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IDENTIFICATION OF THE RIGHT HAND SIDE OF A QUASILINEAR PSEUDOPARABOLIC EQUATION WITH MEMORY TERM

The study of equations of mathematical physics, including inverse problems, is relevant today. This work is devoted to the fundamental problem of studying the solvability and qualitative properties of the solution of the inverse problem for a quasilinear pseudoparabolic equation (also called Sobolev-type equations) with memory term. To date, studies of direct and inverse problems for a pseudoparabolic equations are rapidly developing in connection with the needs of modeling and control of processes in thermal physics, hydrodynamics, and mechanics of a continuous medium. The pseudoparabolic equations similar to those considered in this work arise in the description of heat and mass transfer processes, processes of non-Newtonian fluids motion, wave processes, and in many other areas.

The main types of the inverse problems are: boundary, retrospective, coefficient and geometric. The boundary and retrospective inverse problems lead to the study of linear problems. In turn, the statements related to the study of coefficient and geometric types bring to the nonlinear problems. Coefficient inverse problems are divided into two main groups: coefficient inverse problems, where the unknown is a function of one or several variables, and finite-dimensional coefficient inverse problems.

In this article the existence and uniqueness of a weak and strong solution of the inverse problem in a bounded domain are proved by the Galerkin method. Also we used Sobolev's embedding theorems, and obtained a priori estimates for the solution. Moreover, we get local and global theorems on the existence of the solution.

Key words: Pseudoparabolic equation, inverse problem, existence, uniqueness, local solvability, global solvability, non-local condition.

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Жады бар квазисызықты псевдопараболалық теңдеудің оң жағын анықтау

Математикалық физика теңдеулерін, оның ішінде кері есептерді зерттеу бүгінгі күні өзекті болып табылады. Бұл жұмыс квазисызықтық псевдопараболалық теңдеу үшін кері есептің шешімділігі мен сапалық қасиеттерін зерттеудің іргелі мәселесіне арналған (Соболев типті теңдеулер деп те аталады). Бүгінгі таңда псевдопараболалық теңдеулер үшін тура және кері есептерді зерттеу жылу физикасы, гидродинамика және үздіксіз орта механикасындағы процестерді модельдеу және басқару қажеттіліктеріне байланысты тез дамып келеді. Осы жұмыста қарастырылған псевдопараболалық теңдеулер жылу-масса алмасу процестерін, ньютондық емес сұйықтықтардың қозғалыс процестерін, толқындық процестерді және басқа да көптеген салаларды сипаттау кезінде пайда болады.

Кері есептердің негізгі түрлеріне мыналар жатады: шекаралық, ретроспективті, коэффициенттік және геометриялық. Шекаралық және ретроспективті кері есептер -сызықтық есептерді зерттеуге, ал коэффициенттік және геометриялық есептер -сызықтық емес есептерді зерттеуге алып келеді. Коэффициенттік кері есептер екі негізгі түрге бөлінеді-коэффициенттік кері есептер, онда бір немесе бірнеше айнымалылардан тәуелді функциясы белгісіз және шекті өлшемді коэффициенттік кері есептер. Мақалада Галеркин әдісімен шенелген облыстағы кері есептің әлсіз және әлді шешімінің бар және жалғыздығы дәлелденеді. Соболевтің енгізу теоремалары қолданылып, шешімнің априорлық бағалаулары алынды. Шешімнің локалді және глобалді шешімділігі туралы теоремалар алынды.
Түйін сөздер: Псевдопараболалық теңдеу, кері есеп, шешімнің бар болуы, шешімнің жалғыздығы, локалді шешімділік, глобалді шешімділік, локалді емес шарт.

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Идентификация правой части квазилинейного псевдопараболического уравнения с памятью

Исследование уравнений математической физики, в том числе обратных задач на сегодняшний день является актуальной. Эта работа посвящена фундаментальной проблеме исследованию разрешимости и качественных свойств решения обратной задачи для квазилинейного псевдопараболическогоуравнения (называемых также уравнениями соболевского типа) с памятью. На сегодняшний день исследования прямых и обратных задач для псевдопараболических уравнений бурно развиваются в связи с потребностями моделирования и управления процессами в теплофизике, гидродинамике и механике сплошной среды. Псевдопараболические уравнения подобные рассматриваемым в данной работе возникают при описании процессов тепломассопереноса, процессов движение неньютоновских жидкостей, волновых процессов и во многих других областях. К основным типам обратных задач относятся: граничные, ретроспективные, коэффициентные и геометрические. Граничные и ретроспективные обратные задачи приводят к исследованию линейных задач. В свою очередь, постановки, к которым приводит исследование коэффициентных и геометрических задач, являются нелинейными. Коэффициентные обратные задачи подразделяются на два основных вида — коэффициентные обратные задачи, в которых неизвестной является функция одной или нескольких переменных, и конечномерные коэффициентные обратные задачи. В статье методом Галеркина доказывается существование и единственность слабого и сильного решения обратной задачи в ограниченной области. Использование теорем вложения Соболева, получены априорные оценки решения. Получены локальная и глобальная теорема о существовании решения.
Ключевые слова: Псевдопараболическое уравнение, обратная задача, существования, единственность, локальная разрешимость, глобальная разрешимость, нелокальное условие.

1 Introduction

Let Ω is a bounded area of a space $R^N,\ N \geq 1$ with a sufficiently smooth boundary Γ, Q_T is a cylinder $\Omega \times (0,T)$ of finite height T, $S = \Gamma \times (0,T)$. Let $b(x,t),\ h(x,t),\ u_0(x),\ \varphi(t), \omega(x)$ are given functions, χ, a and β are positive constants. Consider an inverse problem in the cylinder Q_T for a pseudoparabolic equation with a nonlocal overdetermination condition. Find a pair of functions $\{u(x,t),\ f(t)\}$ that satisfy:

$$u_t - \chi\Delta u_t - a\Delta u - \int\limits_0^t g(t-\tau)\Delta u(\tau)d\tau = b(x,t)|u|^{\beta-2}u + f(t)h(x,t), \tag{1}$$

$$u(x,0) = u_0(x), \tag{2}$$

$$u|_S = 0, \tag{3}$$

$$\int_\Omega u(x,t)(\omega(x) - \chi\Delta\omega(x))dx = \varphi(t). \tag{4}$$

The monograph ([1] and see its references) considers a wide class of direct problems for nonlinear Sobolev-type equations. In papers [2-12] some inverse problems similar to our problem statement were studied.

Let us note some papers on inverse problems for Sobolev-type equations with the integral overdetermination condition. Yaman M. [7] obtained sufficient conditions for both destruction and stability of the solution. The work [8] establishes an existence theorem for regular solutions of the inverse problem of recovering coefficients in equations of composite type. A.I. Kozhanov and L.A. Teleshova [9] have proved the existence theorem for regular solutions of a nonlinear inverse problem for nonstationary differential equations of higher order is proved. Kozhanov A.I. and Namsaraeva G.V. [10] showed the existence and uniqueness of the regular solutions of linear inverse problem for equation of Sobolev type. The work [11] is devoted to the investigation of the inverse problem for the equation

$$u_t - \chi\Delta u_t - \Delta u = b(x,t)|u|^{\beta-2}u + f(t)h(x),$$

with the conditions (2)-(4). In this paper the existence of a weak solution is proved, the asymptotic behavior of the solutions is shown at $t \to \infty$. Moreover, sufficient conditions of finite time "blow up"of the solution are obtained. In the work [12] is dedicated to the inverse problem for a pseudoparabolic equation with p-Laplacian. In the present work, we proved the existence of a weak solution and showed the asymptotic behavior of the solutions at $t \to \infty$. We obtained sufficient conditions of finite time "blow up"of the solution, furthermore we get sufficient conditions for the disappearance (vanishing) of the solution in a finite time.

From a physical point of view, the considered initial-boundary value problem is a mathematical model of quasi-stationary processes in semiconductors and magnets allowing for a wide variety of physical factors.

Let the functions $h(x,t)$, $\omega(x)$, $\varphi(t)$, $u_0(x)$ satisfy the following conditions:

$$\begin{aligned} &h_1(t) \equiv \int_\Omega h(x,t)\omega(x)dx \neq 0, \ \ \forall t \in [0,T], \\ &h(x,t) \in L_\infty(0,T;\ L_2(\Omega)) \cap L_2(Q_T) \cap L_{\beta*}(Q_T), \ \ \beta* = \tfrac{\beta}{\beta-1}, \ \ \beta \geq 2. \end{aligned} \tag{5}$$

$$\omega \in L_2(\Omega) \cap L_\beta(\Omega) \cap \overset{0}{W}{}_2^2(\Omega), \ \ \beta \geq 2. \tag{6}$$

$$\int_\Omega u_0(x)\omega(x)dx = \varphi(0), \ \ \varphi(t) \in W_2^1[0,T], \ \ |\varphi'(t)| \leq C, \ \ u_0 \in \overset{0}{W}{}_2^1(\Omega) \cap L_\beta(\Omega). \tag{7}$$

2 Main results

2.1 Reducing the inverse problem (1)-(4) to a direct problem

Lemma 1 *The problem* (1) - (4) *is equivalent to the following problem for a nonlinear pseudoparabolic equation containing a nonlinear nonlocal operator of the function* $u(x,t)$

$$u_t - \chi\Delta u_t - a\Delta u - \int_0^t g(t-\tau)\Delta u(\tau)d\tau = b(x,t)|u|^{\beta-2}u + F(t,u)h(x,t), \ \ x \in \Omega, \ \ \ t > 0, \tag{8}$$

$$u(x,0) = u_0(x), \ \ x \in \Omega, \ \ \ \ u|_S = 0. \tag{9}$$

Here

$$F(t,u) = \frac{1}{h_1(t)}\left(\varphi'(t) + a\int_\Omega \nabla u\nabla\omega dx + \int_0^t g(t-\tau)\int_\Omega \nabla u(\tau)\nabla\omega dxd\tau - \int_\Omega b(x,t)|u|^{\beta-2}u\omega dx\right). \tag{10}$$

Proof. Indeed, from the equation (1) follows that

$$\begin{aligned}&\int\limits_{\Omega} u_t\omega dx-\chi\int\limits_{\Omega}\Delta u_t\omega dx-a\int\limits_{\Omega}\Delta u\omega dx-\int\limits_{\Omega}\int\limits_0^t g(t-\tau)\Delta u(\tau)d\tau\omega dx=\\&=\int\limits_{\Omega} b(x,t)|u|^{\beta-2}u\omega dx+\int\limits_{\Omega} f(t)h(x,t)\omega dx,\end{aligned}\tag{11}$$

next if conditions (4) and (5) are satisfied, then

$$F(t,u)=\frac{1}{h_1(t)}\left(\varphi'(t)+a\int\limits_{\Omega}\nabla u\nabla\omega dx+\int\limits_0^t g(t-\tau)\int\limits_{\Omega}\nabla u(\tau)\nabla\omega dxd\tau-\int\limits_{\Omega} b(x,t)|u|^{\beta-2}u\omega dx\right).\tag{12}$$

Therefore the relation (10) is fulfilled. Now consider the problem (8)-(9). If the relation (10) is performed, then it obviously leads to the equality (12) . Then

$$\begin{aligned}&F(t,u)=\tfrac{1}{h_1}\left(\varphi'(t)+a\int\limits_{\Omega}\nabla u\nabla\omega dx+\int\limits_0^t g(t-\tau)\int\limits_{\Omega}\nabla u(\tau)\nabla\omega dxd\tau-\int\limits_{\Omega} b(x,t)|u|^{\beta-2}u\omega dx\right)=\\&=\tfrac{1}{h_1}\left(\varphi'(t)-a\int\limits_{\Omega}\Delta u\omega dx-\int\limits_0^t g(t-\tau)\int\limits_{\Omega}\Delta u(\tau)\omega dxd\tau-\int\limits_{\Omega} b(x,t)|u|^{\beta-2}u\omega dx\right).\end{aligned}$$

By virtue of (11) we obtain that

$$\begin{aligned}&F(t,u)=\tfrac{1}{h_1}\left(\varphi'(t)+a\int\limits_{\Omega}\nabla u\nabla\omega dx+\int\limits_0^t g(t-\tau)\int\limits_{\Omega}\nabla u(\tau)\nabla\omega dxd\tau-\int\limits_{\Omega} b(x,t)|u|^{\beta-2}u\omega dx\right)=\\&=\tfrac{1}{h_1}\left(\varphi'(t)-\int\limits_{\Omega}(u_t-\chi\Delta u_t)\omega dx+\int\limits_{\Omega} b(x,t)|u|^{\beta-2}u\omega dx+\right.\\&\left.+\int\limits_{\Omega} f(t)h(x,t)\omega dx+\int\limits_{\Omega} b(x,t)|u|^{\beta-2}u\omega dx\right),\end{aligned}$$

$$\varphi'(t)-\int\limits_{\Omega} u_t(\omega-\chi\Delta\omega)dx=0.$$

In this way, $\frac{d}{dt}\left(\varphi(t)-\int\limits_{\Omega} u(\omega-\chi\Delta\omega)dx\right)=0$. We denote by $v(t)=\varphi(t)-\int\limits_{\Omega} u(\omega-\chi\Delta\omega)dx$. Then the function $v(t)$ can be found as a solution of the Cauchy problem: $v'(t)=0,\ \ v(0)=0$. ($v(0)=0$ follows from the agreement condition (7)). The unique solution of the problem is the function $v(t)=0$, consequently, $\int_\Omega u(\omega-\chi\Delta\omega)dx=\varphi(t)$.

Definition 1 *A function $u(x,t)$ from the space $W_2^1(0,T;\overset{0}{W}{}_2^1(\Omega))$ is called a weak solution of the problem* (8)-(9) *which satisfies the integral identity*

$$\begin{aligned}&\int\limits_0^T\int\limits_{\Omega}\left(\tfrac{\partial u}{\partial t}v+\chi\nabla u_t\nabla v+a\nabla u\nabla v\right)dxdt+\int\limits_0^T\int\limits_0^t g(t-\tau)\int\limits_{\Omega}\nabla u(\tau)\nabla v(t)dxd\tau dt-\\&-\int\limits_0^T\int\limits_{\Omega} b(x,t)|u|^{\beta-2}uvdxdt=\int\limits_0^T\int\limits_{\Omega} F(t,u)hvdxdt,\end{aligned}\tag{13}$$

for all $v(x,t)\in L_2(0,T;\overset{0}{W}{}_2^1(\Omega))$.

Definition 2 *A function $u(x,t)$ from the space $u_t,\ \triangle u,\ \triangle u_t,\in L_2(Q_T)$, is called a strong solution of the problem* (8)-(9) *which satisfies the integral identity*

$$\begin{aligned}&\int\limits_0^T\int\limits_{\Omega}\left(\tfrac{\partial u}{\partial t}v-\chi\triangle u_tv-a\triangle uv\right)dxdt+\int\limits_0^T\int\limits_0^t g(t-\tau)\int\limits_{\Omega}\triangle u(\tau)v(t)dxd\tau dt-\\&-\int\limits_0^T\int\limits_{\Omega} b(x,t)|u|^{\beta-2}uvdxdt=\int\limits_0^T\int\limits_{\Omega} F(t,u)hvdxdt,\end{aligned}\tag{14}$$

for all $v(x,t)\in L_2(Q_t)$.

2.2 Existence of a weak solution

Theorem 1 *Let the conditions* (5)-(7) *are performed, and also* $2 < \beta < \frac{2N}{N-2}$, $N \geq 3$. *Then there is a weak solution* $u(x,t)$ *of the problem* (8)-(9) *on the interval* $(0,T)$, $T < T_0$, *and besides accepts the following inclusions:*

$$u \in L_\infty(0,T;\overset{0}{W}{}_2^1(\Omega)), \nabla u \in L_2(Q_T),\ Q_T = \Omega \times (0,T),$$

$$u_t \in L_2(0,T;\overset{0}{W}{}_2^1(\Omega)), |u|^{\beta-2}u \in L_\infty(0,T;L_{\frac{\beta}{\beta-1}}(\Gamma)).$$

Proof. Let us choose in $\overset{0}{W}{}_2^1(\Omega)$ some system of functions $\{\Psi_j(x)\}$ forming a basis in a given space $(\Delta\Psi + \lambda\Psi = 0,\ \Psi|_\Gamma = 0)$. We will look for an approximate solution of the problem (8)-(9) in the form

$$u_m(x,t) = \sum_{k=1}^{m} C_{mk}(t)\Psi_k(x) \tag{15}$$

where the coefficients $C_{mk}(t)$ are searched out from the conditions

$$\begin{aligned} &\sum_{k=1}^{m} C'_{mk}(t)\int_\Omega \left[\Psi_k\Psi_j + \chi\sum_{i=1}^{m}\frac{\partial\Psi_k}{\partial x_i}\cdot\frac{\partial\Psi_j}{\partial x_i}\right]dx + a\sum_{k=1}^{m} C_{mk}(t)\int_\Omega \nabla\Psi_k\nabla\Psi_j dx + \\ &+\sum_{k=1}^{m}\int_0^t g(t-\tau)C_{mk}(\tau)\int_\Omega \nabla\Psi_k\nabla\Psi_j dx d\tau - \\ &-\sum_{k=1}^{m} C_{mk}(t)\int_\Omega b(x,t)|u_m|^{\beta-2}\Psi_k\Psi_j dx = \int_\Omega F(t,u_m)h(x,t)\Psi_j dx. \end{aligned} \tag{16}$$

$$u_{m0} = u_m(0) = \sum_{k=1}^{m} C_{mk}(0)\Psi_k = \sum_{k=1}^{m}\alpha_k\Psi_k \tag{17}$$

and besides

$$u_{m0} \to u_0 \quad strongly \quad in \quad \overset{0}{W}{}_2^1(\Omega) \quad at \quad m\to\infty \tag{18}$$

We introduce a notation

$$\vec{C}_m \equiv \{C_{1m}(t),...,C_{mm}(t)\}^T, \vec{\alpha} \equiv \{\alpha_1,...,\alpha_m\}^T,$$

$$a_{kj} = \int_\Omega [\Psi_k\Psi_j + \chi(\nabla\Psi_k,\nabla\Psi_j)]\,dx, b_{kj} = \int_\Omega \nabla\Psi_k\nabla\Psi_j dx,$$

$$f_{kj} = -a\int_\Omega \nabla\Psi_k\nabla\Psi_j dx + \int_\Omega b(x,t)\,|u_m|^{\beta-2}\Psi_k\Psi_j dx + \int_\Omega F(t,u_m)h(x,t)\Psi_j dx,$$

$$A_m\left(\vec{C}_m\right) \equiv \left\{a_{jk}\left(\vec{C}_m\right)\right\}, B_m\left(\vec{C}_m\right) \equiv \left\{b_{jk}\left(\vec{C}_m\right)\right\}, \vec{F}_m\left(\vec{C}_m\right) \equiv \left\{f_{jk}\left(\vec{C}_m\right)\right\}\vec{C}_m.$$

Then the system of equations (16) and condition (17) take the matrix form.

$$A_m\vec{C}'_m + \int_0^t g(t-\tau)B_m C_m(\tau)d\tau \equiv \vec{F}_m\left(\vec{C}_m\right),\quad \vec{C}_m(0) = \vec{\alpha}. \tag{19}$$

According to the Cauchy theorem, the problem (19) has at least one solution $\vec{C}_m$ on a certain time interval $t \in (0,T_m)$, $T_m > 0$. Below we obtain a priori estimates for u_m which is independent of m and, in some cases, valid for any finite t.

2.3 A priori estimates

To get the first estimate, we multiply both sides of the equality (16) by $C_{mj}(t)$ and summarize both sides of the obtained equality over $j=\overline{1,m}$. As a result, we get the equality

$$\begin{gathered}\frac{1}{2}\frac{d}{dt}\int\limits_{\Omega}[|u_m|^2+\chi|\nabla u_m|^2]dx+a\int\limits_{\Omega}|\nabla u_m|^2dx+\\+\int\limits_0^t g(t-\tau)\int\limits_{\Omega}\nabla u_m(\tau)\nabla u_m(t)dxd\tau=\int\limits_{\Omega}b(x,t)|u_m|^{\beta}dx+\int\limits_{\Omega}F(t,u_m)hu_mdx.\end{gathered}\tag{20}$$

Lemma 2 *If* $u\in\overset{0}{W}{}_2^1(\Omega),\ 2<\beta<\frac{2N}{N-2},\ \ N\geq3,$ *then the next inequality is performed*

$$\|u\|_{\beta,\Omega}^2\leq C_0^2\,\|\nabla u\|_{2,\Omega}^{2\alpha}\,\|u\|_{2,\Omega}^{2(1-\alpha)}\leq\chi\,\|\nabla u\|_{2,\Omega}^2+\frac{(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}C_0^{\frac{2}{1-\alpha}}}{\chi^{\frac{\alpha}{1-\alpha}}}\,\|u\|_{2,\Omega}^2\,,$$

where $C_0=\left(\frac{2(N-1)}{N-2}\right)^{\alpha},\ \ \alpha=\frac{(\beta-2)N}{2\beta},\ \ 0<\alpha<1.$

From the lemma 2 follows the inequality

$$\|u\|_{\beta,\Omega}^{\beta}\leq C_1\left(\|u\|_{2,\Omega}^2+\chi\,\|\nabla u\|_{2,\Omega}^2\right)^{\frac{\beta}{2}},$$

where $C_1=\left(\max\left\{1;\ \frac{(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}C_0^{\frac{2}{1-\alpha}}}{\chi^{\frac{\alpha}{1-\alpha}}}\right\}\right)^{\frac{\beta}{2}}$.

We estimate the right hand side of (20)

$$\left|\int\limits_{\Omega}b(x,t)|u_m|^{\beta}dx\right|\leq b_0\,\|u_m\|_{\beta,\Omega}^{\beta}\leq C_1b_0\left(\|u_m\|_{2,\Omega}^2+\chi\,\|\nabla u_m\|_{2,\Omega}^2\right)^{\frac{\beta}{2}},\tag{21}$$

$$\left|\frac{\varphi'(t)}{h_1(t)}\int\limits_{\Omega}hu_mdx\right|\leq\frac{|\varphi'(t)|}{|h_1(t)|}\,\|h\|_{2,\Omega}\,\|u_m\|_{2,\Omega}\leq\frac{1}{4}\sup_{0\leq t\leq T}\frac{|\varphi'(t)|^2}{|h_1(t)|^2}\,\|h\|_{2,\Omega}^2+\|u_m\|_{2,\Omega}^2\,,\tag{22}$$

$$\begin{gathered}\left|\frac{a}{h_1(t)}\int\limits_{\Omega}hu_mdx\int\limits_0^t g(t-\tau)\int\limits_{\Omega}\nabla u_m(\tau)\nabla\omega dxd\tau\right|\leq\\\leq\frac{a}{|h_1(t)|}\,\|h(x,t)\|_{2,\Omega}\,\|u_m\|_{2,\Omega}\int\limits_0^t g(t-\tau)\,\|\nabla u_m(\tau)\|_{2,\Omega}\,\|\nabla\omega\|_{2,\Omega}\,d\tau\leq\\\leq\frac{a^2}{4}\left(\|\nabla\omega\|_{2,\Omega}\sup_{0\leq t\leq T}\frac{1}{|h_1(t)|}\,\|h(x,t)\|_{2,\Omega}\right)^2\int\limits_0^t\|\nabla u_m(\tau)\|_{2,\Omega}^2\,d\tau+\|u_m\|_{2,\Omega}^2\,.\end{gathered}\tag{23}$$

$$\begin{gathered}\left|\int\limits_0^t g(t-\tau)\int\limits_{\Omega}\nabla u_m(\tau)\nabla u_m(t)dxd\tau\right|\leq\int\limits_0^t g(t-\tau)\,\|\nabla u_m(\tau)\|_{2,\Omega}\,\|\nabla u_m(t)\|_{2,\Omega}\,d\tau\leq\\\leq\frac{g_0}{2a}\int\limits_0^t\|\nabla u_m(\tau)\|_{2,\Omega}^2\,d\tau+\frac{a}{2}\,\|\nabla u_m(t)\|_{2,\Omega}^2\,.\end{gathered}$$

$$\begin{gathered}\left|\frac{1}{h_1(t)}\int\limits_{\Omega}hu_mdx\int\limits_{\Omega}b(x,t)|u_m|^{\beta-2}u_m\cdot\omega dx\right|\leq\frac{b_0}{|h_1(t)|}\,\|u_m\|_{\beta,\Omega}^{\beta}\,\|\omega\|_{\beta,\Omega}\,\|h\|_{\frac{\beta}{\beta-1},\Omega}\leq\\\leq C_1b_0\,\|\omega\|_{\beta,\Omega}\sup_{0\leq t\leq T}\frac{1}{h_1^2(t)}\,\|h(x,t)\|_{\frac{\beta}{\beta-1},\Omega}\left(\|\nabla u_m\|_{2,\Omega}^2+C_1\,\|u_m\|_{2,\Omega}^2\right)^{\frac{\beta}{2}}.\end{gathered}\tag{24}$$

$$\begin{aligned}&\frac{d}{dt}\int\limits_{\Omega}[|u_m|^2+\chi|\nabla u_m|^2]dx+a\int\limits_{\Omega}|\nabla u_m|^2dx\leq\\&\leq C_2\left(\|u_m\|_{2,\Omega}^2+\chi\|\nabla u_m\|_{2,\Omega}^2\right)+C_3\left(\|u_m\|_{2,\Omega}^2+\chi\|\nabla u_m\|_{2,\Omega}^2\right)^{\frac{\beta}{2}}+\\&+C_4\int\limits_0^t\|\nabla u_m(\tau)\|_{2,\Omega}^2\,d\tau+\frac{1}{4}\sup_{0\leq t\leq T}\frac{|\varphi'(t)|^2}{|h_1(t)|^2}\|h\|_{2,\Omega}^2\,.\end{aligned}\tag{25}$$

We denote by $y(t)\equiv\chi\|\nabla u\|_{2,\Omega}^2+\|u\|_{2,\Omega}^2$, then (25) takes the form

$$\frac{dy(t)}{dt}\leq C_4\int\limits_0^t y(\tau)d\tau+C_3[y(t)]^{\frac{\beta}{2}}+C_2y(t)+\frac{1}{4}\sup_{0\leq t\leq T}\frac{|\varphi'(t)|^2}{|h_1(t)|^2}\|h\|_{2,\Omega}^2\,.$$

By integrating from 0 to t, we get

$$y(t)\leq y(0)+\frac{1}{4}\sup_{0\leq t\leq T}\frac{|\varphi'(t)|^2}{|h_1(t)|^2}\int\limits_0^t\|h\|_{2,\Omega}^2\,d\tau+C_5(t^{\frac{2(\beta-1)}{\beta-2}}+t)+C_6\int\limits_0^t[y(\tau)]^{\frac{\beta}{2}}d\tau,$$

Applying the Bihari lemma to (25), if

$$\frac{C_4}{C_3}\left(e^{C_3\frac{p-2}{2}t}-1\right)<\frac{1}{\left(z(0)+\frac{C_2}{C_3}+\frac{C_0}{2(C_3-\gamma)}\right)^{\frac{p-2}{2}}},\quad 0\leq t<T,$$

then the next inequality is true

$$z(t)\leq\frac{z(0)+\frac{C_2}{C_3}+\frac{C_0}{2(C_3-\gamma)}}{\left[1-\left(z(0)+\frac{C_2}{C_3}+\frac{C_0}{2(C_3-\gamma)}\right)^{\frac{p-2}{2}}\frac{C_4}{C_3}\left(e^{C_3\frac{p-2}{2}t}-1\right)\right]^{\frac{2}{p-2}}}.$$

$$\begin{aligned}&\chi\|\nabla u_m\|_{2,\Omega}^2+\|u_m\|_{2,\Omega}^2\leq\\&\leq\frac{\left(\chi\|\nabla u_m(x,0)\|_{2,\Omega}^2+\|u_m(x,0)\|_{2,\Omega}^2+C\right)e^{C_3t}}{\left[1-\left(\chi\|\nabla u_m(x,0)\|_{2,\Omega}^2+\|u_m(x,0)\|_{2,\Omega}^2+C\right)^{\frac{\beta-2}{2}}\frac{C_4}{C_3}\left(e^{C_3\frac{\beta-2}{2}t}-1\right)\right]^{\frac{2}{\beta-2}}}.\end{aligned}\tag{26}$$

From this estimate, we can conclude that there is $T_0>0$ such that

$$\|u_m\|_{2,\Omega}^2+\chi\|\nabla u_m\|_{2,\Omega}^2\leq C_5,\quad for\quad all\quad t\in[0,T],\ T<T_0,\tag{27}$$

where the constant C_5 does not depend on $m\in\mathrm{N}$.

Returning to (25) and taking into account (27), we obtain one more inequality:

$$\int\limits_0^t\int\limits_{\Omega}|\nabla u_m|^2dxdt\leq C_5.\tag{28}$$

Now we multiply equality (16) by $C'_{mj}(t)$ and summarize over $j=\overline{1,m}$. As a result, we get

$$\begin{aligned}&\|\partial_\tau u_m\|_{2,\Omega}^2+\chi\|\nabla\partial_\tau u_m\|_{2,\Omega}^2+\frac{a}{2}\frac{d}{dt}\int\limits_{\Omega}|\nabla u_m|^2dx+\\&+\int\limits_0^t g(t-\tau)\int\limits_{\Omega}\nabla u_m(\tau)\nabla\partial_t u_m(t)dxd\tau=\frac{1}{\beta}\frac{d}{dt}\int\limits_{\Omega}b(x,t)|u_m|^\beta dx-\\&-\frac{1}{\beta}\int\limits_{\Omega}b_t(x,t)|u_m|^\beta dx+\int\limits_{\Omega}F(t,u_m)h(x,t)\partial_t u_m dx.\end{aligned}\tag{29}$$

We integrate with respect to τ from 0 to t, then we get the relation

$$\begin{gathered}\int_0^t\left(\|\partial_\tau u_m\|_{2,\Omega}^2+\chi\|\nabla\partial_\tau u_m\|_{2,\Omega}^2\right)d\tau+\frac{a}{2}\int_\Omega|\nabla u_m|^2dx=\frac{1}{2}\int_\Omega|\nabla u_m(x,0)|^2dx-\\-\frac{1}{\beta}\int_\Omega b(x,0)|u_m(x,0)|^\beta dx+\frac{1}{\beta}\int_\Omega b(x,t)|u_m|^\beta dx-\\-\frac{1}{\beta}\int_0^t\int_\Omega b_\tau(x,\tau)|u_m|^\beta dxd\tau-\int_0^t\int_0^\tau g(\tau-s)\int_\Gamma\nabla u_m(s)\nabla\partial_\tau u_m(\tau)d\Gamma dsd\tau+\\+\int_0^t\int_\Omega F(t,u_m)h(x,t)\partial_\tau u_m dxd\tau.\end{gathered}\tag{30}$$

We estimate the right hand side of (30):

$$\begin{gathered}\left|\frac{1}{\beta}\int_\Omega b(x,t)|u_m|^\beta dx\right|\le\frac{b_0}{\beta}\|u_m\|_{\beta,\Omega}^\beta\le\\\le\frac{C_1b_0}{\beta}\left(\|u_m\|_{2,\Omega}^2+\chi\|\nabla u_m\|_{2,\Omega}^2\right)^{\frac{\beta}{2}}\le\frac{C_1b_0}{\beta}C_5^{\frac{\beta}{2}},\end{gathered}\tag{31}$$

$$\begin{gathered}\left|\frac{1}{\beta}\int_0^t\int_\Omega b_\tau(x,\tau)|u_m|^\beta dxd\tau\right|\le\frac{b_0}{\beta}\int_0^t\|u_m\|_{\beta,\Omega}^\beta d\tau\le\\\le\frac{C_1b_0}{\beta}\int_0^t\left(\|u_m\|_{2,\Omega}^2+\chi\|\nabla u_m\|_{2,\Omega}^2\right)^{\frac{\beta}{2}}d\tau\le\frac{C_1b_0}{\beta}C_5^{\frac{\beta}{2}}t,\end{gathered}\tag{32}$$

$$\begin{gathered}\left|\int_0^t\int_\Omega\frac{\varphi'(\tau)}{h_1(\tau)}h(x,\tau)\partial_\tau u_m dxd\tau\right|\le\\\le\frac{3}{2}\int_0^t\int_\Omega\left|\frac{\varphi'(\tau)h(x,\tau)}{h_1(\tau)}\right|^2dxd\tau+\frac{1}{6}\int_0^t\int_\Omega|\partial_\tau u_m|^2dxd\tau.\end{gathered}\tag{33}$$

$$\begin{gathered}\left|\int_0^t\int_\Omega\frac{h(x,\tau)}{h_1(\tau)}\partial_\tau u_m dx\int_\Omega b(x,\tau)|u|^{\beta-2}u\omega dxd\tau\right|\le\\\le\frac{2}{3}C_1^{\frac{2\beta-2}{\beta}}\left(b_0\|\omega\|_{\beta,\Omega}\sup_{0\le t\le T}\frac{\|h(x,t)\|_{2,\Omega}}{h_1(t)}\right)^2\int_0^t\left(\|u_m\|_{2,\Omega}^2+\chi\|\nabla u_m\|_{2,\Omega}^2\right)^{\frac{2\beta-2}{\beta}}d\tau+\\+\frac{1}{6}\int_0^t\|\partial_\tau u_m\|_{2,\Omega}^2d\tau\le\frac{1}{6}\int_0^t\|\partial_\tau u_m\|_{2,\Omega}^2d\tau+\\+\frac{2}{3}C_1^{\frac{2\beta-2}{\beta}}\left(b_0\|\omega\|_{\beta,\Omega}\sup_{0\le t\le T}\frac{\|h(x,t)\|_{2,\Omega}}{h_1(t)}\right)^2C_5^{\frac{2\beta-2}{\beta}}t.\end{gathered}\tag{34}$$

$$\begin{gathered}\left|\int_0^t\int_0^\tau g(\tau-s)\int_\Gamma\nabla u_m(s)\nabla\partial_\tau u_m(\tau)d\Gamma dsd\tau\right|\le\int_0^t\int_0^\tau g(\tau-s)\|\nabla u_m(s)\|_{2,\Omega}\|\nabla\partial_\tau u_m(\tau)\|_{2,\Omega}d\tau\le\\\le\frac{g_0}{2}\int_0^t(t-\tau)\|\nabla u_m(\tau)\|_{2,\Omega}^2d\tau+\frac{1}{2}\int_0^t\|\nabla\partial_\tau u_m(\tau)\|_{2,\Omega}^2d\tau.\end{gathered}\tag{35}$$

$$\begin{gathered}\left|a\int_0^t\int_\Omega\frac{h(x,\tau)}{h_1(\tau)}\partial_\tau u_m dx\int_0^\tau g(\tau-s)\int_\Omega\nabla u_m(s)\nabla\omega dxdsd\tau\right|\le\\\le\frac{1}{6}\int_0^t\|\partial_\tau u_m\|_{2,\Omega}^2d\tau+\frac{2g_0}{3}\left(a\|\nabla\omega\|_{2,\Omega}\sup_{0\le t\le T}\frac{\|h(x,t)\|_{2,\Omega}}{|h_1(t)|}\right)^2\int_0^t(t-\tau)\|\nabla u_m(\tau)\|_{2,\Omega}^2d\tau.\end{gathered}\tag{36}$$

We substitute the obtained inequalities into the identity (30), and from the estimate (27), we get the second estimate

$$\int_0^t\left(\|\partial_\tau u_m\|_{2,\Omega}^2+\chi\|\nabla\partial_\tau u_m\|_{2,\Omega}^2\right)d\tau\le C_6.\tag{37}$$

2.4 Passage to the limit

From the obtained estimates (27), (28), (31) imply the next assertions respectively:

$$u_m \quad is \quad bounded \quad in \quad L_\infty(0,T;\overset{0}{W}{}_2^1(\Omega)), \tag{38}$$

$$\nabla u_m \quad is \quad bounded \quad in \quad L_2(Q_T),\ Q_T=\Omega\times(0,T), \tag{39}$$

$$\partial_t u_m \quad is \quad bounded \quad in \quad L_2(0,T;\overset{0}{W}{}_2^1(\Omega)), \tag{40}$$

In addition, by virtue of the conditions which set for β:

$$|u_m|^{\beta-2}u_m \quad is \quad bounded \quad in \quad L_\infty(0,T;L_{\frac{\beta}{\beta-1}}(\Omega)), 2<\beta<\frac{2N}{N-2},\ \ N\geq 3. \tag{41}$$

From (38) follows that there exists a subsequence u_{m_k} of the sequence u_m, *-weakly converging to some element $u\in L_\infty(0,T;\overset{0}{W}{}_2^1(\Omega))$, that is $u_{m_k} \ \rightarrow\ u$ *-weakly in $L_\infty(0,T;\overset{0}{W}{}_2^1(\Omega))$. Similarly, from (39)-(41) follows that there exists a sequence such that $\{u_{m_k}\}\subset\{u_m\}$, that $u_{m_k}\ \rightarrow\ u$ weakly in $L_2(0,T;\overset{0}{W}{}_2^1(\Omega))$. By virtue of the Rellich-Kondrashov theorem, the embedding $W_2^1(Q_T)$ into $L_2(Q_T)$ is a compact. This means that we can choose the sequence u_{m_k} in such way that $u_{m_k}\ \rightarrow\ u$ in the norm of $L_2(Q_T)$, therefore it converges almost everywhere [13].

The reasoning above allows us to pass to the limit in (16). But first, we multiply each of the equality (16) by $d_j(t)\in C[0,T]$ and summarize both sides of the obtained equality over $j=\overline{1,m}$. Then we integrate with respect to t from 0 to T, and get

$$\begin{aligned}&\int_0^t\int_\Omega\left(\tfrac{\partial u_m}{\partial t}\mu+\chi\nabla u_{mt}\nabla\mu+a\nabla u_m\nabla\mu\right)dxdt+\int_0^t\int_0^t g(t-\tau)\int_\Omega\nabla u_m(\tau)\nabla\mu(t)dxd\tau dt-\\&-\int_0^t\int_\Omega b(x,t)|u_m|^{\beta-2}u_m\mu dxdt=\int_0^t\int_\Omega F(t,u_m)h\mu dxdt,\end{aligned} \tag{42}$$

where $\mu(x,t)=\sum\limits_{j=1}^m d_j(t)\Psi_j(x)$.

Considering the obtained inclusions and convergences we pass to the limit in (42) at $m\rightarrow\infty$ and get (14) for $v=\mu$. Since the set of all functions $\mu(x,t)$ is dense in $W_2^1(0,T;\overset{0}{W}{}_2^1(\Omega))$, then the limit relation is performed for all $v(x,t)\in L_2(0,T;\overset{0}{W}{}_2^1(\Omega))$.

3 Uniqueness of the weak solution

Theorem 2 *Let* $u_0(x)\in\overset{0}{W}{}_2^1(\Omega)$, $2<\beta\leq\frac{2(N-1)}{N-2}$, $N\geq 3$ *are fulfilled .Then the weak solution of the problem* (8)-(9) *on the interval* $(0,T)$ *is unique (in the sense of Definition 1).*

Proof. Suppose that the problem (8)-(9) has two solutions: $u_1(x,t)$ and $u_2(x,t)$. Then their difference $u(x,t)=u_1(x,t)-u_2(x,t)$ satisfies the condition $u(x,0)=0$ and the identity

$$\begin{aligned}&\int_0^t\int_\Omega(u_\tau v+\chi\nabla u_\tau\nabla v+a\nabla u\nabla v)\,dxd\tau+\int_0^t\int_0^\tau g(\tau-s)\int_\Omega\nabla u(s)\nabla v(\tau)dxdsd\tau=\\&=\int_0^t\int_\Omega b(x,\tau)\left(|u_1|^{\beta-2}u_1-|u_1|^{\beta-2}u_1\right)vdxd\tau+\\&+\int_0^t\int_\Omega(F(\tau,u_1)-F(\tau,u_2))hvdxd\tau,\end{aligned}$$

By virtue of $v(x,t) \in L_2(0,T;\overset{0}{W}{}_2^1(\Omega))$, then as $v(x,t)$ we may take $u(x,t)$, that is we put $v(x,t) = u(x,t)$

$$\begin{gathered}\int\limits_0^t\int\limits_\Omega \left(u_\tau u + \chi\nabla u_\tau \nabla u + a|\nabla u|^2\right)dxd\tau + \int\limits_0^t\int\limits_0^\tau g(\tau-s)\int\limits_\Omega \nabla u(s)\nabla u(\tau)dxdsd\tau = \\ = \int\limits_0^t\int\limits_\Omega b(x,\tau)\left(|u_1|^{\beta-2}u_1 - |u_1|^{\beta-2}u_1\right)udxd\tau + \\ + \int\limits_0^t\int\limits_\Omega (F(\tau,u_1)-F(\tau,u_2))hudxd\tau.\end{gathered} \tag{43}$$

We estimate the right hand side of the inequality (43), using the following inequality $||u_1|^q u_1 - |u_2|^q u_2| \le (q+1)\left(|u_1|^q + |u_2|^q\right)|u_1-u_2|$, $q>0$.

$$\begin{gathered}\left|\int\limits_\Omega b(x,\tau)\left(|u_1|^{\beta-2}u_1 - |u_2|^{\beta-2}u_2\right)udx\right| \le b_1(\beta-1)\int\limits_\Omega\left(|u_1|^{\beta-2}+|u_2|^{\beta-2}\right)u^2dx \le \\ \le b_1(\beta-1)\left(\int\limits_\Omega\left(|u_1|^{\beta-2}+|u_2|^{\beta-2}\right)^2u^2dx\right)^{\frac12}\left(\int\limits_\Omega u^2dx\right)^{\frac12} \le \\ \le b_1(\beta-1)\left(\int\limits_\Omega\left(|u_1|^{\beta-2}+|u_2|^{\beta-2}\right)^{\frac{2r}{r-2}}dx\right)^{\frac{r-2}{2r}}\left(\int\limits_\Omega u^r dx\right)^{\frac1r}\left(\int\limits_\Omega u^2dx\right)^{\frac12} \le \\ \le b_1(\beta-1)\left(\left(\int\limits_\Omega |u_1|^{\frac{2r(\beta-2)}{r-2}}dx\right)^{\frac{r-2}{2r}} + \left(\int\limits_\Omega |u_2|^{\frac{2r(\beta-2)}{r-2}}dx\right)^{\frac{r-2}{2r}}\right)\times \\ \times\left(\int\limits_\Omega u^r dx\right)^{\frac1r}\left(\int\limits_\Omega u^2dx\right)^{\frac12}.\end{gathered}$$

We put $r=\frac{2N}{N-2}$, $2<\beta\le\frac{2(N-1)}{N-2}$, $N\ge3$. Then by the Sobolev embedding theorem $\overset{0}{W}{}_2^1(\Omega)\subset L_r(\Omega)$ and $\overset{0}{W}{}_2^1(\Omega)\subset L_{2r(\beta-2)/(r-2)}(\Omega)$.

In this case, taking into account the smoothness class of the solutions $u_1(x,t)$ and $u_2(x,t)$, we arrive at the estimate

$$\begin{gathered}\left|\int\limits_\Omega b(x,\tau)\left(|u_1|^{\beta-2}u_1 - |u_2|^{\beta-2}u_2\right)udx\right| \le \\ \le \tfrac a8\|\nabla u\|_{2,\Omega}^2 + C_{16}'\|u\|_{2,\Omega}^2 \le \tfrac a8\|\nabla u\|_{2,\Omega}^2 + C_{16}\left(\|u\|_{2,\Omega}^2+\chi\|\nabla u\|_{2,\Omega}^2\right).\end{gathered} \tag{44}$$

Analogically,

$$\begin{gathered}\left|\tfrac{1}{h_1}\int\limits_\Omega b(x,\tau)\left(|u_1|^{\beta-2}u_1 - |u_2|^{\beta-2}u_2\right)\omega dx\int\limits_\Omega hudx\right| \le \\ \le \tfrac a8\|\nabla u\|_{2,\Omega}^2 + C_{17}\left(\|u\|_{2,\Omega}^2+\chi\|\nabla u\|_{2,\Omega}^2\right).\end{gathered}$$

$$\begin{gathered}\left|\tfrac{1}{h_1}a\int\limits_\Omega \nabla u\nabla\omega dx\int\limits_\Omega hudx\right| \le \tfrac{1}{|h_1|}\|\nabla u\|_{2,\Omega}\|\nabla\omega\|_{2,\Omega}\|u\|_{2,\Omega}\|h\|_{2,\Omega} \le \\ \le \tfrac a8\|\nabla u\|_{2,\Omega}^2 + C_{18}\left(\|u\|_{2,\Omega}^2+\chi\|\nabla u\|_{2,\Omega}^2\right).\end{gathered}$$

$$\begin{gathered}\left|\tfrac{a}{h_1(t)}\int\limits_\Omega hu_m dx\int\limits_0^t g(t-\tau)\int\limits_\Omega \nabla u_m(\tau)\nabla\omega dxd\tau\right| \le \\ \le C_{19}\left(\|u\|_{2,\Omega}^2+\chi\|\nabla u\|_{2,\Omega}^2\right) + C_{20}\int\limits_0^t\left(\|u\|_{2,\Omega}^2+\chi\|\nabla u\|_{2,\Omega}^2\right)d\tau.\end{gathered}$$

$$\begin{aligned}
&\left|\int_0^t g(t-\tau)\int_\Omega \nabla u_m(\tau)\nabla u_m(t)dxd\tau\right| \le \int_0^t g(t-\tau)\left\|\nabla u_m(\tau)\right\|_{2,\Omega}\left\|\nabla u_m(t)\right\|_{2,\Omega}d\tau \le \\
&\le \left\|\nabla u_m(t)\right\|_{2,\Omega}\int_0^t g(t-\tau)\left\|\nabla u_m(\tau)\right\|_{2,\Omega}d\tau \le \\
&\le \tfrac{2}{a}\int_0^t g^2(t-\tau)d\tau\int_0^t \left\|\nabla u_m(\tau)\right\|^2_{2,\Omega}d\tau + \tfrac{a}{8}\left\|\nabla u_m(t)\right\|^2_{2,\Omega} \le \\
&\le \tfrac{2g_0}{a}\int_0^t \left\|\nabla u_m(\tau)\right\|^2_{2,\Omega}d\tau + \tfrac{a}{8}\left\|\nabla u_m(t)\right\|^2_{2,\Omega} \le \\
&\le C_{21}\left(\|u\|^2_{2,\Omega} + \chi\left\|\nabla u\right\|^2_{2,\Omega}\right) + C_{22}\int_0^t\left(\|u\|^2_{2,\Omega} + \chi\left\|\nabla u\right\|^2_{2,\Omega}\right)d\tau.
\end{aligned} \tag{45}$$

By virtue of (44)-(45), we obtain

$$\begin{aligned}
&\int_\Omega |u|^2dx + \chi\int_\Omega |\nabla u|^2dx + \tfrac{a}{2}\int_0^t\int_\Omega |\nabla u|^2dxd\tau \le \\
&\le C_{23}\int_0^t\left(\|u\|^2_{2,\Omega} + \chi\left\|\nabla u\right\|^2_{2,\Omega}\right)d\tau + C_{24}\int_0^t\int_0^\tau\left(\|u(s)\|^2_{2,\Omega} + \chi\left\|\nabla u(s)\right\|^2_{2,\Omega}\right)dsd\tau.
\end{aligned} \tag{46}$$

$$\begin{aligned}
&\int_\Omega |u|^2dx + \chi\int_\Omega |\nabla u|^2dx \le \\
&\le C_{25}\int_0^t (t-\tau+1)\left(\|u\|^2_{2,\Omega} + \chi\left\|\nabla u\right\|^2_{2,\Omega}\right)d\tau.
\end{aligned} \tag{47}$$

By virtue of Gronwall's lemma from the inequality (47), we get $\int_\Omega |u|^2dx + \chi\int_\Omega |\nabla u|^2dx = 0$ almost everywhere on the time interval $(0,T)$, which shows the uniqueness of the weak solution.From Lemma 1 we can establish the solvability of the inverse problem (1)-(4). Let $u(x,t)$ be a solution of the initial-boundary value problem (8)-(9) from the space (Theorem 1) $u \in L_\infty(0,T;\overset{0}{W}{}_2^1(\Omega))$,$\nabla u \in L_2(Q_T)$, $Q_T = \Omega\times(0,T)$, $u_t \in L_2(0,T;\overset{0}{W}{}_2^1(\Omega))$,$|u|^{\beta-2}u \in L_\infty(0,T;L_{\frac{\beta}{\beta-1}}(\Gamma))$. Obviously, the function $f(t)$ from the relation (10) belongs to the space $L_\infty(0,T)$. What was proved above means that the found functions $u(x,t)$ and $f(t)$ give a weak solution of the inverse problem.

4 Global solvability of the problem (8)-(9).

Consider the case when $1<\beta\le 2$. Let the conditions

$$\begin{aligned}
&h_1(t) \equiv \int_\Omega h(x,t)\omega(x)dx \ne 0,\ \ \forall t\in[0,T], \\
&h(x,t) \in L_\infty(0,T;\ L_2(\Omega))\cap L_2(Q_T),\ \ 1<\beta\le 2, \\
&\omega \in L_2(\Omega)\cap L_\beta(\Omega)\cap \overset{0}{W}{}_2^2(\Omega),\ \ 1<\beta\le 2, \\
&\int_\Omega u_0(x)\omega(x)dx = \varphi(0),\ \ \varphi(t)\in W_2^1[0,T],\ \ |\varphi'(t)|\le C,\ \ u_0\in \overset{0}{W}{}_2^1(\Omega)\cap L_\beta(\Omega).
\end{aligned} \tag{48}$$

are fulfilled.

Lemma 3 *If $u\in \overset{0}{W}{}_2^1(\Omega)$, $1<\beta\le 2$, then the next inequality is performed*

$$\int_\Omega |u_m|^\beta dx \le \left(\int_\Omega |u|^2dx\right)^{\frac{\beta}{2}}|\Omega|^{\frac{2-\beta}{2}} \le C_1\left(1+\int_\Omega |u|^2dx + \chi\int_\Omega |\nabla u|^2dx\right).$$

For the case when $1<\beta\le 2$, we estimate the right hand side of (20), applying Lemma 3, as well as the Cauchy and Young inequalities, we obtain

$$\left|\int\limits_\Omega b(x,t)|u_m|^\beta dx\right| \le b_1\left\|u_m\right\|_{\beta,\Omega}^\beta \le C_1 b_1\left(1+\left\|u_m\right\|_{2,\Omega}^2+\chi\left\|\nabla u_m\right\|_{2,\Omega}^2\right).$$

$$\left|\frac{1}{h_1}\int\limits_\Omega \varphi'(t)h(x,t)u_m dx\right| \le \frac{|\varphi'|}{|h_1|}\left\|h\right\|_{2,\Omega}\left\|u_m\right\|_{2,\Omega} \le \left\|u_m\right\|_{2,\Omega}^2+\frac{|\varphi'|^2}{4h_1^2}\left\|h\right\|_{2,\Omega}^2.$$

$$\begin{aligned}
&\left|\tfrac{a}{h_1(t)}\int\limits_\Omega h u_m dx\int\limits_0^t g(t-\tau)\int\limits_\Omega \nabla u_m(\tau)\nabla\omega dx d\tau\right| \le \\
&\le \tfrac{a}{|h_1(t)|}\left\|h(x,t)\right\|_{2,\Omega}\left\|u_m\right\|_{2,\Omega}\int\limits_0^t g(t-\tau)\left\|\nabla u_m(\tau)\right\|_{2,\Omega}\left\|\nabla\omega\right\|_{2,\Omega}d\tau \le \\
&\le \tfrac{a}{|h_1(t)|}\left\|h(x,t)\right\|_{2,\Omega}\left\|\nabla\omega\right\|_{2,\Omega}\left\|u_m\right\|_{2,\Omega}\int\limits_0^t g(t-\tau)\left\|\nabla u_m(\tau)\right\|_{2,\Omega}d\tau \le \\
&\le \tfrac{a^2}{4}\left(\left\|\nabla\omega\right\|_{2,\Omega}\sup_{0\le t\le T}\tfrac{1}{|h_1(t)|}\left\|h(x,t)\right\|_{2,\Omega}\right)^2\int\limits_0^t g^2(t-\tau)d\tau\int\limits_0^t\left\|\nabla u_m(\tau)\right\|_{2,\Omega}^2 d\tau+\left\|u_m\right\|_{2,\Omega}^2 \le \\
&\le \tfrac{a^2}{4}\left(\left\|\nabla\omega\right\|_{2,\Omega}\sup_{0\le t\le T}\tfrac{1}{|h_1(t)|}\left\|h(x,t)\right\|_{2,\Omega}\right)^2\int\limits_0^t\left\|\nabla u_m(\tau)\right\|_{2,\Omega}^2 d\tau+\left\|u_m\right\|_{2,\Omega}^2.
\end{aligned}\tag{49}$$

$$\begin{aligned}
&\left|\int\limits_0^t g(t-\tau)\int\limits_\Omega \nabla u_m(\tau)\nabla u_m(t)dxd\tau\right| \le \int\limits_0^t g(t-\tau)\left\|\nabla u_m(\tau)\right\|_{2,\Omega}\left\|\nabla u_m(t)\right\|_{2,\Omega}d\tau \le \\
&\le \left\|\nabla u_m(t)\right\|_{2,\Omega}\int\limits_0^t g(t-\tau)\left\|\nabla u_m(\tau)\right\|_{2,\Omega}d\tau \le \\
&\le \tfrac{1}{2a}\int\limits_0^t g^2(t-\tau)d\tau\int\limits_0^t\left\|\nabla u_m(\tau)\right\|_{2,\Omega}^2 d\tau+\tfrac{a}{2}\left\|\nabla u_m(t)\right\|_{2,\Omega}^2 \le \\
&\le \tfrac{g_0}{a}\int\limits_0^t\left\|\nabla u_m(\tau)\right\|_{2,\Omega}^2 d\tau+\tfrac{a}{4}\left\|\nabla u_m(t)\right\|_{2,\Omega}^2.
\end{aligned}$$

$$\begin{aligned}
&\left|\tfrac{1}{h_1}a\int\limits_\Omega \nabla u_m\nabla\omega dx\int\limits_\Omega h(x,t)u_m dx\right| \le \tfrac{1}{|h_1|}\left\|\nabla u_m\right\|_{2,\Omega}\left\|\nabla\omega\right\|_{2,\Omega}\left\|u_m\right\|_{2,\Omega}\left\|h\right\|_{2,\Omega} \le \\
&\le \tfrac{a}{h_1^2}\left\|\nabla\omega\right\|_{2,\Omega}^2\left\|h\right\|_{2,\Omega}^2\left\|u_m\right\|_{2,\Omega}^2+\tfrac{a}{4}\left\|\nabla u_m\right\|_{2,\Omega}^2.
\end{aligned}$$

$$\begin{aligned}
&\left|\tfrac{1}{h_1}\int\limits_\Omega b(x,t)|u_m|^{\beta-2}u_m\omega dx\int\limits_\Omega h(x,t)u_m dx\right| \le \tfrac{b_1}{|h_1|}\left\|u_m\right\|_{\beta,\Omega}^\beta\left\|\omega\right\|_{\beta,\Omega}\left\|h\right\|_{\frac{\beta}{\beta-1},\Omega} \le \\
&\le C_1\tfrac{b_1}{|h_1|}\left\|\omega\right\|_{\beta,\Omega}\left\|h\right\|_{\frac{\beta}{\beta-1},\Omega}\left(1+\left\|u\right\|_{2,\Omega}^2+\chi\left\|\nabla u\right\|_{2,\Omega}^2\right).
\end{aligned}$$

Then from the obtained estimates follows that

$$\begin{aligned}
&\tfrac{1}{2}\tfrac{d}{dt}\int\limits_\Omega[|u_m|^2+\chi|\nabla u_m|^2]dx+a\int\limits_\Omega|\nabla u_m|^2dx \le \\
&\le C_1 b_1\left(1+\left\|u_m\right\|_{2,\Omega}^2+\chi\left\|\nabla u_m\right\|_{2,\Omega}^2\right)+\left\|u_m\right\|_{2,\Omega}^2+\tfrac{|\varphi'|^2}{4h_1^2}\left\|h\right\|_{2,\Omega}^2+ \\
&\tfrac{a^2}{4}\left(\left\|\nabla\omega\right\|_{2,\Omega}\sup_{0\le t\le T}\tfrac{1}{|h_1(t)|}\left\|h(x,t)\right\|_{2,\Omega}\right)^2\int\limits_0^t\left\|\nabla u_m(\tau)\right\|_{2,\Omega}^2 d\tau+\left\|u_m\right\|_{2,\Omega}^2+ \\
&+\tfrac{g_0}{a}\int\limits_0^t\left\|\nabla u_m(\tau)\right\|_{2,\Omega}^2 d\tau+\tfrac{a}{4}\left\|\nabla u_m(t)\right\|_{2,\Omega}^2+ \\
&+\tfrac{a}{h_1^2}\left\|\nabla\omega\right\|_{2,\Omega}^2\left\|h\right\|_{2,\Omega}^2\left\|u_m\right\|_{2,\Omega}^2+\tfrac{a}{4}\left\|\nabla u_m\right\|_{2,\Omega}^2+ \\
&+C_1\tfrac{b_1}{|h_1|}\left\|\omega\right\|_{\beta,\Omega}\left\|h\right\|_{\frac{\beta}{\beta-1},\Omega}\left(1+\left\|u\right\|_{2,\Omega}^2+\chi\left\|\nabla u\right\|_{2,\Omega}^2\right).
\end{aligned}$$

$$\begin{aligned}
&\tfrac{d}{dt}\int\limits_{\Omega}[1+|u_m|^2+\chi|\nabla u_m|^2]dx+a\int\limits_{\Omega}|\nabla u_m|^2dx\leq\\
&\leq C_2\left(1+\|u_m\|_{2,\Omega}^2+\chi\|\nabla u_m\|_{2,\Omega}^2\right)+\\
&+C_3\int\limits_0^t\left(1+\|u_m\|_{2,\Omega}^2+\chi\|\nabla u_m\|_{2,\Omega}^2\right)d\tau+\tfrac{|\varphi'|^2}{4h_1^2}\|h(x,t)\|_{2,\Omega}^2.
\end{aligned}\tag{50}$$

We denote by $y(t)\equiv 1+\|u_m\|_{2,\Omega}^2+\chi\|\nabla u_m\|_{2,\Omega}^2$, then (50) takes the form

$$\frac{dy(t)}{dt}\leq C_2y(t)+C_2\int\limits_0^t y(\tau)d\tau+\frac{|\varphi'|^2}{4h_1^2}\|h(x,t)\|_{2,\Omega}^2.\tag{51}$$

Using the Gronwall's inequality, we obtain the required estimate

$$\|u_m\|_{2,\Omega}^2+\chi\|\nabla u_m\|_{2,\Omega}^2+\int\limits_0^t\int\limits_{\Omega}|\nabla u_m|^2dxdt\leq C_5.$$

The derivatives $\partial_t u_m$ and $\partial_t\nabla u_m$ are estimated in a similar way, as well as (37). We multiply the equality (16) by $C'_{mj}(t)$ and summarize over $j=\overline{1,m}$. As a result, we get

$$\begin{aligned}
&\|\partial_\tau u_m\|_{2,\Omega}^2+\chi\|\nabla\partial_\tau u_m\|_{2,\Omega}^2+\tfrac{a}{2}\tfrac{d}{dt}\int\limits_{\Omega}|\nabla u_m|^2dx=\tfrac{1}{\beta}\int\limits_{\Omega}b(x,t)|u_m|^{\beta-2}u_m\partial_t u_mdx+\\
&+\int\limits_{\Omega}F(t,u_m)h(x,t)\partial_t u_mdx.
\end{aligned}\tag{52}$$

We integrate with respect to τ from 0 to t, then get the relation

$$\begin{aligned}
&\int\limits_0^t\left(\|\partial_\tau u_m\|_{2,\Omega}^2+\chi\|\nabla\partial_\tau u_m\|_{2,\Omega}^2\right)d\tau+\tfrac{a}{2}\int\limits_{\Omega}|\nabla u_m|^2dx=\int\limits_0^t\int\limits_{\Omega}b(x,t)|u_m|^{\beta-2}u_m\partial_\tau u_mdxd\tau+\\
&+\int\limits_0^t\int\limits_{\Omega}F(t,u_m)h(x,t)\partial_\tau u_mdxd\tau.
\end{aligned}\tag{53}$$

We estimate the right hand side of (53),

$$\begin{aligned}
&\left|\int\limits_{\Omega}b(x,t)|u_m|^{\beta-2}u_m\partial_t u_mdx\right|\leq b_1\int\limits_{\Omega}|u_m|^{\beta-1}|\partial_t u_m|dx\leq\\
&\leq b_1\left(\int\limits_{\Omega}|u_m|^{\beta}dx\right)^{\frac{\beta-1}{\beta}}\left(\int\limits_{\Omega}dx\right)^{\frac{2-\beta}{2\beta}}\left(\int\limits_{\Omega}|\partial_t u_m|^2dx\right)^{\frac12}\leq\\
&\leq b_1\left(\int\limits_{\Omega}|u|^2dx\right)^{\beta-1}|\Omega|^{\frac{(2-\beta)(1+\beta)}{2\beta}}\left(\int\limits_{\Omega}|\partial_t u_m|^2dx\right)^{\frac12}\leq\\
&\leq\tfrac52 b_1^2|\Omega|^{\frac{(2-\beta)(1+\beta)}{\beta}}C_5^{2\beta-2}+\tfrac{1}{10}\|\partial_t u_m\|_{2,\Omega}^2,
\end{aligned}$$

$$\begin{aligned}
&\left|\tfrac{1}{h_1}\int\limits_{\Omega}\varphi'(t)h(x,t)\partial_t u_mdx\right|\leq\\
&\leq\tfrac{|\varphi'|}{|h_1|}\|h\|_{2,\Omega}\|\partial_t u_m\|_{2,\Omega}\leq\tfrac{1}{10}\|\partial_t u_m\|_{2,\Omega}^2+\tfrac{5|\varphi'|^2}{2h_1^2}\|h\|_{2,\Omega}^2.
\end{aligned}$$

$$\begin{aligned}
&\left|\tfrac{1}{h_1}a\int\limits_{\Omega}\nabla u_m\nabla\omega dx\int\limits_{\Omega}h(x,t)\partial_t u_mdx\right|\leq\\
&\leq\tfrac{1}{|h_1|}\|\nabla u_m\|_{2,\Omega}\|\nabla\omega\|_{2,\Omega}\|\partial_t u_m\|_{2,\Omega}\|h\|_{2,\Omega}\leq\\
&\leq\tfrac{1}{12}\|\partial_t u_m\|_{2,\Omega}^2+\tfrac{3\|\nabla\omega\|_{2,\Omega}^2\|h\|_{2,\Omega}^2}{h_1^2}\|\nabla u_m\|_{2,\Omega}^2\leq\\
&\leq\tfrac{1}{10}\|\partial_t u_m\|_{2,\Omega}^2+\tfrac{5\|\nabla\omega\|_{2,\Omega}^2\|h\|_{2,\Omega}^2}{2h_1^2}C_5.
\end{aligned}$$

$$\left|\tfrac{1}{h_1}\int\limits_\Omega b(x,t)|u_m|^{\beta-2}u_m\omega dx\int\limits_\Omega h(x,t)\partial_t u_m dx\right|\le$$
$$\le \tfrac{b_1}{|h_1|}\|u_m\|_{\beta,\Omega}^{\beta-1}\|\omega\|_{\beta,\Omega}\|\partial_t u_m\|_{2,\Omega}\|h\|_{2,\Omega}\le$$
$$\le C_1^{\frac{\beta-1}{\beta}}\tfrac{b_1}{|h_1|}\|\omega\|_{\beta,\Omega}\|h\|_{2,\Omega}\left(1+\|u\|_{2,\Omega}^2+\chi\|\nabla u\|_{2,\Omega}^2\right)^{\frac{\beta-1}{2}}\|\partial_t u_m\|_{2,\Omega}\le$$
$$\le \tfrac{1}{10}\|\partial_t u_m\|_{2,\Omega}^2+\tfrac{5}{2}C_1^{\frac{2\beta-2}{\beta}}C_5^{\beta-1}\tfrac{b_1^2}{h_1^2}\|\omega\|_{\beta,\Omega}^2\|h\|_{2,\Omega}^2.$$

$$\left|\int\limits_0^t\int\limits_0^\tau g(\tau-s)\int_\Gamma \nabla u_m(s)\nabla\partial_\tau u_m(\tau)d\Gamma ds d\tau\right|\le\int\limits_0^t\int\limits_0^\tau g(\tau-s)\|\nabla u_m(s)\|_{2,\Omega}\|\nabla\partial_\tau u_m(\tau)\|_{2,\Omega}d\tau\le$$
$$\le\int\limits_0^t\|\nabla\partial_\tau u_m(\tau)\|_{2,\Omega}\int\limits_0^\tau g(\tau-s)\|\nabla u_m(s)\|_{2,\Omega}d\tau\le$$
$$\le\tfrac12\int\limits_0^t\int\limits_0^\tau g^2(\tau-s)ds\int\limits_0^t\|\nabla u_m(s)\|_{2,\Omega}^2dsd\tau+\tfrac12\int\limits_0^t\|\nabla\partial_\tau u_m(\tau)\|_{2,\Omega}^2d\tau\le$$
$$\le\tfrac{g_0}{2}\int\limits_0^t\int\limits_0^\tau\|\nabla u_m(s)\|_{2,\Omega}^2dsd\tau+\tfrac12\int\limits_0^t\|\nabla\partial_\tau u_m(\tau)\|_{2,\Omega}^2d\tau\le$$
$$\le\tfrac{g_0}{2}\int\limits_0^t(t-\tau)\|\nabla u_m(\tau)\|_{2,\Omega}^2d\tau+\tfrac12\int\limits_0^t\|\nabla\partial_\tau u_m(\tau)\|_{2,\Omega}^2d\tau.$$

$$\left|a\int\limits_0^t\int\limits_\Omega\tfrac{h(x,\tau)}{h_1(\tau)}\partial_\tau u_m dx\int\limits_0^\tau g(\tau-s)\int\limits_\Omega\nabla u_m(s)\nabla\omega dxdsd\tau\right|\le$$
$$\le a\|\nabla\omega\|_{2,\Omega}\sup_{0\le t\le T}\tfrac{\|h(x,t)\|_{2,\Omega}}{|h_1(t)|}\int\limits_0^t\|\partial_\tau u_m\|_{2,\Omega}\int\limits_0^\tau g(\tau-s)\|\nabla u_m(s)\|_{2,\Omega}dsd\tau\le$$
$$\le\tfrac{1}{10}\int\limits_0^t\|\partial_\tau u_m\|_{2,\Omega}^2d\tau+\tfrac52\left(a\|\nabla\omega\|_{2,\Omega}\sup_{0\le t\le T}\tfrac{\|h(x,t)\|_{2,\Omega}}{|h_1(t)|}\right)^2\int\limits_0^t\int\limits_0^\tau g^2(\tau-s)ds\int\limits_0^\tau\|\nabla u_m(s)\|_{2,\Omega}^2dsd\tau\le \quad (54)$$
$$\le\tfrac{1}{10}\int\limits_0^t\|\partial_\tau u_m\|_{2,\Omega}^2d\tau+\tfrac52\left(a\|\nabla\omega\|_{2,\Omega}\sup_{0\le t\le T}\tfrac{\|h(x,t)\|_{2,\Omega}}{|h_1(t)|}\right)^2g_0\int\limits_0^t\int\limits_0^\tau\|\nabla u_m(s)\|_{2,\Omega}^2dsd\tau\le$$
$$\le\tfrac{1}{10}\int\limits_0^t\|\partial_\tau u_m\|_{2,\Omega}^2d\tau+\tfrac{5g_0}{2}\left(a\|\nabla\omega\|_{2,\Omega}\sup_{0\le t\le T}\tfrac{\|h(x,t)\|_{2,\Omega}}{|h_1(t)|}\right)^2\int\limits_0^t(t-\tau)\|\nabla u_m(\tau)\|_{2,\Omega}^2d\tau$$

Substituting the obtained inequality into (53), we get the required estimate

$$\int\limits_0^t\left(\|\partial_\tau u_m\|_{2,\Omega}^2+\chi\|\nabla\partial_\tau u_m\|_{2,\Omega}^2\right)d\tau\le C_6. \quad (55)$$

Theorem 3 *Let the conditions* (48) *be satisfied, and also* $0<b_0\le b(x,t)\le b_1<\infty,\ 1<\beta\le 2,\ N\ge 3.$ *Then there exists a weak solution* $u(x,t)$ *of the problem* (8)-(9) *on the interval* $(0,T)$.

The uniqueness is proved in an analogical way, and for the case when $1<\beta\le 2$. For this case, we formulate the assertion in the form of the theorem.

Theorem 4 *Let the conditions* (48) *be satisfied,* $1<\beta\le 2,\ \ N\ge 3$. *Then the weak solution of the problem* (8)-(9) *on the interval* $(0,T)$ *is unique (in the sense of Definition 1).*

We assume that the next conditions are valid for the case when $b(x,t)=-q(x,t)$:

$$\begin{aligned}
&h_1(t)\equiv\int\limits_\Omega h(x,t)\omega(x)dx\ne 0,\ \ \forall t\in[0,T],\\
&h(x,t)\in L_\infty(0,T;\ L_2(\Omega))\cap L_2(Q_T)\cap L_{\frac{\beta}{\beta-1}}(Q_T),\ \ \beta\ge 2,\\
&\omega\in L_2(\Omega)\cap L_\beta(\Omega)\cap \overset{0}{W}{}_2^2(\Omega),\ \ \beta\ge 2,\\
&\int\limits_\Omega u_0(x)\omega(x)dx=\varphi(0),\ \ \varphi(t)\in W_2^1[0,T],\ \ |\varphi'(t)|\le C,\ \ u_0\in \overset{0}{W}{}_2^1(\Omega)\cap L_\beta(\Omega).
\end{aligned} \quad (56)$$

$$
\begin{array}{l}
0 < q_0 \le q(x,t) \le q_1, \\
2 < \beta < \frac{2N}{N-2},\ N \ge 3, \\
q_0 - q_1 \|\omega\|_{\beta,\Omega} \sup\limits_{0\le t\le T} \frac{1}{|h_1(t)|} \|h(x,t)\|_{\frac{\beta}{\beta-1},\Omega} > 0.
\end{array} \tag{57}
$$

Theorem 5 *Let the conditions* (56) *and* (57) *be satisfied, then there exists a weak solution* $u(x,t)$ *of the problem* (8)-(9) *(in the sense of Definition 1) on the interval* $(0,T)$.

4.1 Existence and uniqueness of a strong solution

Theorem 6 *Let the conditions* (5)-(7) *are performed, and also* $2 < \beta < \frac{2N}{N-2},\ N \ge 3$*, additional conditions* $u_0(x) \in \overset{0}{W}{}_2^2(\Omega)$. *Then there is a strong solution* $u(x,t)$ *(in the sense of Definition 2) of the problem* (8)-(9) *on the interval* $(0,T),\ T < T_0$.

Theorem 7 *Let* $u_0(x) \in \overset{0}{W}{}_2^2(\Omega)$, $2 < \beta \le \frac{2(N-1)}{N-2},\ N \ge 3$ *are fulfilled .Then the strong solution (in the sense of Definition 2) of the problem* (8)-(9) *on the interval* $(0,T)$ *is unique.*

Theorem 8 *Let the conditions*

1) (48) *be satisfied, and also* $0 < b_0 \le b(x,t) \le b_1 < \infty,\ 1 < \beta \le 2,\ N \ge 3$;

2) (56) *and* (57).

Then there exists and unique a strong solution $u(x,t)$ *of the problem* (8)-(9) *on the interval* $(0,T)$.

5 Conclusion

In this article we investigated the solvability of the inverse problem for a quasilinear pseudoparabolic equation with memory. As a result, we proved the existence and uniqueness of a weak and strong solutions of the inverse problem in a bounded domain, and obtained local and global theorems on the existence of the solution. The field of application of the obtained results are inverse problems for pseudoparabolic equations and theory of boundary value problems. These results allow to proceed to the new inverse problems, generalize and systemize research in the given area.

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