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INCEPTION OF GREEN FUNCTION FOR THE THIRD-ORDER LINEAR DIFFERENTIAL EQUATION THAT IS INCONSISTENT WITH THE BOUNDARY PROBLEM CONDITIONS

Regarding the importance of teaching linear differential equations, it should be noted that every physical and technical phenomenon, when expressed in mathematical sciences, is a differential equation. Differential equations are an essential part of contemporary comparative mathematics that covers all disciplines of physics (heat, mechanics, atoms, electricity, magnetism, light and wave), many economic topics, engineering fields, natural issues, population growth and today's technical issues. Used cases. In this paper, the theory of third-order heterogeneous linear differential equations with boundary problems and transforming coefficients into multiple functions $p(x)$ we will consider. In mathematics, in the field of differential equations, a boundary problem is called a differential equation with a set of additional constraints called boundary problem conditions. A solution to a boundary problem is a solution to the differential equation that also satisfies the boundary conditions. Boundary problem problems are similar to initial value problems. A boundary problem with conditions defined at the boundaries is an independent variable in the equation, while a prime value problem has all the conditions specified in the same value of the independent variable (and that value is below the range, hence the term "initial value"). A limit value is a data value that corresponds to the minimum or maximum input, internal, or output value specified for a system or component. When the boundaries of boundary values in the solution of the equation to obtain constants D_1, D_2, D_3 to lay down Failure to receive constants is called a boundary problem. We solve this problem by considering the conditions given for that true Green expression function. Every real function of the solution of a set of linear differential equations holds, and its boundary values depend on the distances.
Key words: Green Function, Boundary Problem, Private Solution, Public Solution, Wronskian Determinant.

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Үшінші ретті сызықтық дифференциалдық теңдеу үшін Грин функциясына сәйкес шекаралық есебі

Математика ғылымында сызықтық дифференциалдық теңдеулерді оқытудың маңыздылығы туралы айта кету керек, көрсетілген әрбір физикалық-техникалық құбылыс дифференциалдық теңдеу болып табылады. Дифференциалдық теңдеулер – бұл барлық физикалық пәндерді (жылу, механика, атомдар, электр, магнетизм, жарық және толқындар), көптеген экономикалық тақырыптарды, инженерлік өрістерді, табиғи мәселелерді, халық санының өсуін және заманауи техникалық мәселелерді қамтитын қазіргі салыстырмалы математиканың ажырамас бөлігі. Бұл мақалада шекаралық есептері бар үшінші ретті біртекті емес сызықтық дифференциалдық теңдеулер және коэффициенттерді бірнеше функцияға айналдыру теориясын қарастырамыз. Шектік есептер бастапқы мәнмен есептерге ұқсас. Шектерінде анықталған шарттары бар шекаралық есеп теңдеудегі тәуелсіз айнымалы болып табылады, ал қарапайым мәні бар есепте барлық шарттар болады. Тәуелсіз айнымалының бірдей мәнінде көрсетілген (және бұл мән ауқымнан төмен, демек, "бастапқы мән" термині).

Шектік мән дегеніміз – жүйеге немесе құрамдас бөлікке көрсетілген минималды немесе максималды кіріс, ішкі немесе шығыс мәндеріне сәйкес келетін деректер мәні. Теңдеу шешіміндегі шекара мәндерінің шекаралары тұрақтыларды алу үшін пайдаланады, содан кейін тұрақтыларды қосамыз. Бұл Грин шекаралық есебі деп аталады. Сызықтық дифференциалдық теңдеулер жүйесін шешудің әрбір нақты функциясы орындалады және оның шекаралық мәндері арақашықтықтарға тәуелді болады.
Түйін сөздер: Грин функциясы, шекаралық есеп, нақты шешім, вронскиян анықтамасы.

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Возникновение функции Грина для линейного дифференциального уравнения третьего порядка, несовместимого с условиями краевой задачи

Что касается важности обучения линейным дифференциальным уравнениям, следует отметить, что каждое физическое и техническое явление, выраженное в математических науках, является дифференциальным уравнением. Дифференциальные уравнения являются неотъемлемой частью современной сравнительной математики, которая охватывает все дисциплины физики (тепло, механику, атомы, электричество, магнетизм, свет и волны), многие экономические темы, области техники, естественные проблемы, рост населения и современные технические проблемы. В данной статье мы рассмотрим теорию неоднородных линейных дифференциальных уравнений третьего порядка с краевыми задачами и преобразованием коэффициентов в кратные функции. В области дифференциальных уравнений краевая задача называется дифференциальным уравнением с набором дополнительных ограничений, называемых условиями краевой задачи. Решение краевой задачи – это решение дифференциального уравнения, которое также удовлетворяет граничным условиям. Краевые задачи аналогичны задачам с начальным значением. Граничная задача с условиями, определенными на границах, является независимой переменной в уравнении, тогда как задача с простым значением имеет все условия, указанные в одном и том же значении независимой переменной (и это значение находится ниже диапазона, отсюда выйдет термин "начальное значение"). Предельное значение – это значение данных, которое соответствует минимальному или максимальному входному, внутреннему или выходному значению, заданному для системы или компонента. Когда границы граничных значений в решении уравнения для получения констант сложить потом получаем констант. Это называется краевой задачей Грина. Каждая действительная функция решения системы линейных дифференциальных уравнений имеет место, и ее граничные значения зависят от расстояний.

Ключевые слова: функция Грина, граничная задача, частное решение, публичное решение, определитель Вронскиана.

1 Introduction

Differential equations are one of the most interesting and widely used mathematical topics that have attracted the attention of many researchers. Differential equations in various disciplines including physics; It is especially useful in the movement of weights attached to springs, electrical circuits and free vibrations. In mathematics, in the field of differential equations, a boundary value problem is a differential equation with a set of additional constraints called boundary conditions. A solution to a boundary value problem is a solution to the differential equation that also satisfies the boundary conditions [3].

Boundary value problems arise in several branches of physics because each equation has a body differential. Wave equation problems, such as determining normal states, are often referred to as boundary value problems. A large category of important problems in the boundary value are the Storm-Liouville problems. The analysis of these problems involves special functions, the Green functions of a differential operator [4].

2 Objectives of this research

Topics for introducing heterogeneous linear differential equations of the third order with boundary conditions, and obtaining the Green function are discussed.

3 Methodology

Information has been collected in the form of libraries, websites, domestic and foreign scientific articles, undergraduate and doctoral research dissertations.

3.1 Literature review

Differential equations have been developed for nearly 300 years, and the relationship between evolutions is functions and derivatives of functions, so its history naturally dates back to the discovery of the derivative by the English scientist Isaac Newton (1772-1642) and Gottfried Gottfried Wilhelm Leibniz (Germany (1716-1646)) began. Newton worked on differential equations, including first-order differential equations, into forms. Jacob proposed the Bernoulli differential equation in 1674, but failed to prove it until Euler proved it in 1705.

In the linear differential equations of the boundary problem, Sturm-Lowville first worked, the Sturm-Lowville theory in mathematics and its applications, the classical Sturm-Lowville theory, named after Jacques François Sturm (1803-1855) and Joseph Lowville (1809-1882), the theory of linear differential equations is the second real order of form. In 1969, the Russian scientist Nymark wrote in his book Linear Differential Operators about the Green function to solve differential equations with boundary problem conditions.

4 Green function of an unperturbation boundary value problem

Problem statement. Consider the general solution of a third-order heterogeneous linear differential equation with boundary problem conditions

It should be noted that in the space $L_2(0,1)$ the operator generated by a linear differential expression of the third order with constant coefficients is considered.

$$y^{(3)}(x) + P_1(x)y^{(1)}(x) + P_0(x)y(x) = f(x) \tag{1}$$

Here $P_0(x)$ multiple functions are limited in $[0,1]$ interval $P_1(x)$ multiple functions can be changed in the interval $[0,1]$ are on. Number 3 is the order of differential expression and three times different.

In this section, we recall the known features of these operators, which weconsider with the following boundary conditions

$$U_j(y) = \alpha_j y^{(\gamma_j)}(0) + \beta_j y^{(\gamma_j)}(1) = 0, \quad j = 1,2,3, \tag{2}$$

where $\gamma_1 = 0, \ \ \gamma_2 = 1, \ \ \gamma_3 = 2$ chosen according to the Mikhailov-Keselman theorem are often called the strongly regular boundary conditions [4]. Therefore, the eigenvalues of the operator S_0 are asymptotic simple and separated [3], that is, there is a positive number δ for which any two eigenvalues of the operator S_0 are separated from each other by a distance greater than δ. It also follows from works [1, 2] that the system of eigenfunctions and associated functions of the operator S_0 forms a Rises basis in the space $L_2(0,1)$.

We assume

$$3 > \gamma_3 \geq \gamma_2 \geq \gamma_1 \geq 0$$

The general form of the heterogeneous linear differential equation using differential operators can also be written as follows

$$L(y) = \lambda y(x) + f(x) \tag{3}$$

We consider the general solution of equation (1), (2) to be a third-order differential

$$y(x) = y_0(x) + D_1\varphi_1(x) + D_2\varphi_2(x) + D_3\varphi_3(x) \tag{4}$$

where

$$y_0(x) = \int\limits_0^x g(x,t)f(t)dt$$

$y_0(x)$ The specific solution is a heterogeneous equation and here $\varphi_1(x), \ \varphi_2(x), \ \varphi_3(x)$ the basic system of solutions of the equation is homogeneous when the conditions $L(\varphi_1) = 0$, $L(\varphi_2) = 0$, $L(\varphi_3) = 0$ and satisfaction with heterogeneous border conditions $\varphi_j^{(k-1)}(0) = \delta_{kj}$ function $g(x,t)$ Determined by the following formula, which I call the Green function

$$g(x,t) = \frac{P(x,t)}{W(t)}$$

Where $\delta_{kj} = \begin{cases} 1 \ , k = j \\ 0 \ , \ k \neq j \end{cases}$ and $W(t)$ determinative Wronskian

$$W(t_1,t_2,t_3) = \begin{vmatrix} y_1(t) & y_2(t) & y_3(t) \\ y_1^{(1)}(t) & y_2^{(1)}(t) & y_3^{(1)}(t) \\ y_1^{(2)}(t) & y_2^{(2)}(t) & y_3^{(2)}(t) \end{vmatrix}$$

and it should be known that $P(x,t)$ is equal to becomes

$$P(x,t) = \begin{vmatrix} y_1(t) & y_2(t) & y_3(t) \\ y_1^{(1)}(t) & y_2^{(1)}(t) & y_3^{(1)}(t) \\ y_1(x) & y_2(x) & y_3(x) \end{vmatrix}.$$

So you should know $g(x,t) = P(x,t)$ because $g(x,t)$ the following formula can be defined.

$$g(x,t) = \begin{vmatrix} y_1(t) & y_2(t) & y_3(t) \\ y_1^{(1)}(t) & y_2^{(1)}(t) & y_3^{(1)}(t) \\ y_1(x) & y_2(x) & y_3(x) \end{vmatrix}.$$

From here we propose a specific inhomogeneous solution, below

$$y_0(x)=\int_0^x \begin{vmatrix} y_1(t) & y_2(t) & y_3(t) \\ y_1^{(1)}(t) & y_2^{(1)}(t) & y_3^{(1)}(t) \\ y_1(x) & y_2(x) & y_3(x) \end{vmatrix} f(t)dt$$

function $y_0(x)$ the heterogeneous solution is equation (1), (2) and to investigate it we take the first-order derivative from the specific (inhomogeneous) solution.

$$y_0^{(1)}(x)=\int_0^x \begin{vmatrix} y_1(t) & y_2(t) & y_3(t) \\ y_1^{(1)}(t) & y_2^{(1)}(t) & y_3^{(1)}(t) \\ y_1^{(1)}(x) & y_2^{(1)}(x) & y_3^{(1)}(x) \end{vmatrix} f(t)dt+\begin{vmatrix} y_1(x) & y_2(x) & y_3(x) \\ y_1^{(1)}(x) & y_2^{(1)}(x) & y_3^{(1)}(x) \\ y_1(x) & y_2(x) & y_3(x) \end{vmatrix} f(x)$$

now we take the second-order derivative

$$y_0^{(2)}(x)=\int_0^x \begin{vmatrix} y_1(t) & y_2(t) & y_3(t) \\ y_1^{(1)}(t) & y_2^{(1)}(t) & y_3^{(1)}(t) \\ y_1^{(2)}(x) & y_2^{(2)}(x) & y_3^{(2)}(x) \end{vmatrix} f(t)dt$$

now we take the third order derivative

$$y_0^{(3)}(x)=\int_0^x \begin{vmatrix} y_1(t) & y_2(t) & y_3(t) \\ y_1^{(1)}(t) & y_2^{(1)}(t) & y_3^{(1)}(t) \\ y_1^{(3)}(x) & y_2^{(3)}(x) & y_3^{(3)}(x) \end{vmatrix} f(t)dt+\begin{vmatrix} y_1(x) & y_2(x) & y_3(x) \\ y_1^{(1)}(x) & y_2^{(1)}(x) & y_3^{(1)}(x) \\ y_1^{(2)}(x) & y_2^{(2)}(x) & y_3^{(2)}(x) \end{vmatrix} f(x)$$

now to solve a given heterogeneous problem, we must examine Equation (1)and we $y(x)$ offer solutions

$$L(y)={y_0}^{(3)}(x)+P_1(x){y_0}^{(1)}(x)+P_0(x)y_0(x)$$

Solve the received function $y(x)$ Prove the following

$$L(y)=\int_0^x \begin{vmatrix} y_1(t) & y_2(t) & y_3(t) \\ y_1^{(1)}(t) & y_2^{(1)}(t) & y_3^{(1)}(t) \\ y_1^{(3)}(x) & y_2^{(3)}(x) & y_3^{(3)}(x) \end{vmatrix} f(t)dt+f(x)+P_1(x)\int_0^x \begin{vmatrix} y_1(t) & y_2(t) & y_3(t) \\ y_1^{(1)}(t) & y_2^{(1)}(t) & y_3^{(1)}(t) \\ y_1^{(1)}(x) & y_2^{(1)}(x) & y_3^{(1)}(x) \end{vmatrix} f(t)dt+$$
$$+P_0(x)\int_0^x \begin{vmatrix} y_1(t) & y_2(t) & y_3(t) \\ y_1^{(1)}(t) & y_2^{(1)}(t) & y_3^{(1)}(t) \\ y_1(x) & y_2(x) & y_3(x) \end{vmatrix} f(t)dt$$

from here we have to add the matrices,

$$L(y)=$$
$$=\int_0^x \begin{vmatrix} y_1(t) & y_2(t) & y_3(t) \\ y_1^{(1)}(t) & y_2^{(1)}(t) & y_3^{(1)}(t) \\ y_1^{(3)}(x)+P_1(x)y_1^{(1)}(x)+P_0(x)y_1(x) & y_2^{(3)}(x)+P_1(x)y_2^{(1)}(x)+P_0(x)y_2(x) & y_3^{(3)}(x)+P_1(x)y_3^{(1)}(x)+P_0(x)y_3(x) \end{vmatrix}$$
$$f(t)dt+f(x)$$

Homogeneous equation condition $L(y) = y_1^{(3)}(x) + P_1(x)y_1^{(1)}(x) + P_0(x)y_1(x) = 0$ to be so we can function $f(x)$ and as a result we can say that we have obtained the solution of the heterogeneous part.

we get the Green function for the proposed problem and prove it given the problem Equation (1), (2) Heterogeneous linear differential with boundary value problem can also be considered as follows

$$L(y) = f(x), \quad 0 < x < 1 \tag{5}$$

With border conditions

$$U_1(y) = 0, \; U_2(y) = 0, \; U_3(y) = 0. \tag{6}$$

the kind of frontier conditions defined for us in advance

$$U_1(y) = \alpha_1 y(0) - \beta_1 y(1) = 0$$

$$U_2(y) = \alpha_2 y(0) - \beta_2 y(1) = 0$$

$$U_3(y) = \alpha_3 y(0) - \beta_3 y(1) = 0$$

we can say that we can solve the equation and function of Green (5), (6) using differential operators as follows

$$y(x,t) = (L_0 - \lambda I)^{-1} f = \int_0^1 G_0(x,t,\lambda) f(t) dt$$

where

$$G_0(x,t,\lambda) = -\frac{\begin{vmatrix} y_1(x,\lambda) & y_2(x,\lambda) & y_3(x,\lambda) & g(x,t) \\ U_1(y_1) & U_1(y_2) & U_1(y_3) & U_1(g) \\ U_2(y_1) & U_2(y_2) & U_2(y_3) & U_2(g) \\ U_3(y_1) & U_3(y_2) & U_3(y_3) & U_3(g) \end{vmatrix}}{\begin{vmatrix} U_1(y_1) & U_1(y_2) & U_1(y_3) \\ U_2(y_1) & U_2(y_2) & U_2(y_3) \\ U_3(y_1) & U_3(y_2) & U_3(y_3) \end{vmatrix}}$$

$G_0(x,t,\lambda)$ – is a Green function.

If $x > t$ function $g(x,t)$ It has the following form that has been proven before

$$g(x,t) = \begin{vmatrix} y_1(t) & y_2(t) & y_3(t) \\ y_1^{(1)}(t) & y_2^{(1)}(t) & y_3^{(1)}(t) \\ y_1(x) & y_1(x) & y_1(x) \end{vmatrix}$$

If $x \leq t$ then$g(x,t) = 0$.

$$\Delta_0(\lambda) = \begin{vmatrix} U_1(y_1) & U_1(y_2) & U_1(y_3) \\ U_2(y_1) & U_2(y_2) & U_2(y_3) \\ U_3(y_1) & U_3(y_2) & U_3(y_3) \end{vmatrix}$$

5 Discussion

Since we have obtained the Green function of the solution of the third-order heterogeneous linear differential equation, everything in this system is technically solvable. To solve that, we proposed the proposed method and showed that the third-order heterogeneous linear differential equation with the boundary problem does not have a solution but has an infinite solution.

6 Conclusion

From the subject of the research, it is concluded that the problem we studied in the third-order heterogeneous linear differential equations is a set of Green's function. Every real function holds in the solution of the set of linear differential equations, and such equations not only a definite solution but also infinitely solvable. Its field of application in physics, for example, finding the temperature at all points of an iron rod with one end at absolute zero and the other at the freezing point of water, is a boundary value problem. If the problem depends on both place and time, the value of the problem can be specified at a certain point for all times or at a certain time for the whole space, and another example of a linear differential equation with boundary conditions can be given. The boundary condition that specifies the value of the function is the Dirichlet boundary condition. For example, if one end of an iron bar is kept at absolute zero, then the problem value at that point in space is specified.

7 Result

Based on our findings and analysis of our research we found that the proposed problem does not have a solution for each parameter λ a solution can be obtained. So the proposed problem for each parameter λ has an infinite Green function.

References

[1] Berezin F.A., Faddeev L.D. "Zamechanie ob uravnenii Shredingera s singulyarnyim potentsialom [Remark on the Scrodinger equation with singular potential]", *Izv. Vuzov. Dokl.* 2 (1961): 372–375.

[2] Berezin F.A. "O modeli Li [About Lee 's model]", *Matematicheskiy sbornik* 4 (60) (1963): 425–446.

[3] Goloschapova N.I., Zastavnyiy V.P., Malamud M.M. "Polozhitelno opredelennyie funktsii i spektralnyie svoystva operatora Shredingera s tochechnyimi vzaimodeystviyami [Positive definite functions and spectral properties of the Schrodinger operator with point interactions]", *Matematicheskie zametki* 1 (90) (2011): 149–154.

[4] Kanguzhin B.E., Dairbaeva G., Madibayulyi Zh. "Identifikatsiya granichnyih usloviy differentsialnogo operatora [Identification of boundary conditions of a differential operator]", *Vestnik KazNU, ser. mat., mech., inf.* 3 (103) (2019): 13-18

[5] Keselman G.M. "Bezuslovnaya shodimost razlozheniy po sobstvennyim funktsiyam nekotoryih differentsialnyih operatorov [Unconditional convergence of expansions in eigenfunctions of some differential operators]", *Izv. Universitetyi SSSR, ser. mat.* 2 (2017): 82-93.

[6] Kanguzhin B.E., Aniyarov A.A. "Korrektnaya zadacha dlya operatora Laplasa v prokolotom diske [Well-Posed Problem for the Laplace Operator in a Punctured Disk]", *Matem. zametki* 6 (89) (2011): 856–867.

[7] Kokebaev B.K., Otelbaev M., Shyinyibekov A.N. "K voprosam rasshireniy i ogranicheniy operatorov [On the questions of extensions and restrictions of operators]", *Dokl. Akademiya nauk SSSR* 6 (271) (1983): 1307-1313.

[8] Landau L.D., Lifshits E.M. *Kurs teoreticheskoy fiziki* [Course of theoretical physics] (M.: Nauka, 1989).

[9] Mihaylov V.P. "O bazah Rissa [About Riesz bases]", *Dokl. Akad. AN SSSR* 2 (144) (2018): 981-984.

[10] Minlos R.A., Faddeev L.D. "Tochechnoe vzaimodeystvie dlya trehchastichnoy sistemyi v kvantovoy mehanike [Point interaction for a three-particle system in quantum mechanics]", *Dokl. Akad. AN SSSR* 6 (141) (1961): 1335–1338.

[11] Neyman-Zade M.I., Shkalikov A.A. "Operatoryi Schrödingera s singulyarnyimi potentsialami iz prostranstv multiplikatorov [Schrödinger operators with singular potentials from spaces of multipliers]", *Matematicheskie zametki* 5 (66) (1999): 599–607.

[12] Naymark M.A. *Lineynyie differentsialnyie operatory* [Linear differential operators] (M., 2019): 528.

[13] Neyman-Zade M.I., Savchuk A.M. "Operatoryi Schrödingera s singulyarnyimi potentsialami [Schrödinger Operators with Singular Potentials]", *Tr. Mat. inst. im. V.A. Steklova* 236 (2002): 262–371.

[14] Shkalikov A.A. "O bazisnosti sobstvennyih funktsiy obyiknovennyih differentsialnyih operatorov s integralnyimi granichnyimi usloviyami [On the basis property of the eigenfunctions of ordinary differential operators with integral boundary conditions]", *Vestnik MGU. Ser. Mat. Meh.* 6 (2019): 12-21.