IRSTI 27.39.21 DOI: https://doi.org/10.26577/JMMCS.2021.v112.i4.07

Zh.K. Akasheva*, B.K. Assilbekov, K.A. Soltanbekova, A.A. Kudaikulov
Satbayev University, Kazakhstan, Almaty
*e-mail: zhibek_akasheva@mail.ru

NUMERICAL SIMULATION OF CARBONATE ROCKS DISSOLUTION NEAR THE WELLBORE

This paper examines the process of wormholes formation during hydrochloric acid treatment of well bottom-hole zone in carbonate formations. An algorithm for solving the problem of wormhole formation in a porous medium for a two-dimensional case was developed. Two-scale model (pore-scale and Darcy's scale) taking into account convection, diffusion, and chemical reaction were used in this work in order to describe the dissolution of carbonates with hydrochloric acid (hydrochloric acid treatment). The initial distribution of the porosity field was generated as a distribution of random numbers around some mean value. Based on the distribution of the initial porosity field, the initial permeability field of the rock was calculated. The random distribution was used to describe the heterogeneity of the actual rock. The rest of the study parameters were taken from known experiments on the dissolution of carbonate cores. The numerical model was built for solving the system of equations for acid dissolution, and carbonate dissolution modes with hydrochloric acid were obtained depending on the Damkohler number, Thiele modulus on the pore-scale and Darcy's scale as a result of this research. Also, the optimal Damkohler numbers (injection rates) were found. The computer code for the problem of the development/growth of wormholes in a porous medium based on the developed algorithm was built using the C++ programming language.
Key words: acid treatment, carbonate core, dissolution mode, Damkohler number, Thiele modulus.

Ж.К. Акашева*, Б.К. Асилбеков, К.А. Солтанбекова, А.А. Кудайкулов
Сәтбаев университеті, Қазақстан, Алматы қ.
*e-mail: zhibek_akasheva@mail.ru

Ұңғыманың түптік аймағындағы карбонатты тау жыныстарының еру процесін сандық моделдеу

Бұл жұмыста карбонатты жер қабаттарындағы тұз қышқылымен өңдеу кезінде канал тесіктерінің пайда болуы зерттеледі. Екі өлшемді жағдайға арналған кеуекті ортада канал тесіктерінің пайда болу мәселесін шешу алгоритмі жасалды. Бұл жұмыста тұз қышқылымен карбонаттардың еруін сипаттау үшін (тұз қышқылымен өңдеу) конвекция, диффузия және химиялық реакцияны ескеретін екі масштабты модельді қолданылды (кеуек шкаласы мен Дарси шкаласы бойынша). Кеуектілік өрісінің бастапқы таралуы кездейсоқ сандардың орташа мәнінің айналасында таралуы ретінде құрылды. Бастапқы кеуектілік өрісінің таралуы негізінде жыныстың бастапқы өткізгіштік өрісі есептелді. Кездейсоқ бөлу нақты жыныстың біртекті емпестігін сипаттау үшін қолданылды. Зерттеудің қалған параметрлері карбонатты керндердің еруі бойынша белгілі тәжірибелерден алынды. Тұз қышқылымен еру процесін сиппаттайтын теңдеулер жүйесін шешу үшін сандық модель құрылды және осы зерттеудің нәтижесінде карбонаттардың тұз қышқылымен еру режимдері Дамколлер санына, кеуек масштабында және Дарси масштадбында Тиле модуліне байланысты алынды. Дамколердің оңтайлы сандары (айдау жылдамдығы) да табылды. Құрылған алгоритмге негізделген кеуекті ортада канал тесіктерінің құрылуы мен өсуі мәселесінің компьютерлік коды C ++ программалау тілінің көмегімен құрылды.
Түйін сөздер: қышқылмен өңдеу, карбонатты керн, еру режимі, Дамколлер саны, Тиле модулі.

Ж.К. Акашева*, Б.К. Асилбеков, К.А. Солтанбекова, А.А. Кудайкулов
Сатбаев Университет, Казахстан, г.Алматы
*e-mail: zhibek_akasheva@mail.ru

Численное моделирование растворения карбонатных пород вблизи скважины

В данной работе исследуется процесс образования червоточин при соляно-кислотной обработке призабойной зоны скважины в карбонатных пластах. Был разработан алгоритм решения задачи образования червоточин в пористой среде для двумерного случая. Для описания растворения карбонатов соляной кислотой использовалась модель двух масштабов (в масштабе пор и в масштабе Дарси), учитывающая конвекцию, диффузию и химическую реакцию. Начальное распределение поля пористости генерировалось как распределение случайных чисел вокруг некоторого среднего значения. На основании распределения начального поля пористости было рассчитано начальное поле проницаемости породы. Случайное распределение использовалось для описания неоднородности реальной породы. Остальные параметры исследования были взяты из известных экспериментов по растворению карбонатных кернов. Численная модель была построена для решения системы уравнений растворения, и в результате данного исследования были получены режимы растворения карбонатов соляной кислотой в зависимости от числа Дамколера, модуля Тиле в масштабе пор и в масштабе Дарси. Также были найдены оптимальные числа Дамколера (скорости закачки). Компьютерный код для задачи развития/роста червоточин в пористой среде был построен с использованием языка программирования C++.

Ключевые слова: кислотная обработка, карбонатный керн, режим растворения, число Дамколера, модуль Тиле.

1 Introduction

Evidently that over the time, the bottom-hole zone of production and injection wells may be damaged, as a result of the reduction in productivity/injectivity of wells. Well drilling, perforating, cementing, fine particles migration and other industrial operations can cause pore plugging. In this case, an acid treatment is one of the most effective and widely used methods in stimulation techniques for the restore of permeability in carbonate reservoirs. After the acid treatment, the improved formation permeability can increase by several orders of magnitude, for this reason the topic of this study is actual for oil industry.

The literature review on mathematical and numerical modeling of the formation of dissolution channels in a porous medium during the treatment with hydrochloric acid was done in this work. The mathematical and numerical formulation of a two-scale model, dissolution process modeling in a rectangular region and the influence of core size on dissolution were studied as well.

Also, an algorithm for solving the system of equations for the dissolution problem of carbonate cores in a two-scale was developed. Moreover, the computer code based on the created solution algorithm for the 2D case was developed in order to study the modes of carbonate cores dissolution at different injection rates (from 5×10^{-5} cm/s to 5×100 cm/s).

2 Literature review

The experimental work on the study of core dissolution process by acid in laboratory conditions was carried out by many authors [1-8]. These works were devoted to determining the optimal injection rate (or the optimal value of the Damkohler number), developing a

mechanistic model of wormhole growth, studying the effect of rock and acid types on core dissolution modes at different injection rates, and developing a 2D network model to describe the wormholes formation. As a result of these experiments, the change in the pressure drop until the moment of an acid breakthrough at the right end of the core was determined. In the work [6], the Darcy-Brinkman and Stokes equations, together with the reagent concentration equation are used to describe the rock dissolution process, and the influence of the injection rate, temperature and the emulsion use on the dissolution modes is determined. The work [7] is devoted to the development of a model for the wormholes growth in the case of using a self-diverting acid together with an ordinary acid during the cylindrical core testing initially filled with a solution. Authors formally divided the core into three potential zones: the wormholes are formed in the first zone; the second zone is contaminated by the ingress of acid; and the third zone is remaining untouched by acid. And also, the influence of the end effect on the pressure drop change along the core length was studied.

There are several methods for the acidizing process modeling. The work [9] studied the formation of wormholes in carbonate rocks. The concentration equation without a chemical reaction was used to describe the process. The reaction rate was taken into account in the form of a boundary condition on the walls of wormholes, and the acid transfer rate inside the wormhole obeyed the Poiseuille's parabolic law. Research results show that the growth rate of wormholes and the geometry of wormholes during the breakthrough depends on the duration of the contact of the rock with the acid and the acid injection rate.

A semi-empirical model for a quick assessment of the growth rate of wormholes during the acidizing of carbonate rocks is described in [10]. This model includes such two empirical parameters, as an efficiency factor and a parameter that is the inverse of the second power of the optimal averaged fluid velocity in pores (which is experimentally determined). According to this model, the growth rate of wormholes is proportional to the average flow rate to the power of $2/3$ for high values of the acid injection rate. This model does not take into account the diffusion of a chemical reaction. It is possible to translate the results of rock dissolution from laboratory conditions to industrial conditions by using this semi-empirical model.

In the work [11] the strong influence of heterogeneity of a carbonate formation on its filtration characteristics during the acid treatment was proved. A model of acid dissolution of the carbonate matrix was constructed, taking into account possible sedimentation. The mathematical model includes an empirical expression for determining the rate of change in the concentration of deposited nanosized particles and the permeability changes of a porous medium. The model also takes into account the moving boundary of the dissolution channel, on which the particles are deposited. The permeability increases with increasing pore size and pore opening due to the mineral dissolution. In the presented work, macroscopic equations can be used to simulate the acidizing process of wells at the field scale.

The results of work [12] are in a good agreement with the experimental data obtained in the work [4]. Numerical calculations were carried out in a cylindrical region using the Navier-Stokes equations taking into account the resistance of the porous medium instead of the Darcy's equation. The influence of mineral composition of the core on its solubility was investigated by authors. It was found by numerical experiments that in the case of a homogeneous mineral, the acid breakthrough faster than in the case of a sample with inclusions of a less soluble mineral.

In work [13], the influence of medium heterogeneity on the formation of wormholes was

studied using a two-scale continuous model, which was first described in [14-16]. It was determined that rock heterogeneity affects not only the structure of the patterns formed during reactive dissolution, but also the amount of acid required to achieve a given increase in permeability. The acid volume decreases with an increase in the heterogeneity degree, as well as with the decrease of length scale. This is especially noticeable at high acid injection rates.

In work [14], an averaged two-scale model of carbonate rocks dissolution by hydrochloric acid in 1D formulation is described. The model describes the relationship between transport processes and reactions occurring at the pore and Darcy's scale. The coefficients of mass transfer and dispersion are estimated and the local equation and criteria are derived.

In [15], the two-scale model was extended for a 2D region. The paper studied the influence of Thiele modules for pore and Darcy's scales, acid concentration in solution, height-to-length ratio, and acid number on dissolution modes. The results of the two-scale model are compared with the results of the experiment and early work.

A detailed study of the carbonates dissolution process is given in [16]. In this work, a criterion for the qualitative prediction of the wormholes formation was found and it was shown that it is approximately equal to 1. Also, the asymptotic values of the breakthrough volume for low and high injection rates were found by authors. To study the effect of rock heterogeneity on dissolution, the concepts of the magnitude of heterogeneity and the dimensionless scale of heterogeneity were introduced. In addition, this paper shows that the optimal breakthrough volume decreases from the 1D case to the 3D case.

Study of the dissolution modes of carbonates with hydrochloric acid in works [17-19] were carried out in a polar coordinate system. In work [17], a criterion is found for qualitatively predicting the wormholes formation for a radial flow similar to the plane case. And also, the fractal size of the wormholes has been determined. In the work [18], the normal distribution was used to generate the initial porosity distribution. Thus, the influence of the normal distribution of porosity on the dissolution modes was investigated. It has been shown that in the case of a normal distribution of porosity, the acid breakthrough volume will be lower than in the case of an uniform distribution, and it has been proven that such distribution of porosity is close to the real distribution. The influence of the perforation length and the presence of cavities on the dissolution modes were investigated in the work as well.

In the work [19], the two-scale model expands from the laboratory scale to the scale of well bottom-hole zone. The study region consists of such two adjacent areas, as the contaminated area in the well bottom-hole zone and the area behind it, in which the fluid is considered to be weakly compressible. And also, a new criterion for acid breakthrough is introduced for calculating the concentration values at the right end of the region. If this concentration is 10% higher than the initial value, then the breakthrough is considered to occur. And also, the influence of the fluid compressibility, which is contained in the second region, on the growth of wormholes was investigated. It was found that the use of the normal distribution of porosity brings the results closer to the experimental data.

The influence of the region geometry on the rock dissolution was investigated in [20]. Based on the two-scale model, it was determined that if the core height is increased, the breakthrough volume decreases. The dependence of the optimal velocity on the shape function for dominant wormholes was found. The shape function is defined as the ratio of the height of the area of interest to its length.

3 Materials and methods

3.1 Physicochemical formulation of the problem

In acidizing, the acid reaction rate competes with the acid injection rate (inverse of the Damkohler number) and diffusion. Usually the diffusion coefficient is small compared to the injection rate [3]. When wormholes are formed, this mode is effective, since in this case the minimum acid volume leads to the wormholes formation connecting the well with the undisturbed region, and the permeability in the reaction zone increases by several orders of magnitude. The wormholes formation mode is observed at average values of the injection rate. When the injection rate is low or high, surface and uniform dissolution modes are observed, in these cases the injected acid volume will be much larger than in the case of wormholes.

Dissolution modes (surface, wormhole and uniform) are related to the Damkohler number, defined as the ratio of the rate of a chemical reaction to the rate of acid injection. Consequently, at large and small values of the Damkohler number, surface and uniform modes are observed, respectively, and at medium values, the formation mode. As a result of experiments, it was found that the optimal Damkohler number is 0.29 [4].

Important components in determining the optimal mode of hydrochloric acid treatment are the kinetic parameters of the reaction, which can be determined only from experiments. The main process taking place in the rock during acidizing is the dissolution of carbonates. It is known that more than half of the minerals that make up carbonate strata are represented by dolomites ($CaMg(CO_3)_2$) and calcite ($CaCO_3$). The chemical reactions of calcite and dolomite with hydrochloric acid are represented by the equations:

$$CaCO_3 + 2HCl = CaCl_2 + CO_2 + H_2O$$

$$CaMg(CO_3)_2 + 4HCl = CaCl_2 + MgCl_2 + 2H_2O + 2CO_2$$

3.2 Mathematical formulation of the problem

The acidizing process of carbonate formations is considered for single-phase multicomponent isothermal filtration in a heterogeneous formation, provided that the reaction products are completely dissolved in the fluid phase. The simulation takes into account the processes of convection and diffusion, and a chemical reaction. Acid, getting into the pores, corrodes their walls, thereby increasing the pore space. An increase in pore space leads to an increase in permeability. The mathematical model of the dissolution process includes Darcy's law, the equations of continuity, concentration, porosity in dimensionless form (Figure 1):

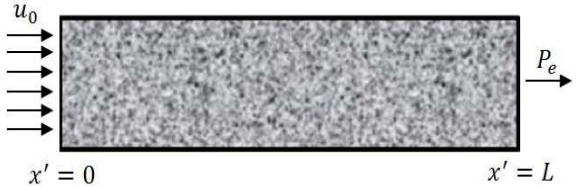

Figure 1: Region of study

$$\nabla \cdot (\boldsymbol{k} \nabla p) = Da_{eff} N_{ac} c_f, \tag{1}$$

$$\vec{u} = -\boldsymbol{k} \nabla p, \tag{2}$$

$$\frac{\partial \pi}{\partial t} + \nabla \cdot (c_f \vec{u}) = \nabla \cdot (\boldsymbol{D_e} \nabla c_f), \tag{3}$$

$$\frac{\partial \varepsilon}{\partial t} = Da_{eff} N_{ac} c_f, \tag{4}$$

$$k_x = k_y = k = \frac{\varepsilon}{\varepsilon_0} \left(\frac{\varepsilon(1-\varepsilon_0)}{\varepsilon_0(1-\varepsilon)} \right)^{2\beta}, \tag{5}$$

$$r = \sqrt{\frac{k\varepsilon_0}{\varepsilon}}, \tag{6}$$

$$A_\nu = \frac{\varepsilon}{\varepsilon_0 r}, \tag{7}$$

$$Sh = Sh_\infty + \frac{0.7}{m^{1/2}} Re_p^{1/2} Sc^{1/3}, \tag{8}$$

$$D_{eX} = \frac{\alpha_{0s} Da \varepsilon}{\Phi^2} + \lambda_X |\vec{u}| r \eta, \tag{9}$$

$$D_{eT} = \frac{\alpha_{0s} Da \varepsilon}{\Phi^2} + \lambda_T |\vec{u}| r \eta, \tag{10}$$

$$Da_{eff} = \frac{Da A_\nu}{\left(1 + \dfrac{\phi^2 r}{Sh}\right)}, \pi = \varepsilon \left(c_f + \frac{1}{N_{ac}} \right), \tag{11}$$

$$-k \frac{\partial p}{\partial x}\bigg|_{x=0} = 1, \tag{12}$$

$$p|_{x=1} = 0, \tag{13}$$

$$\frac{\partial p}{\partial y}\bigg|_{y=0} = \frac{\partial p}{\partial y}\bigg|_{y=\alpha_0} = 0, \tag{14}$$

$$\left(c_f - \varepsilon D_{eX} \frac{\partial c_f}{\partial x} \right)\bigg|_{x=0} = 1, \tag{15}$$

$$\frac{\partial c_f}{\partial x}\bigg|_{x=1} = \frac{\partial c_f}{\partial y}\bigg|_{y=0} = \frac{\partial c_f}{\partial y}\bigg|_{y=\alpha_0} = 0, \tag{16}$$

$$c_f|_{t=0} = 0, \tag{17}$$

$$\varepsilon|_{t=0} = \varepsilon_0 + \hat{f}, \tag{18}$$

where α_0 is the ratio of the height and length of the core, where $\vec{u}$ is the velocity vector, $\boldsymbol{k} = (k_x, k_y)$ is the permeability tensor, p is the pressure, ε is the porosity, t is the time, c_f is the acid concentration in the fluid phase, $\boldsymbol{D_e} = (D_{eX}, D_{eT})$ is the effective dispersion tensor, A_ν is the surface area per unit volume of rock available for reaction, Sh is the Sherwood

number; Sh_∞ is the asymptote of the Sherwood number; m is the ratio of the pore length to its diameter; $Re_p = 2|\vec{u}|r_p/\nu$ is the pore Reynolds number; $|\vec{u}|$ is the velocity module; ν is the kinematic viscosity of the fluid; $S_c = \nu/D_m$ is the Schmidt number; λ_x, λ_T, and a_{0s} are the constants that depend on the type of rock, $\hat{f}$ is a fluctuation that artificially creates porosity heterogeneity of the reservoir, and lies in the interval $[-\Delta\varepsilon_0, \Delta\varepsilon_0]$, where $\Delta\varepsilon_0$ is a given number.

Fugure 1 presents the region of study. For the pressure equation the Neumann condition is set on the left, upper and lower boundaries, the Dirichlet condition is set on the right boundary. For the equation of acid concentration the Dankvert condition is set on the left boundary, and the Neumann condition is set on the other boundaries. We assume that at the initial moment of time, there is no acid in the study region. The porosity is given by the initial distribution. To create the rock heterogeneity, the initial distribution of porosity is set as random with an uniform distribution law around the average value.

Dimensionless parameters that are included in the mathematical model are defined as follows:

$$\eta = \frac{2r_0}{L}; \phi^2 = \frac{2k_s r_0}{D_m}; N_{ac} = \frac{\alpha C_0}{\rho_s}; Da = \frac{k_s \alpha_0 L}{u_0}; Pe = \frac{u_0 L}{D_m}; \Phi^2 = DaPe = \frac{k_s \alpha_0 L^2}{D_m}, \quad (19)$$

where L is the core length; C_0 is the inlet acid concentration; α is the degree of dissolution of the acid; ρ_s is the density of the solid phase; D_m is the effective molecular diffusion coefficient; u_0 is the injection rate; ϕ^2, Φ^2 are the Thiele modulus for the scale of pores and core; N_{ac} is the acid number; Da is the Damkohler number; Pe is the Peclet number.

3.3 Numerical formulation of the problem

To numerically solve the system of equations (1)-(4), together with equations (5)-(11) and the initial-boundary conditions (12)-(18), we use the difference grid, which is shown in Figure 2a. We integrate equations over the control volume, which is shown in Figure 2b, taking into account the boundary conditions.

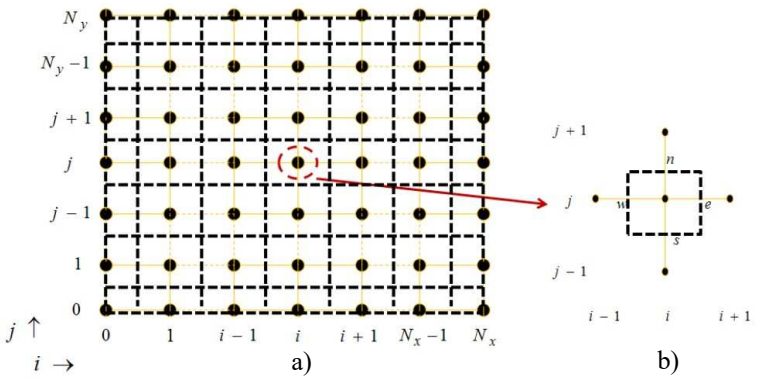

Figure 2: The difference grid: a) (circles are the area where the pressure, concentration and porosity are determined, the dotted lines are the area where the velocity components are determined); b) the control volume

The algorithm for solving the acid treatment problem: from the known initial distribution of concentration and porosity, we iteratively find the distribution of the pressure field, then from the known pressure field we determine the components of the filtration rate according to Darcy's law.

Next, we determine the intermediate (c^*_{fij}, $\varepsilon_{ij}{}^*$) and final (c^{n+1}_{fij}, ε^{n+1}_{ij}) values of concentration and porosity. Then the increase in concentration and porosity is compared with a given small number, if the largest of them does not exceed a given number, then we go to a new time layer. Otherwise, the time step is artificially reduced and all equations are re-solved on the same layer until the above increments are less than a given number.

Thus, this procedure continues until the acid breakthrough at the right end of the core. The criterion for stopping the entire calculation: there is an acid breakthrough at the right end of the core if the average permeability increases 100 times over its original value.

4 Numerical results

This section presents the results of numerical calculations of the development/growth of wormholes in porous media, depending on the parameters of the porous medium, acid type and acid injection rate. The dimensions of carbonate cores are L×H=10cm×4cm and 4cm×10cm, respectively. The other parameters are shown in tables 1 and 2.

4.1 Results for the carbonate core with dimensions L×H=10cm×4cm

Below in Figures 3-6, the results of numerical calculations of acid treatment of a carbonate core with the size L×H=10cm×4cm are shown.

The acid injection rate in all calculations varied from 5×10^{-5} cm/s to 5×100 cm/s. Depending on the above parameters, the optimal injection rates were determined, at which the smallest volume of acid is required to achieve the desired increase in the average core permeability. And also, the modes of rock dissolution were determined.

Таблица 1: The values of the dimensionless parameters used

Parameter	**Value**	**Parameter**	**Value**
Da	40-40000	ϕ^2	0,1; 2; 10
N_{ac}	0,05; 0,1; 0,5	S_c	10^3
Φ^2	10^4; 10^5; 10^6	η	2×10^{-5}
ε_0	0.2	$\Delta\varepsilon_0$	0,01; 0,15

Figures 3 and 4 show the results of the numerical solution at different injection rates, when the Thiele modulus for the pore-scale ϕ^2 takes the value 10^{-2}, 2×10^{-1} and 10^1, and the remaining problem parameters remain unchanged.

The Thiele pore-scale modulus ϕ^2 characterizes the relationship between diffusion and reaction times.

Figure 3 shows the fields of porosity and pressure at the breakthrough moment at different values of ϕ^2. Figure 4 shows acid breakthrough curves depending on the change in injection rate for different values of ϕ^2.

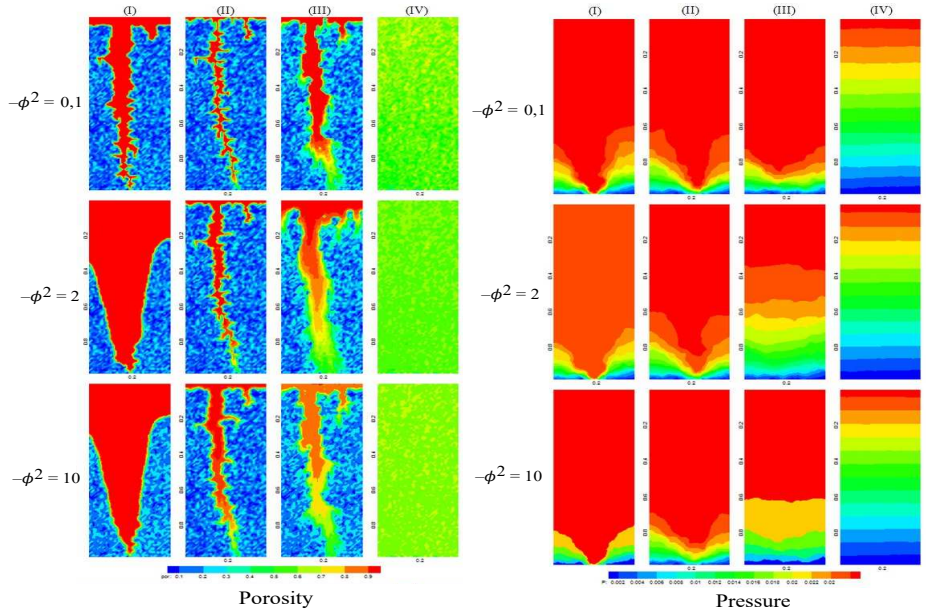

Figure 3: Distribution of porosity and pressure at the breakthrough moment at different values of ϕ^2

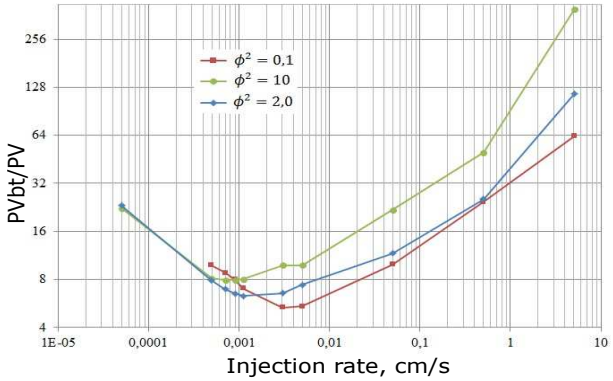

Figure 4: Acid breakthrough curves for different values of ϕ^2

When the reaction at rock surface begins to proceed faster than the diffusion process, a minimum acid volume is required to breakthrough. This process can be seen from Figure 3a. For ϕ^2=0.1 at the same values of the injection rate, the width of the dissolution channels will be less than for ϕ^2=2 and ϕ^2=10.

When changing the Thiele modulus for a pore scale ϕ^2, the pressure field straightens faster (Fig. 3b). In the case of ϕ^2=0.1, the perturbation created by the acid injection reaches the right end faster than for the cases of ϕ^2=2 and ϕ^2=10.

From Figure 4 it can be seen that for different values of ϕ^2, the values of the optimal injection rate lie in different intervals (for ϕ^2=0.1 ϕ^2=2 and ϕ^2=10, the optimal injection rates lie in the intervals 1.1×10^{-3}, -5×10^{-3}, 9×10^{-4}, -3×10^{-3} and 5×10^{-4}, -9×10^{-4}). With a decrease in ϕ^2, the optimal point shifts to the right and is located below.

The results of studying the effect of changing the Thiele modulus for the core scale on the carbonate dissolution process are shown below in Figures 5 and 6.

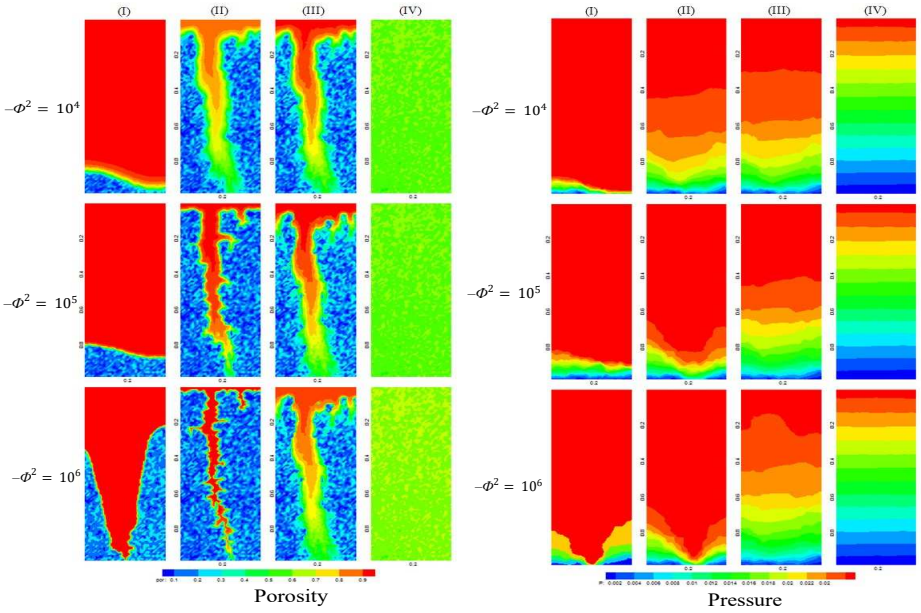

Figure 5: Distribution of porosity and pressure at the breakthrough moment at different values of Φ^2

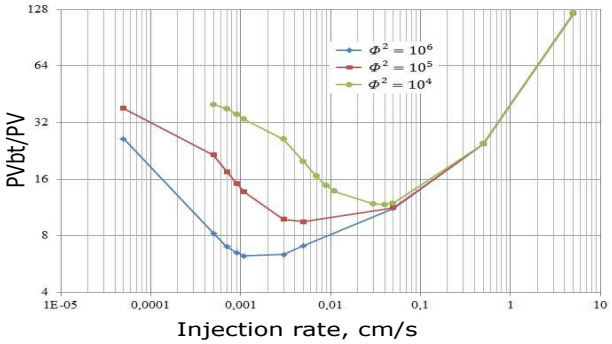

Figure 6: Acid breakthrough curves for different values of Φ^2

Figures 5a and 5b show the distribution of the porosity field and pressure for different values of Φ^2, when the acid breakthrough at the right end of the core. From the distribution of the porosity field (Fig. 5a), it can be seen that with a decrease in Φ^2, the dissolution channels, especially under optimal conditions (i.e., in the case of wormholes), become wider and branched.

Figure 6 shows the distribution of acid breakthrough curves versus injection rate. In this case, the Thiele modulus for the core scale takes the values $\Phi^2 = 10^4$, $\Phi^2 = 10^5$ and $\Phi^2 = 10^6$, respectively.

In work [8], analytical formulas for the asymptote for low and high acid injection rates were found, which are given below. The surface dissolution for low velocity is:

$$PV_F = \frac{1-\varepsilon_0}{N_{ac}\varepsilon_0}, \tag{20}$$

The uniform dissolution for high velocity is:

$$PV_U = \frac{1}{DaN_{ac}\varepsilon_0}\int\limits_{\varepsilon_0}^{\varepsilon_f} 1+\frac{\phi^2 r}{Sh}{A_\nu}d\varepsilon$$

where PV_F and PV_U are the volume of acid breakthrough in surface and uniform modes, respectively, ε_f is the porosity at the breakthrough moment. In accordance with the equation (19), it is possible to approximately calculate the breakthrough volume during surface dissolution. For example, it can be seen from Figure 6 that as the acid injection rate decreases, the breakthrough volume tends to the horizontal asymptote, in this case to $PV_F = 40$. For the uniform dissolution, you can also notice this phenomenon.

4.2 Results for the carbonate core with dimensions L×H=4cm×10cm

This choice can be explained by the fact that, in practice, acidizing is usually carried out in an area whose height (the length of the perforated zone of the well from which the acid is injected) is much greater than its length (the radius of the contaminated zone). Therefore, research in this area is of greatest interest. In this regard, the following are the results of numerical calculations of the dissolution of a carbonate core with dimensions L×H=4cm×10cm with a change in various operating parameters, such as injection rate, N_{ac}, ϕ^2, Φ^2. The parameters used in the calculations are shown in Table 2 below.

Таблица 2: The values of the dimensionless parameters used

Parameter	**Value**	**Parameter**	**Value**
Da	40-40000	ϕ^2	0,1; 2; 10
N_{ac}	0,05; 0,1; 0,5	S_c	10^3
Φ^2	1.6×10^4; 8×10^4; 1.6×10^5	η	2×10^{-5}
ε_0	0.2	$\Delta\varepsilon_0$	0,15

In work [20], the influence of the transition from the scale of the core to the scale of the bottomhole zone of the well on the dissolution process is investigated by changing the geometry of the study region (changing the length and height). As shown by the study results in this work, the density of wormholes increases with an increase in injection rate and with a decrease in the distance calculated from the wall where the acid is injected. The authors of this work, summarizing the results, obtained a correlation between the density of wormholes and the injection rate. This dependency is given below.

$$\rho_{wh}(x) = a(x)\ln(u_0) + b(x)$$

where $\rho_{wh}(x)$ is the number of wormholes per meter, $a(x)$, $b(x)$ are correlation parameters, u_0 is the acid injection rate.

The results of the study of the influence of ϕ^2 on the dissolution of carbonate rock at different injection rates are shown below in Figures 7 and 8.

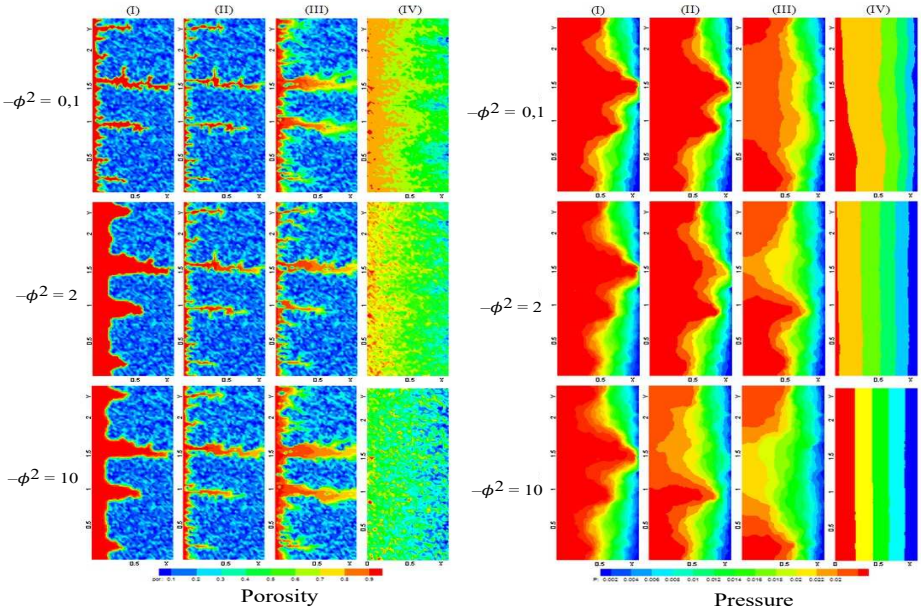

Figure 7: Distribution of porosity and pressure at the breakthrough moment at different values of ϕ^2

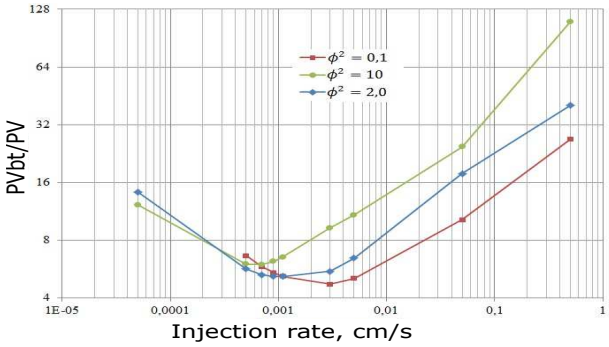

Figure 8: Acid breakthrough curves for different values of ϕ^2

From Figure 7, it is noticeable that with an increase in the height of the core sample from 4 cm to 10 cm, the density of the dissolution channels, i.e. the number of channels per unit height increased several times for the surface mode (I), the mode of formation of thin channels (wormholes) (II) and the mode of branched dissolution (III). Moreover, this tendency can be seen for all considered values of the microscopic Thiele modulus ϕ^2= 0.1; 2; 10. In addition, comparing Figures 8 and 4, we note that the amount of required volume of acid breakthrough became lower in the case of a height of 10 cm (Fig. 8) compared to the case of a height of 4 cm (Fig. 4). This can be explained by the shorter length in the case of the 10 cm height, which contributed to the rapid breakthrough of the acid.

Below in Figures 9 and 10 the dissolution results are shown at different values of Φ^2=1.6×10^4, 8×10^4, 1.6×10^5.

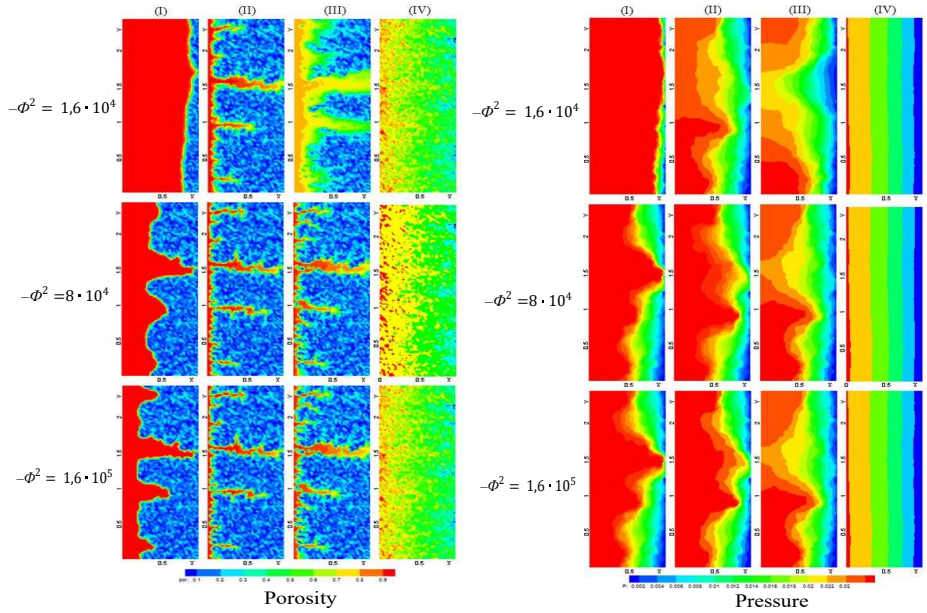

Figure 9: Distribution of porosity and pressure at the moment of breakthrough at different values of Φ^2

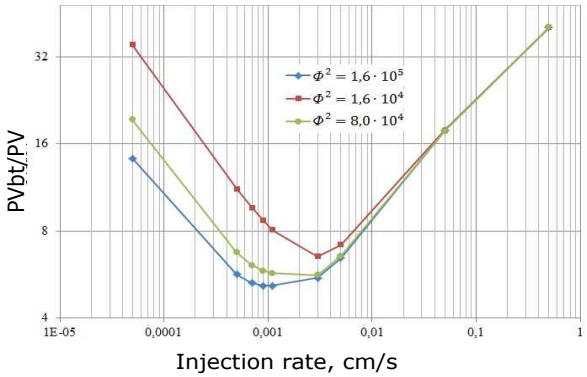

Figure 10: Acid breakthrough curves for different values of Φ^2

Figure 9 shows that with an increase in the macroscopic Thiele modulus, the shape of the dissolution channel changes: if at a low injection rate (I) of acid for Φ^2=1.6×10^4 the dissolution front is almost even, then for Φ^2= 8×10^4 and Φ^2=1.6×10^5 already clearly manifests itself in the formation of several dissolution channels, i.e. dissolution channels become thinner. This is also confirmed by Figure 10, in which the graph for Φ^2=1.6×10^4 is located above the graphs for the remaining values of Φ^2. It can also be seen from Figure 10 that the optimal injection rate, therefore, the optimal breakthrough acid volume shifts to the left with an increase in the macroscopic Thiele modulus Φ^2.

5 Conclusions

The process of wormholes formation during hydrochloric acid treatment of well bottomhole zone in carbonate formations was examined in this paper. Numerical calculations were performed to study the dissolution modes of carbonate cores at different injection rates. The mathematical and numerical formulation of the two-scale model (pore-scale and Darcy's scale) has been implemented and the computer code of the problem was built on the basis of the created solution algorithm for the two-dimensional case using the C++ programming language. The dependence of the acid breakthrough volumes on the injection rate (Damkohler number Da) was obtained for various values of the acid capacity number N_{ac} and Thiele modulus (ϕ^2 and Φ^2). Also, the influence of the core size on the dissolution process was investigated. It has been shown that there are horizontal and oblique asymptotes at low and high injection rates. It was found that in the case when the core height increases, the density of dominant wormholes increases.

6 Acknowledgement

This research is funded by the Science Committee of the Ministry of Education and Science of the Republic of Kazakhstan (Grant No. AP09058419), which is gratefully acknowledged by the authors.

References

[1] Wang Y., Hill A.D., Schechter R.S., "The Optimum Injection Rate for Matrix Acidizing of Carbonate Formations" *SPE,* (1993): 675-687, https://doi.org/10.2118/26578-MS.

[2] Hung K.M., Hill A.D., Sepehrnoori K., "A Mechanistic Model of Wormhole Growth in Carbonate Matrix Acidizing and Acid Fracturing" *J. of Petr. Tech.,* 41 (1989): 59-66, https://doi.org/10.2118/16886-PA.

[3] Fredd C.N., Fogler H.S., "Alternative Stimulation Fluids and Their Impact on Carbonate Acidizing" *SPE,* 3 (1998): 34-41, https://doi.org/10.2118/31074-PA.

[4] Fredd C.N., Fogler H.S., "Influence of Transport and Reaction on Wormhole Formation in Porous Media" *AIChE J.,* 44:9 (1998): 1933-1949, https://doi.org/10.1002/aic.690440902.

[5] Bazin B., Abdulahad G., "Experimental Investigation of Some Properties of Emulsified Acid Systems for Stimulation of Carbonate Formations" *SPE,* (1999): 1-10, https://doi.org/10.2118/53237-MS.

[6] Golfier F., Bazin B., Zarcone C., Lernormand R., Lasseux D., Quintard M., "Acidizing Carbonate Reservoirs: Numerical Modeling of Wormhole Propagation and Comparison to Experiments" *SPE,* (2001): 1-11, https://doi.org/10.2118/68922-MS.

[7] Tardy P., Lecerf B., Christanti Y., "An Experimentally Validated Wormhole Model for Self-Diverting and Conventional Acids in Carbonate Rocks Under Radial Flow Conditions" *SPE,* (2007): 1-17, https://doi.org/10.2118/107854-MS.

[8] Shedid Sh.A., "An Experimental Approach of Matrix Acidizing of Permeability-Damaged Carbonate Reservoirs" *SPE,* (2007): 1-9, https://doi.org/10.2118/106956-MS.

[9] Buijse M.A., "Understanding Wormholing Mechanisms Can Improve Acid Treatments in Carbonate Formations" *SPE Production&Facilities,* 15 (2000): 168-175, https://doi.org/10.2118/65068-PA.

[10] Buijse M., Glasbergen G., "A Semiempirical Model To Calculate Wormhole Growth in Carbonate Acidizing" *SPE,* (2005): 1-14, https://doi.org/10.2118/96892-MS.

[11] Izgec O., Zhu D., Hill A.D., "Numerical and experimental investigation of acid wormholing during acidization of vuggy carbonate rocks" *Elsevier: J. of Petr. Sci.& Eng.,* 74 (2010): 51-66, https://doi.org/10.1016/j.petrol.2010.08.006.

[12] De Oliveira T.J.L., Melo A.R., Oliveira J.A.A., Pereira A.Z.I., "Numerical Simulation of the Acidizing Process and PVBT Extraction Methodology Including Porosity/Permeability and Mineralogy Heterogeneity" *SPE,* (2012): 1-9, https://doi.org/10.2118/151823-MS.

[13] Kalia N., Balakotaiah V., "Effect of medium heterogeneities on reactive dissolution of carbonates" *Elsevier: J. of Chem. Eng. Sci.,* 64 (2009): 376-390, https://doi.org/10.1016/j.ces.2008.10.026.

[14] Panga M.K.R., Balakotaiah V., Ziauddin M., "Modeling, Simulation and Comparison of Models for Wormhole Formation during Matrix Stimulation of Carbonates" *SPE,* (2002): 1-19, https://doi.org/10.2118/77369-MS.

[15] Panga M.K.R., Ziauddin M., Gandikota R., Balakotaiah V., "A New Model for Predicting Wormhole Structure and Formation in Acid Stimulation of Carbonates" *SPE,* (2004): 1-11, https://doi.org/10.2118/86517-MS.

[16] Panga M.K.R., Ziauddin M., Balakotaiah V., "Two-Scale Continuum Model for Simulation of Wormholes in Carbonate Acidization" *AIChE J.* 51:12 (2005): 3231-3248, https://doi.org/10.1002/aic.10574.

[17] Kalia N., Balakotaiah V., "Modeling and analysis of wormhole formation in reactive dissolution of carbonate rocks" *Elsevier: J. of. Chem. Eng. Sci.,* 62 (2007): 919-928, https://doi.org/10.1016/j.ces.2006.10.021.

[18] Liu M., Zhang Sh., Mou J., "Effect of normally distributed porosities on dissolution pattern in carbonate acidizing" *Elsevier: J. of. Petr. Sci.& Eng.,* 94-95 (2012): 28-39, https://doi.org/10.1016/j.petrol.2012.06.021.

[19] Liu M., Zhang Sh., Mou J., "Wormhole Propagation Behavior Under Reservoir Condition in Carbonate Acidizing" *Springer: J. of. Trans. in Porous Media,* 96 (2013): 203-220, https://doi.org/10.1007/s11242-012-0084-z.

[20] Cohen Ch.E., Ding D., Quintard M., Bazin B., "From pore scale to wellbore scale: Impact of geometry on wormhole growth in carbonate acidization" *Elsevier: J. of. Chem. Eng. Sci.,* 63 (2008): 3088-3099, https://doi.org/10.1016/j.ces.2008.03.021.