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LINEAR STOCHASTIC DISTRIBUTED MODEL OF MONEY ACCUMULATION IN THE FORM OF A STATE SPACE

The article deals with the problem of the passive parametric identification of systems for modeling the evolution of money savings income and expenses of one household using a linear stochastic distributed model in the form of a state space taking into account white noises model of the investigated object dynamics' and white noises of the linear model measuring system of a distributed type. The use of the finite difference method allowed reducing the solution of partial differential equations to the solution of linear finite difference system with private derivatives to be reduced to the solution of a system of linear finite-difference and algebraic equations represented by models in the form of state space. It was proposed the use of a Kalman filtering algorithm for reliable evaluation of object behavior. The statement of the problem of estimating the coefficients of the equation of evolution of money savings income and expenses of one household is given. The structure of household income and expenses is described, taking into account additional additive white noise meters. An algorithm for numerical approbation of method for solving the problem of estimating the coefficients of an equation in the form of the state space for the evolution of money savings income and expenses of one household is considered. Calculations were carried out using the Matlab mathematical system based on statistical data for five years, taken from the site "Agency for Strategic planning and reforms of the Republic of Kazakhstan Bureau of National statistics". The proposed method for solving the problem of coefficients assessment's passive identification using the equations of money savings for one household in the form of a state space is sufficiently universal.

Key words: linear finite-difference equation, model in the form of a state space, evolution of one household money savings, passive identification, Kalman filter, prediction estimates, filtering estimates.

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Ақшалай жинақталуының күй кеңістігі түріндегі сызықтық стохастикалық үлестірілген моделі

Мақалада зерттелетін объектінің динамика моделінің ақ шуларын және таратылған типтегі өлшеу жүйесінің сызықтық моделінің ақ шуларын есекере отырып, күй кеңістігі түріндегі сызықтық стохастикалық үлестірілген модельдің көмегімен, жеке үй шаруашылығының кірістері мен шығыстарының ақшалай жинақталуының эволюциясын модельдеу үшін жүйелердің пассивті параметрлік идентификациясының есебі қарастырылады. Ақырғы айырымдар әдісін қолдану дербес туындылы теңдеулер шешімін күй кеңістігі түріндегі модельдермен ұсынылған сызықтық ақырғы-айырымдық және алгебралық теңдеулер жүйесінің шешіміне келтіруге мүмкіндік береді. Объектінің әрекетін дұрыс бағалау үшін Калман сүзгісінің алгоритмін қолдану ұсынылды. Бір үй шаруашылығының кірістері мен шығыстарының ақшалай жинақталу эволюциясы теңдеуінің коэффициенттерін бағалау мәселесінің тұжырымдамасы келтірілген. Есептегіштердің қосымша ақ шуын ескере отырып, үй шаруашылығының кірістері мен шығыстардың құрылымы сипатталған. Бір үй шаруашылығының кірістері мен шығыстардың ақшалай жинақтары эволюциясы күйінің кеңістігі түріндегі теңдеу коэффициенттерін бағалау мәселесін шешудің әдістемесін сандық апробациялау алгоритмі қарастырылған. Есептеулер Matlab математикалық жүйесін қолдана отырып, "Қазақстан Республикасы Стратегиялық жоспарлау және реформалар агенттігі ұлттық статистика бюросы"сайтынан алынған бес жылдағы бақылау деректері негізінде жүргізілді. Күй кеңістігі түрінде бір үй шаруашылығы үшін ақша жинақтау теңдеулерінің коэффициенттерін бағалауды пассивті идентификациялау мәселесін шешудің ұсынылған әдісі жеткілікті түрде әмбебап болып табылады.

Түйін сөздер: сызықтық ақырғы-айырымдық теңдеу, күй кеңістігіндегі модель, бір үй шаруашылығының ақша жинақтарының эволюциясы, пассивті идентификация, Калман сүзгісі, болжауды бағалау, сүзуді бағалау.

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Линейная стохастическая распределенная модель денежных накоплений в форме пространства состояний

В статье рассматривается задача пассивной параметрической идентификации систем для моделирования эволюции денежных накоплений доходов и расходов одного домохозяйства, с помощью линейной стохастической распределенной модели в форме пространства состояний с учетом белых шумов модели динамики исследуемого объекта и белых шумов линейной модели измерительной системы распределенного типа. Использование метода конечных разностей позволило свести решение уравнений с частными производными к решению системы линейных конечно-разностных и алгебраических уравнений, представленных моделями в форме пространства состояний. Для достоверного оценивания поведения объекта было предложено использование алгоритма калмановской фильтрации. Приведена постановка задачи оценивания коэффициентов уравнения эволюции денежных накоплений доходов и расходов одного домохозяйства. Описана структура доходов и расходов домохозяйства с учетом дополнительных аддитивных белых шумов измерителей. Рассмотрен алгоритм численной апробации методики по решению задачи оценивания коэффициентов уравнения в форме пространства состояний эволюции денежных накоплений доходов и расходов одного домохозяйства. Осуществлены расчеты с помощью математической системы Matlab на основе данных наблюдений за пять лет, взятых с сайта “Бюро национальной статистики Агентства по стратегическому планированию и реформам Республики Казахстан”. Предложенная методика решения задачи пассивной идентификации оценивания коэффициентов уравнений денежных накоплений для одного домохозяйства в форме пространства состояний в достаточной степени универсальна.

Ключевые слова: линейное конечно-разностное уравнение, модель в пространстве состояний, эволюция денежных накоплений одного домохозяйства, пассивная идентификация, фильтр Калмана, оценки предсказания, оценки фильтрации.

1 Introduction

The identification of dynamic objects is one of the main directions of modern control theory. In this area, there are many works devoted mainly to the identification of linear dynamic objects [1]- [8]. Moreover, the well-known works cover a variety of situations that arise during identification: the presence of additive noise at the input and output of the object [9, 10], or the impossibility of submitting test signals to the input [6], discrete or continuous form of signals [7], correlation or uncorrelatedness of signals and interference [11, 12], etc. Naturally, these methods generally give good results when analyzing objects in the vicinity of "standard"modes. In all other cases, objects are presented as essentially nonlinear, and at present there are few or no general identification methods to describe them. But recently, partial differential equations are often used to describe the dynamics of the object under study.

Thus, in [13], a number of partial differential equations (PDE) are studied. They are based on models developed to study some of the most important economic issues. At the same time, they are very interesting for mathematicians, because their structure is often quite complex. This paper shows a number of examples of such PDEs, discusses what is known about their properties, and lists some open questions for future research. The paper [14,15] introduces and discusses a nonlinear market equation of the Boltzmann type, which describes the influence of knowledge on the evolution of wealth in a system of agents who interact through binary transactions. The article [16] presents a semigroup approach to the mathematical analysis of problems with inverse parameters when identifying unknown parameters in a linear parabolic equation with mixed boundary conditions. In [17], an estimate of the parameters of stochastic differential equations of the return to mean type caused by Brownian motion is shown. At the same time, when identifying dynamic objects and systems, models in the state space are used with the use of a modified Kalman filter. For example [18], a new bilinear model is introduced in the form of a state space. The development of this model is linear-bilinear with respect to the state of the system. The classical Kalman filter is not applicable to this model, and therefore a new Kalman filter is introduced. The identification of systems described by partial differential equations is considered in papers [19]- [24].

In that research, for modeling of one household money savings dynamics we are going to use a linear stochastic distributed model in the form of a state space (SS) that describes the dynamics of money savings of income and expenses in the form of linear differential equations with partial derivatives, but the model of measuring system in the form of linear distributed algebraic equations with additive white noises in both the dynamics model and the model of the measuring system. Then, we are going to present an economic interpretation of the values included in the proposed model in the form of SS [25, 26].

2 Materials and Methods

In reality, the household money savings have a discrete character: the household receives a salary and household savings in a form that increases spasmodically and does not change further until the nearest waste of money. With expenses (we will take into account the total expenses by the end of the month), household savings are abruptly decreased, that is savings are determined by a piecewise constant function of time.

As time passes, the point moves through the space of savings with rate $\frac{dx}{dt} = \dot{x}$. Suppose that rate can be calculated in another way, using additional terms, for example, as additive white noise dynamics of the savings income or expenses. The possibility of calculations in another way arises in a detailed study in the process of earnings and costs in the household. Let rate is expressed as a function like $F(x, t)$ - the function of two variables x, t and the additive white noise of money savings dynamics $w(t)$. As a result, we get the following relation:

$$\frac{dx(t)}{dt} = F(x(t), t) + w(t). \tag{1}$$

Equation (1) is a stochastic ordinary differential equation that describes the dynamics of household income or expenses; $x(t)$ is an unknown function; $F(x, t)$ - given function. If at the initial moment of time $t = 0$ savings of fixed household are known, then we have the initial condition:

$$x(t)\,|_{t=0} = \bar{x}_0 \ , \tag{2}$$

where $x(t)\,|_{t=0}$ is the white Gaussian value with mathematical expectation $\bar{x}_0$ and unknown variance P_0.

Relations (1) and (2) allow us to formulate the Cauchy problem for a stochastic ordinary differential equation. The type $F\,(x,t)$ of function in (1) depends on the particular household and on its additive white noises of the economic activity. The more accurately we write its analytical function $F\,(x,t)$ (based on statistical data), with the most reliable characteristics of additive noise, the more accurate will be the mathematical model. The function F can be represented globally in the form like $F = D - R$, $(D \geq 0, R \geq 0)$, where $D(x,t)$ is the function that describes the household income and the function $R(x,t)$ is the expenses of the household. In [25, 26], examples of defining the functions D and R are given.

1. The income structure of the household $D(x(t),t)$ taking into account the dynamics of the additive white noise of the investigated object: $D(x(t),t) = D_0(x(t),t) + D_1(x(t),t) + w_1(x(t),t)$, where

a) $D_0(x(t),t)$ - household wages. Suppose that additive noise $w_1(x(t),t)$ is some white distributed Gaussian noise with zero expectation and unknown variance Q.

b) $D_1(x(t),t)$ - solid income from investments in money savings in the bank.

$$D_1(x(t),t) = \alpha \cdot x(t) \cdot \theta(x(t),x_0). \tag{3}$$

Suppose a household invests all available money $x(t)$ in a bank on $p\frac{\%}{month}$.

Function $\theta(x(t),x_0)$ - threshold function (θ-function):

$$\theta(x(t),x_0) = \left\{ \begin{array}{ll} 0, & at\ x < x_0 \\ 1, & at\ x > x_0 \end{array} \right\}, \tag{4}$$

where x_0 is the minimum amount of savings that allows you to make an investment in the bank.

As a result, we get the household income function:

$$D(x(t),t) \approx D_0(x(t),t) + \alpha \cdot x(t) \cdot \theta\left(x\left(t\right), x_0\right) + w_1(x(t),t). \tag{5}$$

2. The structure of the household expenses $R(x(t),t)$, taking into account the additional additive white noise meters $w_2(t)$, can be written in the form: $R(x(t),t) \approx R_0(x(t),t) + R_1(x(t),t) + R_2(x(t),t) + w_2(x(t),t)$, where

a) $R_0(x(t),t)$ - average daily expenses to ensure the existence of the household. This part of the costs includes utility bills, average food costs, expenses for necessary clothes, transportation costs.

b) $R_1(x(t),t)$ - daily expenses that ensure the well-being of the household. This category of expenses is connected with the fact that if a household has surplus money, then it increases the cost of improving the quality of life.

c) $R_2(x(t),t)$ - expenses of elite goods. With sufficiently large savings, a household can allow the purchase of goods which are not the essential goods.

Thus, equation (1) has the form

$$\frac{dx}{dt} \approx D_0\left(x(t),t\right) + \alpha \cdot x(t) \cdot \tilde{\theta}\left(x, x_0\right) - R_0(x(t),t) - C_1 \frac{x(t)}{x(t)+y_1} \tilde{\theta}\left(x, y_0\right) -$$

$$-C_2 \frac{x(t)-z_1}{(x(t)-z_1)+(z_2-z_1)} \tilde{\theta}\left(x, z_1\right) + w(x(t),t), \tag{6}$$

where $w(x(t),\ t) = w_1(x(t),\ t) + w_2(x(t),\ t)$ - the additive generalized white noise.

The given mathematical description, taking into account white Gaussian income and expenses, based on statistical data, can be refined with a more detailed study of the economic activity of the household.

We write equation (6) in the differentials:

$$dx(t) = F(x(t),t)dt + w(x(t),t)dt. \tag{7}$$

In reality, a household, in addition to guaranteed income and expenses, may have random income and expenses. For a mathematical description of such a phenomenon, we introduce a random variable $X(t)$ that means the total amount of money where the household will save from random sources by that time t. The value $X(t+dt)$ is the total random savings of the household at the time $t+dt$, where dt is an infinitely small time interval.

$$dX = X(t+dt) - X(t), \tag{8}$$

which means random household income for an elementary period dt of time at $dX > 0$ and random expenditure $dX < 0$. We are going to call this value the stochastic differential of the random process $X(t)$. If we add the quantity (7) to equation (8), we obtain

$$dx = F\left(x,t\right) dt + dX + wdt. \tag{9}$$

Equation (9) is called a *stochastic differential equation*, where dX is not a differential in the usual sense. In a more general case, taking into account the additive noise of household money savings, relation (9) can be written as

$$dx = F\left(x,t\right) dt + G\left(x,t\right) dX + wdt, \quad G \geq 0. \tag{10}$$

Further, we suppose, that $F\left(x,t\right),\ \ G\left(x,t\right)$ - nonrandom functions, but X - Markov's stochastic process.

We divide the time interval into elementary time intervals Δt_i. We denote it as $t_{i+1} = t_i + \Delta t_i$, $t_0 = 0$, $x_i = x\left(t_i\right)$, $X\left(t_i\right) = z_i$, then

$$x_{i+1} = F\left(x_i,t_i\right) \Delta t_i + G\ \left(x_i,t_i\right)\ z_{i+1} + w(x_i,t_i), \tag{11}$$

where random variables z_{i+1} are determined by probability density function and can be implemented numerically.

Households at each moment of time are distributed unevenly along the axis Ox. In [25], based on the application of the principle of continuous media and the introduction of a function, a linear equation with partial derivatives of a distributed type is obtained, which is satisfied by the density of households in the space of savings $u(x,t)$.

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}\left(F \cdot u\right) = d \cdot f + p_1 \cdot w, \tag{12}$$

Equation (12) is called the linear equation of monetary accumulations of the ensemble of households, consisting of m classes taking into account the white additive noise of the dynamics of the object under study, the initial conditions (2) regarding the process of monetary accumulation, as well as the equations of the measuring system taking into account the white additive noise (12), in a complex is called a stochastic distributed equation in the form of a SS for each i-class, where $i = 1, 2, ..., m$.

Using the initial condition (2) and the difference scheme (11), we determine the approximate values $x\left(t\right)$ of the quantity at time instants $t = t_i$, then we determine one of the possible trajectories of a random variable $x\left(t\right)$.

To calculate a discrete analog of any equations in ordinary or partial derivatives is constantly used. Therefore, the task is to estimate the coefficients of equation (12) from observations of discrete input $z(x_k, t_s)$ and a discrete output $y(x_k,\ t_s)$ of a measuring system of appropriately distributed types, which can be written as

$$y(x_k, t_s) = h \cdot x(x_k, t_s) + p_2 \cdot \varepsilon(x_k, t_s), \quad k = \overline{1, n}; \quad s = \overline{0, m}, \tag{13}$$

where $y(x_k, t_s)$ is the output of the measuring system in which the indices k and s mean that the spatio-temporal state function $x(x,t)$ can be measured only at discrete spatial points x_k and at discrete time instants t_s, i.e $\left\{x(x,t) \approx x(x_k, t_s) = x_{k,s}, \quad k = \overline{1,\ n}, \quad s = \overline{1,\ m}\right\}$, h - a given weight coefficient to the measuring system; $\left\{y(x_k, t_s) = h \cdot x(x_k, t_s) = y_{k,s}, \quad k = \overline{1,\ n}, \quad s = \overline{1,\ m}\right\}$ - output of the measuring system; $\left\{\varepsilon(x_k, t_s) = \varepsilon_{k,s}, \quad k = \overline{1,\ n}, \quad s = \overline{1,\ m}\right\}$ - white Gaussian noise of a distributed type measuring system with zero mathematical expectation and unknown variance $p_2 = Q_2(x_k, t_s)$.

Under these conditions, the task is to estimate the parameters F, d, p_1, p_2 based on a distributed discrete input signal $\left\{u(x_k, t_s) = u_{k,s}, \quad k = \overline{1,\ n}, \quad s = \overline{1,\ m}\right\}$, initial conditions (2), as well as a distributed discrete output of the measuring system $\left\{y(x_k, t_s), \quad k = \overline{1,\ n}, \quad s = \overline{1,\ m}\right\}$.

3 Results and Discussion

In order to make the most reliable calculations for researching one household money savings based on the coefficients of the equations of one household money income and expenses, and subsequently, to get the most reliable estimates of the prediction and filtering behavior to the researched object, the scheme of the Kalman filter algorithm is used. We are going to look at the statistical data accumulated over the five years of 2014-2018 [27]. We give the brief calculation data for this example, which were carried out using the Matlab mathematical system based on the following algorithm:

1. The total number of research dates in months for five years - $n = 60$. At the first step of the algorithm, we conduct a linear regression for calculating the scatter based on research data and linear regression [28, 29]. The regression equation is constantly supplemented by a close coupling index, which allows making the most reliable variance calculations for dynamic models and measuring system. For $n = 60$, the coefficients of the regression model will be $a = -0.0089$; $b = 2474000$. The regression line on the graph is chosen so that the sum of the squares of the vertical distances between the points of the regression line and the observational data is minimal. Data on regular household solid income D_0: y_1 - observational data accumulated over five years and calculated data on household income z (based on $n = 60$ points) with calculated coefficients of the regression model made it possible to solve the problem of constructing a linear regression with minimal variance of residuals (see Fig. 1).

y_1=[56330 56419 59929 60913 61887 63025 64126 62873 61956 63107 63329 73362 61913 61824 61770 66499 66384 66320 68193 68106 68053 72985 72913 72895 71652 71638 71549 76263 76162 76084 76291 76200 76097 82343 82285 82339 79187 79111 79013 82485 82393 82307 83346 83286 83161 90188 90049 89992 86508 86425 86299 91191 91025 90960 94894 94789 94706 100374 100262 100184];

z=[56857 57526 58196 58865 59534 60204 60873 61542 62212 62881 63550 64220 64889 65558 66228 66897 67566 68236 68905 69574 70244 70913 71582 72252 72921 73590 74260 74929 75598 76268 76937 77606 78276 78945 79615 80284 80953 81623 82292 82961 83631 84300 84969 85639 86308 86977 87647 88316 88985 89655 90324 90993 91663 92332 93001 93671 94340 95009 95679 96348].

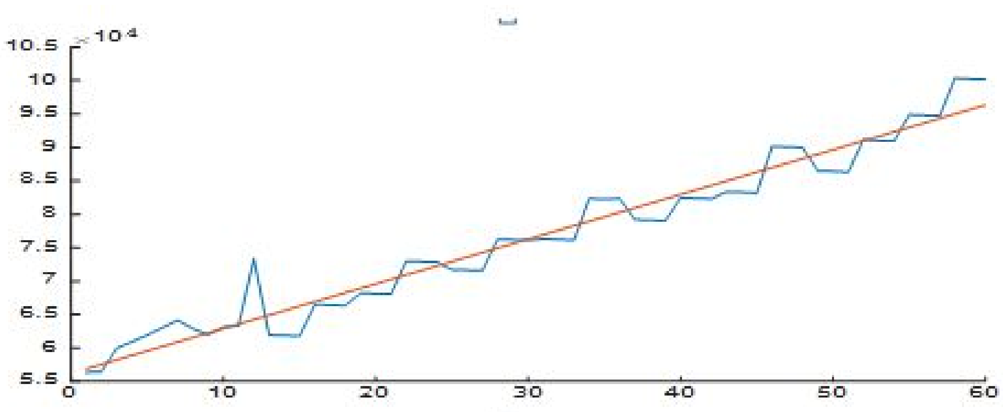

Figure 1: Regression line with the minimal residue variance

2. Then, we are going to reflect on a series of data, where each member of the series makes up the difference between the calculated values of the regression equation on the abscissa axis and the research data about $y = 0$ axes. The maximum line deviation of the regression equation from the research data is $x(12); max = 9142.3$ (see Fig. 2).

3. Further, the observational data is divided into two data sets $n_1 = 48$ and $n_2 = 12$. 48 observations were used to calculate the coefficients of the differential equation ($a = 24687$;

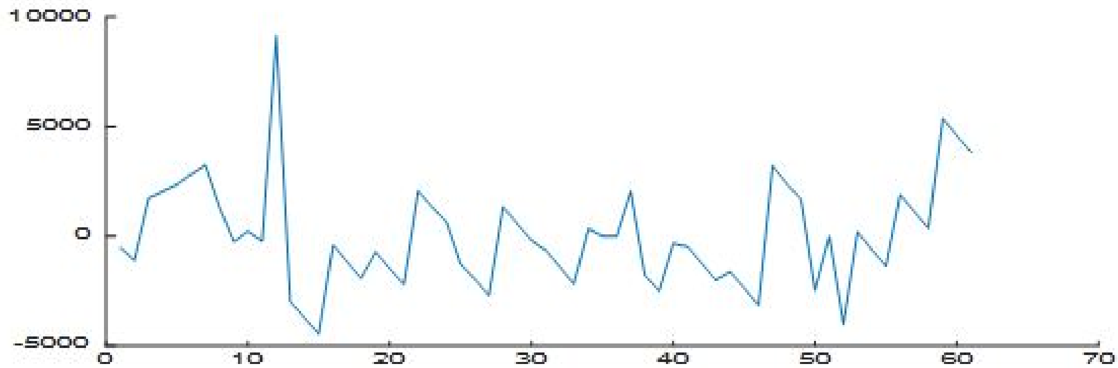

Figure 2: Calculation graph $x(12)$; $max = 9142.3$

$b = 0.652$;), as well as to calculate the variances of the model in the form of SS ($p_1 = Q_1 = 25171$ - variance for the dynamics model; variance for the model of the measuring system $p_2 = Q_2 = 50999$; variance for the initial moment of time $P_0 = Q_1$ in the Kalman filter algorithm) [30,31]. The remaining 12 observations are used to calculate prediction estimates and adjusted filtering estimates based on equations from the Kalman filter algorithm under initial conditions with respect to the state $x_0 = \bar{x}_0$ and variance at the initial time $P_0 = Q_1$ [31,32].

The calculation data of prediction and filtering assessment according to the Kalman scheme (see Fig. 3):

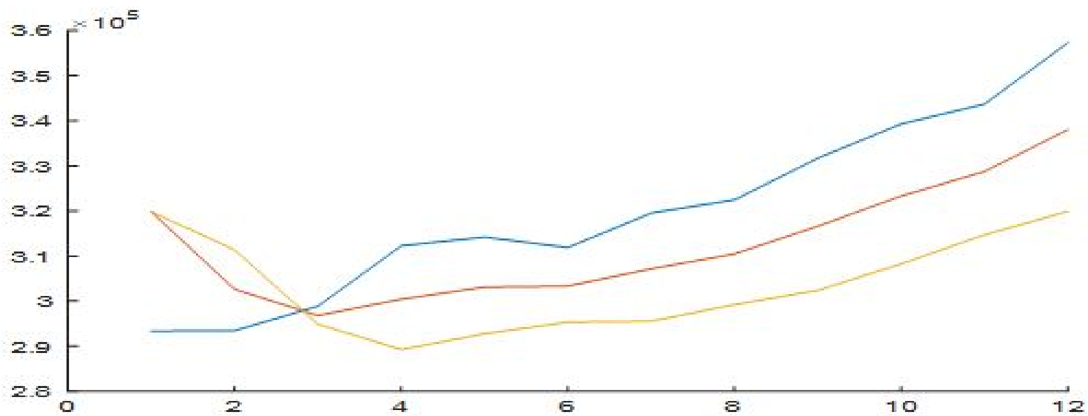

Figure 3: Graphs of prediction (x_p), filtration (x_f) and observational estimates (y_s)

y_s=[79187 86425 86299 91191 91025 90960 94894 94789 94706 100374 100262 100184]: observational data;

x_f=[79187 80491 80828 82895 83635 83899 85571 86184 86392 88733 89606 89917]: estimates of filtration;

x_p=[79187 76317 77167 77387 78735 79217 79389 80479 80879 81015 82541 83110]: prediction estimates.

4 Conclusion

The proposed techniques for solving the problem of coefficient assessment's passive identification using the equations of money savings for one household in the form of SS is sufficiently simple and universal. By analogy with the research techniques that was used for the data from the section «Regular solid household income D_0», other researches can be carried out in other sections: «Total solid expenses R», «Household living expenses R_0», «Household well-being expenses R_1», «Expenses of elite goods R_2». Regarding the last section, we note that in section R_2, it is constantly important to conduct a regular calculation to the total amount of savings at the time of purchase of the elite item and if this total amount exceeds the cost of the elite item R_2, then we must make the purchase of this item.

To clarify the values of the coefficients included in the regression equation, as well as in linear differential equations, there are many other possibilities that refine these coefficients. In particular, to such leverages, which can increase the accuracy coefficient assessment, according to the Kalman scheme, the following items can be attributed:

1. Increase in sample size relative to research data due to the increase in sample size by the growth of the data amount in months;

2. For interior points, more accurate approximation formulas can be used

$$\left(\frac{\partial u}{\partial x}\right)_k \approx \frac{u_{k+1} - u_{k-1}}{2 \cdot h}$$

3. The accumulated information about the estimates of the desired parameters, which ultimately would allow using the ideas of Bayesian parameter estimation [32, 33].

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