function (radial inhomogeneity of strength characteristics) in the known criteria of material plasticity for axisymmetric and plane problems [9-11]. In [12], modified plasticity criteria were used that can take into account the accumulation of material damage under difficult boundary conditions, when the plastic inhomogeneity changes in accordance with the change in the elastoplastic boundary. In this work, we use just such plasticity criteria.

3 Solution of the problem

The element of a thick-walled underground pipeline with ovalization is in plane de-formation [14]. The equations of the cross-section of the oval pipeline element in the polar coordinate system r, θ are written for the inner contour in the form $a_0 + f_1(r, \theta)$, and for the outer contour in the form $1 + f_2(r, \theta)$. Here, $f_1(r, \theta)$, $f_2(r, \theta)$ are some functions of coordinates, $a_0 < 1$.

The pipeline material is taken to be ideally elastoplastic, obeying the Prandtl loading diagram [15].

Equilibrium equations in general form are written as $\sigma_{ij,j} = 0$.

In the considered formulation, the equilibrium equations of the pipeline in the polar coordinate system r, θ take the form :

$$\frac{\partial \sigma_r}{\partial r} + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta} + \frac{\sigma_r - \sigma_\theta}{r} = 0, \qquad \frac{\partial \tau_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_\theta}{\partial \theta} + 2\frac{\tau_{r\theta}}{r} = 0 \tag{1}$$

Here $\sigma_r, \sigma_\theta, \tau_{r\theta}$ are the components of the stress tensor.

In the elastic region, Hooke's law is valid for a homogeneous, isotropic linear elastic material:

$$\varepsilon_{ij} = \frac{1}{E}\left((1+\mu)\sigma_{ij} - \mu\delta_{ij}\sigma_{kk}\right), \tag{2}$$

where σ_{ij} and ε_{ij} are the components of the stress and strain tensors, E is the modulus of elasticity, μ is the Poisson's ratio, and δ_{ij} is the Kronecker symbol.

The stress function in the elastic region $\Phi(r, \theta)$ must satisfy the biharmonic equation (here ∇^2 is the Laplace operator)

$$\nabla^2\nabla^2\Phi = 0. \tag{3}$$

The solution of equation (3) at $r, m\theta$ can be presented in the general form

$$\Phi_m = (C_1 r^m + C_2 r^{-m} + C_3 r^{m+2} + C_4 \varphi_m(r)) \cos m\theta, \ \ m = 0, 1, 2, \ldots, \tag{4}$$

where $\varphi_m(r) = r^m \ln r$ at $m = 0, 1$; $\varphi_m(r) = r^{-m+2}$ at $m \geq 2$. The constants $C_1 - C_4$ in (4) are found in the course of the solution from the boundary conditions.

The plasticity condition is generally written as follows:

$$f_* \left(\sigma_{ij}, \sigma_{s*} \left(x_i, \chi_j\right)\right) = 0. \tag{5}$$

In condition (5), σ_{ij} are the components of the stress tensor, $\sigma_{s*}(x_i, \chi_j)$ are the strength characteristics of the material in the plastic zone, which are continuous and differentiable functions of the coordinates x_i and loading parameters χ_j [12].

As a condition for the transition of a material into a plastic state, we take the Tresca-Saint-Venant condition, which is widely used in calculations of plastically deformable metal structures and constructions:

$$(\sigma_\theta - \sigma_r)^2 + 4\tau_{r\theta}^2 = 4K_*^2 \tag{6}$$

where K_* is the adhesion coefficient of material.

The material strength parameter K_* in condition (6) characterizes the plastic inhomogeneity formed as a result of varying degrees of damage to the material (the presence of many defects and microcracks in it) due to the force-corrosion effect and distributed over the thickness of the plastic zone, similar to the outline of its boundary. $K_* = K_1$ At the border of the plastic zone, the value K_* is constant: $K_* = K_1$. The quantity K_* is a special softening function that depends on the coordinates r, θ and loading parameters r_O, δ [12]:

$$K_* = K_*(r, r_0, \theta, \delta) \tag{7}$$

Here r_0, δ are the axisymmetric and non-axisymmetric loading parameters: $r_0 = r_0(P_0, P_1 + P_2)$, $\delta = \delta(P_1 - P_2)$.

The problem is solved by the method of joint use of static and physical equations for the considered elastoplastic material.

The problem also uses the perturbation method in the theory of an elastoplastic body [13]. The solution by the perturbation method is determined near the known "zero" solution at $\delta = 0$. As an initial state, we will take the solution we obtained earlier for a circular thick-walled element in an axisymmetric formulation [1]:

$$\begin{gathered} \sigma_{[r]}^0 = P_0 + 2\int_{a_o}^{r_o} r^{-1} K_*^0 dr, \quad \sigma_{[\theta]}^0 = \sigma_{[r]}^0 + 2K_*^0, \quad \tau_{[r\theta]}^0 = 0, \\ \left.\begin{matrix} \sigma_{(r)}^0 \\ \sigma_{(\theta)}^0 \end{matrix}\right\} = P + K_1 r_o^2 \left(1 \mp \frac{1}{r^2}\right), \quad \tau_{(r\theta)}^0 = 0. \end{gathered} \tag{8}$$

Axisymmetric boundary conditions on the inner and outer contour a_0 and 1 of thick-walled circular element and the conjugation conditions on the contour r_0 have the form (Figure 1):

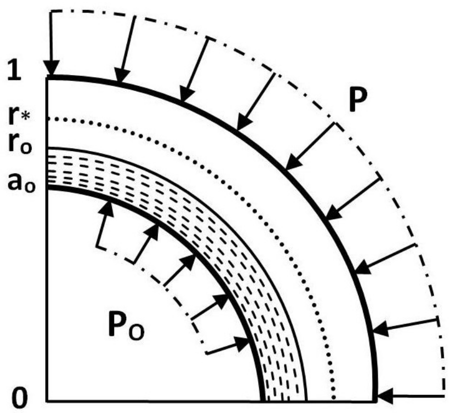

Figure 1: Design scheme for a circular thick-walled pipeline element

$$\sigma^0_{[r]} = P_0 \text{ at } r = a_0;\ \sigma^0_{(r)} = P$$

$$\text{at } r = 1; [\sigma^0_r] = [\sigma^0_\theta] = 0 \text{ at } r = r_0$$

In expressions (8), square (round) brackets at the indices mean belonging to the plastic (elastic) zone. The symbol K^0_* denotes the softening function (7) in the axisymmetric case, which depends only on the current radius r and the boundary radius r_0 :

$$K^0_* = K_*(r, r_0) = (K_0 - K_1)\overline{f}(r, r_0) + K_1. \tag{9}$$

Here K_0 and K_1 is the value of the strength of the material on the inner contour a_0 and on the elastoplastic radius r_0, $\overline{f}(r, r_0)$ – some kernel with the properties $\overline{f}(a_0,\, 1) = 1$, $\overline{f}(r_0, r_0) = 0$. In [10], the kernel $\overline{f}(r, r_0)$ was taken as a kernel that describes well the decrease in the value of K^0_* during loading both along the radius r and depending on the position of the boundary radius r_0 (n is the nonlinearity parameter): $\overline{f}(r, r_0) = \dfrac{a_0^n\,(r_0^n - r^n)}{r^n\,(1 - a_0^n)}$.

Elastoplastic radius r_0 in our "zero" solution is implicitly determined from the transcendental equation

$$P_0 - P + 2\int\limits_{a_o}^{r_o} r^{-1} K^0_* dr + K_1(1 - r_0^2) = 0 \tag{10}$$

In the absence of corrosion damage, the parameter $K^0_* = K_1$ and the radius of the plastic zone r_0 are found from the equation

$$P_0 - P + 2K_1\left(\ln\left(\frac{r_0}{a_0}\right) + \frac{1}{2}\left(1 - r_o^2\right)\right) = 0. \tag{11}$$

According to the perturbation method, the solution is sought in the form of rows of the sought components in powers of a small parameter, which is δ

$$\begin{aligned}
\sigma_{ij} &= \sum_0^\nu \delta^\nu \sigma_{ij}^{(\nu)} = \sigma^0_{ij} + \sum_1^\nu \delta^\nu \sigma_{ij}^{(\nu)}, \\
K_* &= \sum_0^\nu \delta^\nu K_*^{(\nu)} = K^0_* + \sum_1^\nu \delta^\nu K_*^{(\nu)}, \\
r_s &= \sum_0^\nu \delta^\nu r_\nu = r_0 + \sum_1^\nu \delta^\nu r_\nu
\end{aligned} \tag{12}$$

where r_s is the sought elastoplastic boundary.

For this, the initial equations, boundary conditions and conjugation conditions are linearized. Equilibrium equations (1) retain their form for any approximation

$$\frac{\partial\sigma_r^{(\nu)}}{\partial r}+\frac{1}{r}\frac{\partial\tau_{r\theta}^{(\nu)}}{\partial\theta}+\frac{\sigma_r^{(\nu)}-\sigma_\theta^{(\nu)}}{r}=0,\qquad \frac{\partial\tau_{r\theta}^{(\nu)}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_\theta^{(\nu)}}{\partial\theta}+2\frac{\tau_{r\theta}^{(\nu)}}{r}=0. \tag{13}$$

The linearization of the relations of the theory of ideal plasticity consists in the combined use of equations (1), (6). Introducing the stress function $F=F(r,\theta)$ according to (1)

$$\sigma_r^{(\nu)}=\frac{1}{r}\frac{\partial F^{(\nu)}}{\partial r}+\frac{1}{r^2}\frac{\partial^2F^{(\nu)}}{\partial\theta^2},\quad \sigma_\theta^{(\nu)}=\frac{\partial^2F^{(\nu)}}{\partial r^2},\quad \tau_{r\theta}^{(\nu)}=-\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial F^{(\nu)}}{\partial\theta}\right),\quad \nu=0,1,2,\dots \tag{14}$$

and linearizing equation (6) we obtain in the plastic zone an inhomogeneous differential equation in partial derivatives for the function $F^{(\nu)}(r,\theta)$:

$$r^2\frac{\partial^2F^{(\nu)}}{\partial r^2}-r\frac{\partial F^{(\nu)}}{\partial r}-\frac{\partial^2F^{(\nu)}}{\partial\theta^2}=r^2f^{(\nu)}(r,\theta),\qquad \nu\geq 0. \tag{15}$$

Here $f^{(\nu)}$ is the right side of the corresponding linearized relation: $f^0=2K_*^0$, $f^{(I)}=2K_*^{(I)}$, $f^{(II)}=-\dfrac{1}{K_O}(\tau_{r\theta}^{(I)})^2+2K_*^{(II)}$.

The solution to equation (15) $F^{(\nu)}$ is determined taking into account static or geometric boundary conditions.

The linearization of the boundary conditions depends on the given forces on the initial contour, and the linearization of the conjugation conditions on the elastoplastic boundary is determined by the nature of the initial conjugation conditions.

4 Thick-walled pipeline element with ovalization under uniform external and internal pressure

Consider a thick-walled pipeline element with ovalization, loaded with uniform internal P_0 and external P pressures, under conditions of plane deformation (Fig. 2)

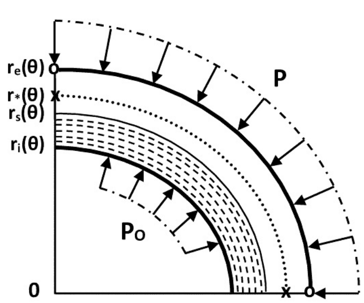

Figure 2: Design scheme for a thick-walled pipeline element with ovalization

The equations of the outer and inner contours of the pipeline element have the form

$$\begin{aligned} r_e&=1+\delta d_1\cos 2\theta,\\ r_i&=a_0(1+\delta d_2\cos 2\theta), \end{aligned} \tag{16}$$

where δ is the parameter of deviation of the oval contour from the circular one, d_1 and d_2 are geometric coefficients.

We construct a solution to the problem in the form (12) at $\nu\geq 0$. The zero solution for $\nu=0$ is given in the previous part of the work. Let us find a solution for $\nu=1$.

Consider the plastic zone of a thick-walled element. We represent the stress function $F^{(1)}$ in equation (15) based on the geometric boundary conditions of the outer and inner contours (16): $F^{(1)} = R(r)\cos 2\theta$.

Solving equation (15), we find the function $F^{(1)}$ in the plastic zone:

$$F^{(1)} = \left(A_i R_i + R_i \int\limits_{a_0}^{r} \frac{V_i(r)}{V(r)} dr \right) \cos 2\theta, \ , \quad i = \overline{1,2}, \tag{17}$$

where $A_i R_i = A_1 R_1 + A_2 R_2 = r(A_1 \cos(\sqrt{3}\ln r) + A_2 \sin(\sqrt{3}\ln r))$, $V(r)$ is the Wronskian of the system of solutions R_i, $V_i(r)$ is the determinant obtained from the Wronskian by replacing the i^{th} column with a column with a single nonzero element $2K_*^{(I)} \cos^{-1} 2\theta$ located at its end.

The stress components in the plastic zone under the Tresque-Saint-Venant condition based on (14), (17) are written as:

$$\begin{gathered}
\sigma_{[r]}^{(I)} = \frac{1}{r}[A_1 \cos(\sqrt{3}\ln r) + A_2 \sin(\sqrt{3}\ln r) - 2(\sqrt{3}\cos(\sqrt{3}\ln r) + \sin(\sqrt{3}\ln r))\times \\
\times \int\limits_{a_0}^{r} \frac{-\sin(\sqrt{3}\ln r)K_*^{(I)}}{\cos 2\theta} dr + 2(\cos(\sqrt{3}\ln r) - \sqrt{3}\sin(\sqrt{3}\ln r))\times \\
\times \int\limits_{a_0}^{r} \frac{\cos(\sqrt{3}\ln r)K_*^{(I)}}{\cos 2\theta} dr] \cos 2\theta, \quad \sigma_{[\theta]}^{(I)} = \sigma_{[r]}^{(I)} + 2K_*^{(I)}, \\
\tau_{[r\theta]}^{(I)} = \frac{1}{2r}[(A_1 - \sqrt{3}A_2)\cos(\sqrt{3}\ln r) + (A_2 - \sqrt{3}A_1)\sin(\sqrt{3}\ln r) - \\
-8\sin(\sqrt{3}\ln r) \cdot \int\limits_{a_0}^{r} \frac{-\sin(\sqrt{3}\ln r)K_*^{(I)}}{\cos 2\theta} dr + 8\cos(\sqrt{3}\ln r)\times \\
\times \int\limits_{a_0}^{r} \frac{\cos(\sqrt{3}\ln r)K_*^{(I)}}{\cos 2\theta} dr] \sin 2\theta.
\end{gathered} \tag{18}$$

Find constants A_1 and A_2. For this, we have linearized boundary conditions on the inner contour

$$\sigma_{[r]}^{(I)} + a_0 d_2 \frac{d\sigma_{[r]}^0}{dr} \cos 2\theta = 0, \quad \tau_{[r\theta]}^{(I)} + 2d_2(\sigma_{[\theta]}^0 - \sigma_{[r]}^0)\sin 2\theta = 0, \quad \text{at} \quad r = a_0. \tag{19}$$

Substituting (18) into (19), we obtain

$$A_1 = -2(\sqrt{3}\sin(\sqrt{3}\ln a_0) + \cos(\sqrt{3}\ln a_0))a_0 d_2 K_*(a_0, r_0),$$

$$A_2 = 2(\sqrt{3}\cos(\sqrt{3}\ln a_0) - \sin(\sqrt{3}\ln a_0))a_0 d_2 K_*(a_0, r_0).$$

Substituting A_1 and A_2 in (18), we can obtain expressions for the components $\sigma_{[r]}^{(I)}$, $\sigma_{[\theta]}^{(I)}$, $\tau_{[r\theta]}^{(I)}$ in their final form.

Consider the elastic region of a thick-walled element. The components $\sigma_{(r)}^{(I)}$, $\sigma_{(\theta)}^{(I)}$, $\tau_{(r\theta)}^{(I)}$ of the stress tensor in the elastic region are found from the stress function $\Phi^{(1)}$ in equations (4), (14):

$$\begin{aligned} \sigma_{(r)}^{(I)} &= (-2C_1 - 6C_2 r^{-4} - 4C_4 r^{-2})\cos 2\theta = 0, \\ \sigma_{(\theta)}^{(I)} &= (2C_1 + 6C_2 r^{-4} + 12C_3 r^2)\cos 2\theta = 0, \\ \tau_{(r\theta)}^{(I)} &= (2C_1 - 6C_2 r^{-4} + 6C_3 r^2 - 2C_4 r^{-2})\sin 2\theta \end{aligned} \tag{20}$$

To determine C_1, C_2, C_3, C_4 we have linearized boundary conditions on the outer contour of the element

$$\sigma_{(r)}^{(I)} + d_1 \frac{d\sigma_{(r)}^0}{dr}\cos 2\theta = 0, \quad \tau_{(r\theta)}^{(I)} + 2d_1(\sigma_{(\theta)}^0 - \sigma_{(r)}^0)\sin 2\theta = 0, \quad \text{at} \quad r = 1 \tag{21}$$

and two linearized conditions for conjugation of stresses at the boundary of the plastic zone

$$[\sigma_r^{(I)}] = 0, \ \ [\tau_{r\theta}^{(I)}] = 0 \qquad \text{at} \quad r = r_0 \tag{22}$$

Then, from conditions (21), (22), we obtain the boundary value problem for the elastic region of a thick-walled oval element:

$$\begin{aligned} \sigma_{(r)}^{(I)} &= M_1(r)\cos 2\theta, \quad \tau_{(r\theta)}^{(I)} = M_2(r)\sin 2\theta, \quad \text{at } r = r_0, \\ \sigma_{(r)}^{(I)} &= -M_0(r)\cos 2\theta, \quad \tau_{(r\theta)}^{(I)} = -2M_0(r)\sin 2\theta, \ \text{at } r = 1. \end{aligned} \tag{23}$$

Here under the condition of Tresque-Saint-Venant

$$M_0 = 2r_0^2 d_1 K_1,$$

$$M_1 = \frac{2a_0 d_2 K_*(a_0, r_0)}{r}\left(\sqrt{3}\sin(\sqrt{3}\ln\frac{r_0}{a_0}) + \cos(\sqrt{3}\ln\frac{r_0}{a_0})\right) -$$

$$-\frac{2}{r_0}(\sqrt{3}\cos(\sqrt{3}\ln r_0) + \sin(\sqrt{3}\ln r_0)) \cdot \int_{a_0}^{r_0} \frac{-\sin(\sqrt{3}\ln r)K_*^{(I)}}{\cos 2\theta} dr +$$

$$+\frac{2}{r_0}(\cos(\sqrt{3}\ln r_0) - \sqrt{3}\sin(\sqrt{3}\ln r_0)) \cdot \int_{a_0}^{r_0} \frac{\cos(\sqrt{3}\ln r)K_*^{(I)}}{\cos 2\theta} dr,$$

$$M_2 = \frac{4a_0 d_2 K_*(a_0, r_0)}{r}\cos(\sqrt{3}\ln\frac{r_0}{a_0}) - \frac{8}{r_0}\sin(\sqrt{3}\ln r_0) \cdot \int_{a_0}^{r_0} \frac{-\sin(\sqrt{3}\ln r)K_*^{(I)}}{\cos 2\theta} dr +$$

$$+\frac{8}{r_0}\cos(\sqrt{3}\ln r_0)\int_{a_0}^{r_0} \frac{\cos(\sqrt{3}\ln r)K_*^{(I)}}{\cos 2\theta} dr$$

Solving the boundary value problem (23), we find all constants $C_1 - C_4$:

$$C_1 = \frac{1}{2}(1-r_0^{-2})^{-3}\left(M_0 r_0^{-2}(2-r_0^{-2}-r_0^{-4}) - M_1(1+r_0^{-2}+2r_0^{-4}) + 2M_2 r_0^{-4}\right),$$

$$C_2 = \frac{1}{2}(1-r_0^{-2})^{-3}\left(3M_0(1-r_0^{-2}) - M_1(3+r_0^{-2}) + 2M_2 r_0^{-2}\right),$$

$$C_3 = \frac{1}{6}r_0^{-2}(1-r_0^{-2})^{-3}\left(3M_0(r_0^{-4}-r_0^{-2}) + M_1(1+3r_0^{-2}) + M_2(1-3r_0^{-2})\right),$$

$$C_4 = \frac{1}{2}(1-r_0^{-2})^{-3}\left(-M_0(1+r_0^{-2}-2r_0^{-4}) + M_1(2+r_0^{-2}+r_0^{-4}) - M_2(r_0^{-2}+r_0^{-4})\right)$$

Substituting C_1, C_2, C_3, C_4 in (20), we obtain the stress components in the elastic region of the thick-walled element $\sigma^{(I)}_{(r)}$, $\sigma^{(I)}_{(\theta)}$, $\tau^{(I)}_{(r\theta)}$ in the final form.

We seek the equation for the boundary of the plastic zone r_s in the form $r_s = r_0 + \delta r_1$. To determine the value of r_1, we use the linearized conditions for the conjugation of the components σ_θ and K_* on r_0 :

$$\left[\sigma_\theta^{(I)} + \frac{d\sigma_\theta^0}{dr}r_1\right] = 0, \qquad \left[K_*^{(I)} + \frac{dK_*^0}{dr}r_1\right] = 0 \quad \text{at} \quad r = r_0, \tag{24}$$

From conditions (24) we obtain

$$r_1 = (\sigma^{(I)}_{(\theta)} - \sigma^{(I)}_{[\theta]})/\left(\frac{d\sigma^0_{[\theta]}}{dr} - \frac{d\sigma^0_{(\theta)}}{dr}\right), \quad K_*^{(I)} = -\frac{dK_*^0}{dr}r_1 \ \text{at} \ r = r_0$$

Then we finally have

$$r_1 = \psi(r_0) r_0 \cos 2\theta,$$

where $\psi(r_0)$ is some function of $r_0 : \psi(r_0) = \dfrac{Y_1+Y_2+Y_3}{Y_4+Y_5+Z}$.

Here under the condition of Tresque-Saint-Venant

$$Y_1 = -4(2-7r_0^{-2}+5r_0^{-4})(1-r_0^{-2})^{-3}d_1K_1,$$

$$Y_2 = (1+5r_0^{-2}-11r_0^{-4}-3r_0^{-6}-(1-r_0^{-2})^3)\frac{a_0 d_2 K_*(a_0,r_0))}{r_0(1-r_0^{-2})^3}\times$$

$$\times\left(\sqrt{3}\sin\left(\sqrt{3}\ln\frac{r_0}{a_0}\right) - \cos\left(\sqrt{A}3\ln\frac{r_0}{a_0}\right)\right),$$

$$Y_3 = -8\left(1-3r_0^{-2}+r_0^{-4}+3r_0^{-6}\right)\frac{a_0 d_2 K_*(a_0,r_0))}{r_0(1-r_0^{-2})^3}\times\cos\left(\sqrt{3}\ln\frac{r_0}{a_0}\right),$$

$$Y_4 = -2(1+5r_0^{-2}-11r_0^{-4}-3r_0^{-6}-(1-r_0^{-2})^3)\frac{2\widetilde{K}_*}{(1-r_0^{-2})^3}\times$$

$$\times((\sqrt{3}\cos(\sqrt{3}\ln r_0)+\sin(\sqrt{3}\ln r_0))B_1 + (\cos(\sqrt{3}\ln r_0) - \sqrt{3}\sin(\sqrt{3}\ln r_0))B_2),$$

$$Y_5 = -4(1-3r_0^{-2}+r_0^{-4}+3r_0^{-6})\frac{\widetilde{K}_*}{(1-r_0^{-2})^3}\times(\sin(\sqrt{3}\ln r_0)B_1 + \cos(\sqrt{3}\ln r_0)B_2),$$

$$Z = 4K_1, \quad \widetilde{K}_* = (K_0 - K_1)\frac{a_0^n}{1 - a_0^n}\frac{n}{r_0},$$

$$B_1 = \frac{r_0}{2}\sin(\sqrt{3}\ln r_0 - \frac{\pi}{3}) - \frac{a_0}{2}\sin(\sqrt{3}\ln a_0 - \frac{\pi}{3}),$$

$$B_2 = \frac{r_0}{2}\cos(\sqrt{3}\ln r_0 - \frac{\pi}{3}) - \frac{a_0}{2}\cos(\sqrt{3}\ln a_0 - \frac{\pi}{3})$$

In the homogeneous case, the expression Y_1 is the same, the expressions Y_2 and Y_3 are preserved, but instead of $K_*(a_0, r_0)$, K_1, $Y_4 = Y_5 = 0$ should be written. In this case, the radius r_0 corresponds to the homogeneous case.

The equation for the boundary of the plastic zone r_s is written in the form:

$$r_s = r_0(1 + \delta\psi(r_0)\cos 2\theta).$$

The obtained solution area is as follows

$$r_0(1 - \delta\psi(r_0)) \geq a_0(1 - \delta d_2).$$

The bearing capacity of a thick-walled pipeline element with ovalization is determined as follows: If $\psi(r_0) = d_1$, then the boundary equation r_s has the form $r_s = r_0(1 + \delta d_1 \cos 2\theta)$. In this case, the ellipses bounding the outer contour of the element and the plastic zone will be similar. Consequently, the plastic zone will reach the critical contour at once in all its points and the bearing capacity is determined from a simple condition $r_* = r_0$.

In the absence of corrosion damage ($r_* = 1$), the condition for determining the bearing capacity of the element will take the form from $r_0 = 1$. Here r_* is the numerically determined critical radius of the element ($a_0 < r_* \leq 1$), corresponding to the maximum point on the loading diagram $\Delta P = \Delta P(r_0)$, at which a thick-walled element is destroyed [1].

In all other cases ($\psi(r_0) \neq d_1$), the bearing capacity of a pipeline element with ovalization can be researched as follows.

From the equation

$$r_*(1 \pm \delta d_1) = r_0(1 \pm \delta\psi(r_0)), \qquad (25)$$

at $\theta = 0$ or $\theta = \pi/2$, we find the value r_0, and then from (10) we obtain the critical value of external loads, at which the plastic zone will reach some "critical" points of the thick-walled element.

In the absence of corrosion damage to the element, equation (11) should be adopted and $r_* = 1$ used in equation (25). In this case, critical points (marked with zeros in Fig. 2) are located on the outer contour of the element $1 + \delta d_1 \cos 2\theta$ at the points of its greatest ($\theta = 0$) or least ($\theta = \pi/2$) curvature.

In the presence of corrosion damage of the element, the critical points are located inside the element (marked with crosses in Fig. 2) on the contour $r_*(1 + \delta d_1 \cos 2\theta)$ in the directions of its greatest ($\theta = 0$ or least ($\theta = \pi/2$) curvature (this is determined through the coefficients d_1 and d_2). Reaching these crosses by any point of the plastic zone will lead to the destruction of the element. Note that in an oval element, the plastic zone in the presence of damage to the material becomes larger in size and somewhat elongated in the directions of its greatest curvature. Wherein, in the above directions, the element acquires the greatest damage.

In this case $d_1 = 0$, $d_2 = 1$, we have a thick-walled circular element with an oval hole. The bearing capacity of such an element is determined from the equation $r_* = r_0(1 + \delta\psi(r_0))$ at $Y_1 = 0$. Consequently, the "critical" points are located inside the element on the contour r_* in the directions of the greatest curvature ($\theta = \pi/2$) of the hole.

In this case $d_1 = 1$, $d_2 = 0$, we have a thick-walled oval element with a circular hole. The bearing capacity of such element is determined from equation (25) at $Y_2 = Y_3 = 0$. "Critical" points are located inside the element on the contour $r_*(1 + \delta d_1 \cos 2\theta)$ in the directions of the greatest ($\theta = 0$) or least ($\theta = \pi/2$) curvature of its outer contour.

All the obtained solutions at $\gamma = 1$ go to the homogeneous case, and at $\delta = 0$ go to the axisymmetric case.

5 Conclusion

The stress state of an elastoplastic element of a thick-walled pipeline when ovalizing a cross section is studied under conditions of power and corrosion effect using a special softening function (plastic inhomogeneity) in the plasticity condition of Tresca-Saint-Venant. An elastoplastic problem is considered for a thick-walled pipeline element with ovalization under the action of uniform external and internal pressure in a nonaxisymmetric formulation. The problem is solved by the method of joint use of equilibrium equations and physical equations in each zone and their "sewing" through the conjugation conditions on the elastoplastic boundary, as well as by the method of perturbations in the theory of an elastoplastic body.

An assessment of the strength and bearing capacity of a loaded thick-walled pipeline element with ovalization in the presence and absence of corrosion damage is given.

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