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AN INVERSE PROBLEM OF RECOVERING THE RIGHT HAND SIDE OF 1D PSEUDOPARABOLIC EQUATION

Inverse problems of finding the right-hand side of a partial differential equations arise when an external source is unknown or impossible to measurement, for example, the source is in a high-temperature environment or underground. The partial differential equations with mixed derivatives by time and space variables are usually called pseudo-parabolic equations or Sobolev type equations. Pseudo-parabolic equations occur in mathematical modeling of many physical phenomena such as the motion of non-Newtonian fluids, thermodynamic processes, filtration in a porous medium, unsteady flow of second-order fluids, etc. This paper is devoted to investigate the unique solvability of two inverse problems for a linear one dimensional pseudoparabolic equation. The inverse problems consist of recovering the right hand side of the equation depending on the space variable. An additional information for the first inverse problem is given by an final overdetermination condition and for the second is given by an integral overdetermination condition. Under the suitable conditions on the initial data of the problem, the existence and uniqueness of a classical solution to these inverse problems are established. By using the Fourier method, the explicit formulas of a solutions are presented in the form of a series, which make it possible to perform the necessary numerical calculations with a given accuracy.
Key words: inverse problem, pseudoparabolic equations, overdetermination condition, existence, uniqueness, classical solution.

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Бір өлшемді псевдопараболалық теңдеудің оң жағын құру кері есебі

Дербес туындылы дифференциалдық теңдеулердің оң жағын табу кері есептері әдетте сыртқы жылу көзі белгісіз болғанда немесе өлшеу мүмкін болмаған кезде туындайды, мысалы, жылу көзі өте жоғары температуралы ортада немесе жер астында орналасса. Уақыт және кеңістіктік айнымалылар бойынша аралас туындылары бар дербес туындылы дифференциалдық теңдеулер көбінде псевдопараболалық теңдеулер немесе Соболев типіндегі теңдеулер деп аталады. Псевдо-параболалық теңдеулер көптеген физикалық құбылыстарды математикалық модельдеу кезінде кездеседі, мысалға, ньютондық емес сұйықтықтардың қозғалысында, термодинамикалық процесстерде, кеуекті ортадағы сүзгілеуде, екінші ретті сұйықтықтардың тұрақсыз ағыны және т.б. Бұл мақала сызықты бір өлшемді псевдопараболалық теңдеу үшін қойылған екі кері есептің бірмәнді шешімділігін зерттеуге арналған. Кері есептер болуы кеңістіктік айнымалыдан ғана тәуелді теңдеудің оң жағын қалпына келтіруімен сипатталады. Бірінші кері есеп үшін қосымша ақпарат финальдық қосымша шартпен берілсе, екінші есеп үшін - интегралдық қосымша шартпен беріледі. Берілген есептің берілгендері қандай да бір шарттарды қанағаттандырған кезде, осы кері есептердің классикалық шешімінің бар болуы мен жалғыздығы көрсетіледі. Фурье әдісі көмегімен сандық есептеулерді берілген дәлдікпен жүргізуге мүмкіндіктер беретін ізделінді шешімнің қатар түріндегі айқын формулалары алынды.

Түйін сөздер: Кері есеп, псевдопараболалық теңдеу, қосымша шарт, шешімнің бар және жалғыз болуы, классикалық шешім.

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Обратная задача восстановления правой части одномерного псевдопараболического уравнения

Обратные задачи нахождения правой части дифференциального уравнения в частных производных возникают когда внешний источник неизвестен или невозможен для измерения, например, источник находится в высокотемпературной среде или под землей. Уравнения в частных производных с смешанным производными по времени и по пространственным переменным обычно называются псевдо-параболическим уравнениям или уравнениям типа Соболева. Псевдо-параболические уравнения встречаются при математическом моделировании многих физических явлений как движения неньютоновских жидкостей, термодинамические процессы, фильтрация в пористой среде, нестационарный поток жидкостей второго порядка и т. д. Настоящая работа посвящена исследованию однозначной разрешимости двух обратных задач для линейного одномерного псевдопараболического уравнения. Обратные задачи состоят из восстановления правой части, зависящего от пространственной переменной. Дополнительная информация для первой обратной задачи задается финальным условием переопределения, а для второй задачи - интегральным условием переопределения. При подходящих условиях на данные первоначальной задачи, устанавливаются существования и единственности классического решения этих обратных задач. С помощью методом Фурье, представлены в виде ряда явные формулы искомых функции, которые позволяют производить необходимые численные расчеты с заданной точностью.

Ключевые слова: Обратная задача, псевдопараболические уравнения, условие переопределение, существования и единственность решения, классическое решение.

1 Introduction

The partial differential equations like (1) with a mixed derivative by time and space variables are called pseudo-parabolic or Sobolev type equations [1], [2], and arise in many areas of mathematics and physics. For instance, they have been used, to model thermodynamics processes [3], filtration in porous media [4], and nonsteady flow of second order fluids [5], the motion of non-Newtonian fluids [2], [6], and many other physical phenomena.

The present paper is devoted to study two inverse problems of determining the right hand side of an one dimensional pseudoparabolic equation with an additional information, which given by a final overdetermination condition for first, and an integral overdetermination condition for second. The inverse problems of recovering the right-hand side of a differential equation arise in the mathematical modeling of many physical phenomena, when an external source is unknown or unacceptable for measurement, for example, the source is in a high-temperature environment or underground, etc. [7–10] To our knowledge, the inverse problems for pseudoparabolic equations have not been studied a lot. The inverse problem for pseudoparabolic equation (1) with periodic boundary condition have been studied by Khompysh and Shakir in [11] by same method as we have used here and in [12]. The inverse problems of finding the right hand side for pseudoparabolic equation with fractional derivatives by time have been studied by Ruzhanksy and et. in [13]. We refer the readers to see [14], [15], [16], [17], [18], [19], [20, 21] for other some inverse problems for classical pseudoparabolic equations and [22], [23], [24,25], [26] for modified pseudoparabolic equations with p-Laplacian and other related equations.

2 Material and methods

2.1 The formulation of the problem

Let $Q_T := \{0 < x < l < \infty, 0 < t < T < \infty\}$ be a rectangle domain. In this present paper, we consider the following inverse problems of determining a pair of functions $(u(x,t), f(x))$, which satisfy the linear 1D pseudoparabolic equation

$$u_t - u_{xxt} - u_{xx} = f(x), \text{ in } \ Q_T := \{0 < x < l, 0 < t < T\}, \tag{1}$$

the initial condition

$$u(x,0) = \varphi(x), \quad 0 \le x \le l, \tag{2}$$

the Dirichlet boundary condition

$$u(0,t) = u(l,t) = 0, \quad 0 < t < T, \tag{3}$$

and the final overdetermination condition

$$u(x,T) = h(x), \quad 0 \le x \le l \tag{4}$$

or the integral overdetermination condition

$$\int_0^T u(x,t)dt = e(x), \quad 0 \le x \le l. \tag{5}$$

We study the following two inverse problems:
Inverse problem I: Given the initial data $\varphi(x)$ and $h(x)$, to find the pair of functions $(u(x,t), f(x))$, satisfying to (1)-(4).
Inverse problem II: Given the initial data $\varphi(x)$ and $e(x)$, to find the pair of functions $(u(x,t), f(x))$, satisfying to (1)-(3), (5).

Definition 1 *The pair of the functions $(u(x,t), f(x))$ is called a classical solution to the inverse problem I (inverse problem II), if*

$$u(x,t) \in C^{2,1}_{x,t}(Q_T), \quad f(x) \in C(0,l)$$

and satisfies the every relation of the system (1)-(4) *(*(1)-(3), (5) *for inverse problem II) at every point of the corresponding their domain.*

3 Results and discussion

For these possed inverse problems the following theorems are valid.

Theorem 1 *Assume that the conditions*

$$\varphi(x), h(x) \in C^3[0,l] \quad \textit{and}$$

$$\varphi(0) = \varphi(l) = 0, \quad h(0) = h(l) = 0 \quad \varphi''(0) = \varphi''(l) = 0, \; h''(0) = h''(l) = 0$$

hold. Then the inverse problem I has a unique classical solution.

The analogical statement is hold for the inverse problem II.

Theorem 2 *Assume that the conditions*

$$\varphi(x), e(x) \in C^3[0,l] \quad \textit{and}$$

$$\varphi(0)=\varphi(l)=0, \quad e(0)=e(l)=0 \quad \varphi''(0)=\varphi''(l)=0, \;\; e''(0)=e''(l)=0$$

hold. Then the inverse problem II has a unique classical solution.

Let us start to prove the Theorem 1. The proof of the Theorem 2 is analogical as first.
Look for a solution to the inverse problem 1 in the following form

$$u(x,t)=\sum_{k=1}^{\infty} u_k(t)\cdot \sin\frac{\pi k}{l}x, \tag{6}$$

$$f(x)=\sum_{k=1}^{\infty} f_k\cdot \sin\frac{\pi k}{l}x \tag{7}$$

where the system $\left\{\sin\frac{\pi k}{l}x\right\}$ is the eigenfunctions of the Sturm-Liouville problem corresponding to the boundary condition (3)

$$X''(x)-\lambda^2 X(x)=0, \quad X(0)=X(l)=0,$$

and $u_k(t)$ and f_k are unknowns.

Substituting (6) and (7) into the equation (1) - (2), we obtain the following Cauchy problem for ordinary differential equation respect to $u_k(t)$ with an unknown parameter f_k

$$u_k'(t)+\frac{\lambda_k}{1+\lambda_k}u_k(t)=\frac{1}{1+\lambda_k}f_k, \quad \lambda_k=\left(\frac{\pi k}{l}\right)^2, \quad k=1,2,3... \tag{8}$$

$$u_k(0)=\varphi_k, \quad k=1,2,3... \tag{9}$$

where

$$\varphi_k=\frac{2}{l}\int_0^l \varphi(x)\sin\frac{\pi k}{l}x dx, \quad \text{and} \quad \varphi(x)=\sum_{k=1}^{\infty}\varphi_k\cdot\sin\frac{\pi k}{l}x. \tag{10}$$

Solving the Cauchy problem (8)-(9), we find the solution

$$u_k(t)=e^{-\frac{\lambda_k}{1+\lambda_k}t}\cdot\left(\varphi_k-\frac{f_k}{\lambda_k}\right)+\frac{f_k}{\lambda_k} \tag{11}$$

and plugging it into (6), we obtain the solution $u(x,t)$

$$u(x,t)=\sum_{k=1}^{\infty}\left(e^{-\frac{\lambda_k}{1+\lambda_k}t}\cdot\left(\varphi_k-\frac{f_k}{\lambda_k}\right)+\frac{f_k}{\lambda_k}\right)\sin\frac{\pi k}{l}x \tag{12}$$

where f_k is an unknown number. It will be defined by using the final overdetermination condition (4), i.e. substituting (12) into (4) we have

$$e^{-\frac{\lambda_k}{1+\lambda_k}T}\cdot\left(\varphi_k-\frac{f_k}{\lambda_k}\right)+\frac{f_k}{\lambda_k}=h_k,\quad k=1,2,3,..., \tag{13}$$

where

$$h_k=\frac{2}{l}\int_0^l h(x)\sin\frac{\pi k}{l}xdx,\quad \text{and}\quad h(x)=\sum_{k=1}^{\infty}h_k\cdot\sin\frac{\pi k}{l}x. \tag{14}$$

It follows from (13) that

$$f_k=\frac{\lambda_k\left(h_k-\varphi_k e^{-\frac{\lambda_k}{1+\lambda_k}T}\right)}{1-e^{-\frac{\lambda_k}{1+\lambda_k}T}},\quad k=1,2,3,\ldots. \tag{15}$$

Substituting these defined coefficients $u_k(t)$ and f_k into (6) and (7), we obtain the explicit formulas of the solution (u,f) to the inverse problem 1:

$$u(x,t)=\sum_{k=1}^{\infty}\left(\frac{1-e^{-\frac{\lambda_k}{1+\lambda_k}t}}{1-e^{-\frac{\lambda_k}{1+\lambda_k}T}}h_k+\frac{1-e^{-\frac{\lambda_k}{1+\lambda_k}(T-t)}}{1-e^{-\frac{\lambda_k}{1+\lambda_k}T}}e^{-\frac{\lambda_k}{1+\lambda_k}t}\varphi_k\right)\sin\frac{\pi k}{l}x, \tag{16}$$

$$f(x)=\sum_{k=1}^{\infty}\frac{\lambda_k\left(h_k-\varphi_k e^{-\frac{\lambda_k}{1+\lambda_k}T}\right)}{1-e^{-\frac{\lambda_k}{1+\lambda_k}T}}\sin\frac{\pi k}{l}x. \tag{17}$$

Now, in order to complete the proof of existence of a classical solution, we investigate these formal series (16) and (17) of $u(x,t)$ and $f(x)$, and their derivatives $u_t(x,t)$, $u_x(x,t)$, $u_{xx}(x,t)$, $u_{xxt}(x,t)$ involving in the equation (1) for the uniformly convergence.

Let us consider the series (16). It can be majorated by the following number series

$$|u(x,t)|=\left|\sum_{k=1}^{\infty}\left(\frac{1-e^{-\frac{\lambda_k}{1+\lambda_k}t}}{1-e^{-\frac{\lambda_k}{1+\lambda_k}T}}h_k+\frac{1-e^{-\frac{\lambda_k}{1+\lambda_k}(T-t)}}{1-e^{-\frac{\lambda_k}{1+\lambda_k}T}}e^{-\frac{\lambda_k}{1+\lambda_k}t}\varphi_k\right)\sin\frac{\pi k}{l}x\right|\le$$

$$\sum_{k=1}^{\infty}\left(\frac{e^{\frac{\lambda_k}{1+\lambda_k}T}}{e^{\frac{\lambda_k}{1+\lambda_k}T}-1}\left(|h_k|+|\varphi_k|\right)\right)\le e^T\sum_{k=1}^{\infty}\left(|h_k|+|\varphi_k|\right). \tag{18}$$

This series converges if the conditions

$$\varphi(x),h(x)\in C^1[0,l]\quad\text{and}\quad\varphi(0)=\varphi(l)=0,h(0)=h(l)=0 \tag{19}$$

hold. In fact, integrating by parts the integrals in (10) and (14), we replace the Fourier coefficients φ_k and h_k of the functions $\varphi(x)$ and $h(x)$, by the Fourier coefficients of their derivatives $\varphi'(x)$ and $h'(x)$, respectively

$$\varphi_k=\frac{l}{\pi k}\varphi'_k,\quad h_k=\frac{l}{\pi k}h'_k \tag{20}$$

where

$$\varphi'_k = \frac{2}{l} \cdot \int_0^l \varphi'(x) \cdot \cos\frac{\pi k}{l} x dx, \quad h'_k = \frac{2}{l} \cdot \int_0^l h'(x) \cdot \cos\frac{\pi k}{l} x dx.$$

Plugging (20) into (18) then by using the Cauchy inequality, we have

$$|u(x,t)| \leq \frac{\pi e^T}{l} \sum_{k=1}^{\infty} \frac{1}{k} \left(|h'_k| + |\varphi'_k|\right) \leq C \sum_{k=1}^{\infty} \left(\frac{1}{k^2} + |h'_k|^2 + |\varphi'_k|^2\right)$$

These series converge since the first is a harmonic series, the second and third series due to Bessel inequality

$$\sum_{k=1}^{\infty} |\varphi'_k|^2 \leq \|\varphi'(x)\|_{L^2[0,l]} < \infty, \quad \sum_{k=1}^{\infty} |h'_k|^2 \leq \|h'(x)\|_{L^2[0,l]} < \infty.$$

Let us now consider the series (17). It can be majorated by the number series

$$|f(x)| \leq C \sum_{k=1}^{\infty} k\left(|h_k| + |\varphi_k|\right). \tag{21}$$

In order to show the convergence of the series (21), we assume that

$$\varphi(x), h(x) \in C^2[0,l] \text{ and } \varphi(0) = \varphi(l) = 0, \; h(0) = h(l) = 0 \tag{22}$$

and replace the coefficients φ_k and h_k by the Fourier coefficients of their second order derivatives $\varphi''(x)$ and $h''(x)$, respectively

$$\varphi_k = -\frac{l^2}{\pi^2 k^2}\varphi''_k, \quad h_k = -\frac{l^2}{\pi^2 k^2} h''_k \tag{23}$$

where

$$\varphi''_k = \frac{2}{l} \cdot \int_0^l \varphi''(x) \cdot \cos\frac{\pi k}{l} x dx, \quad h''_k = \frac{2}{l} \cdot \int_0^l h''(x) \cdot \cos\frac{\pi k}{l} x dx.$$

Plugging (23) into (21) then by using the Cauchy and Bessel inequalities, we obtain

$$|f(x)| \leq C \sum_{k=1}^{\infty} k\left(|h_k| + |\varphi_k|\right) \leq C \sum_{k=1}^{\infty} \frac{1}{k}\left(|h''_k| + |\varphi''_k|\right) \leq C \sum_{k=1}^{\infty} \left(\frac{1}{k^2} + |h''_k|^2 + |\varphi''_k|^2\right)$$

The last series are converge, therefore, by the Veiershtrass theorem, the series (16) and (17) are uniformly converge.

Analogical way, we see that the series of derivatives $u_t(x,t)$, $u_x(x,t)$, $u_{xx}(x,t)$, $u_{xxt}(x,t)$ can be majorated by the series

$$|u_t, u_x| \leq C \sum_{k=1}^{\infty} k(|\varphi_k| + |e_k|) \leq C \sum_{k=1}^{\infty} \left(\frac{1}{k^2} + |\varphi''_k| + |e''_k|\right) \tag{24}$$

$$|u_{xx}(x,t)| \leq C \sum_{k=1}^{\infty} k^2(|\varphi_k| + |e_k|) \leq C \sum_{k=1}^{\infty} \left(\frac{1}{k^2} + |\varphi'''_k| + |e'''_k|\right) \tag{25}$$

$$|u_{xxt}(x,t)| \le C\sum_{k=1}^{\infty} k^2(|\varphi_k| + |e_k|) \le C\sum_{k=1}^{\infty}\left(\frac{1}{k^2} + |\varphi_k'''| + |e_k'''|\right) \tag{26}$$

where φ_k''' and e_k''' are the Fourier coefficients of the functions $\varphi'''(x)$ and $e'''(x)$, respectively.

The series in (24) are converge due to the conditions (19), (22) and the series (25) and (26) are converge due to the conditions (19), (22) and

$$\varphi(x), h(x) \in C^3[0,l] \ \text{ and } \ \varphi''(0) = \varphi''(l) = 0,\ h''(0) = h''(l) = 0,$$

The uniqueness of the solution inverse problem I can be proved by standard method as [11], [12], i.e. assuming that the problem (1)-(4) has two different solutions (u_1, f_1) and (u_2, f_2) we obtain for $U = u_1 - u_2$, $F = f_1 - f_2$ the following inverse problem with zero data

$$U_t - U_{xxt} - U_{xx} = F(x), \text{ in } Q_T := \{0 < x < l, 0 < t < T\},$$

$$U(x,0) = 0, \quad 0 \le x \le l,$$

$$U(0,t) = U(l,t) = 0, \quad 0 < t < T,$$

$$U(x,T) = 0, \quad 0 \le x \le l.$$

Next, repeating above arguments, we get that the Fourier coefficients of U and F are zero $U_k = F_k = 0$, therefore, $U = F = 0$.

4 Conclusion

In this paper, the unique solvability of two inverse problems of recovering the right hand side of the a linear one dimensional pseudoparabolic equation is studied. An additional information for the first inverse problem is given by an final overdetermination condition and for the second is given by an integral overdetermination condition. Under the suitable conditions on the initial data of the problem, the existence and uniqueness of a classical solution to these inverse problems are established. By using the Fourier method, the explicit formulas of a solutions are presented in the form of a series, which make it possible to perform the necessary numerical calculations with a given accuracy.

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